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M. J. Breining ton
Berwyn, Pa.

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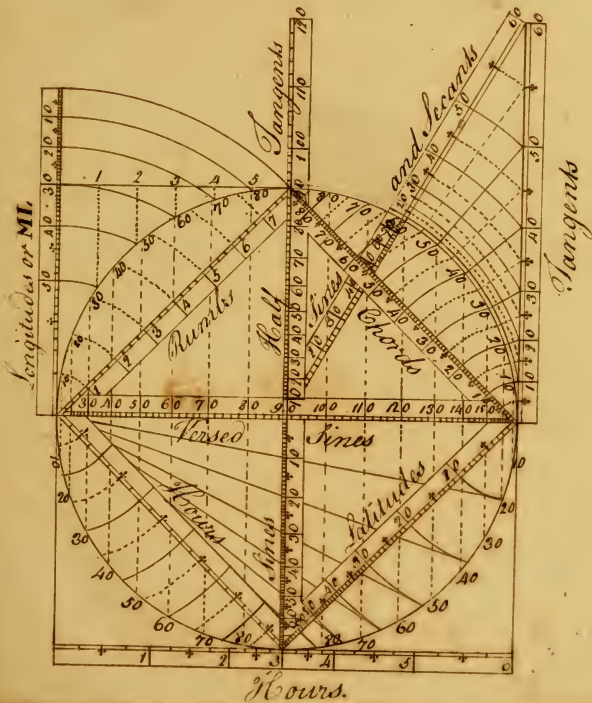
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The Diagram.



Found in *Archaeologia* *Vol. 1*
Memoirs: *Treatise on Shipbuilding*,
 London, 1750, by *M. & G. G. G.*, 1750.
 The diagram shows the *main* of
Planes *Scale*.



Joshua Humphreys

A
T R E A T I S E
O N
S H I P - B U I L D I N G
A N D
N A V I G A T I O N.
I N T H R E E P A R T S

The Theory, Practice, and Application of all the necessary
Instruments are perspicuously handled.

W I T H

The Construction and Use of a new invented Shipwright's Sector, for readily
laying down and delineating Ships, whether of similar or dissimilar Forms.

A L S O

Tables of the Sun's Declination, of Meridional Parts, of difference of Latitude and Departure, of Logarithms, and of artificial Sines, Tangents and Secants.

By M U N G O M U R R A Y.

Shipwright, in his Majesty's Yard, DEPTFORD.

To which is added by way of Appendix,

An English Abridgment of another Treatise on Naval Architecture, lately published at Paris by M. DUHAMEL, Mem. of the R. Acad. of Sciences, Fellow of the Royal Society of London, and Surveyor General of the French Marine.

The whole illustrated with eighteen COPPER PLATES.

L O N D O N :

Printed by D. HENRY and R. CAVE, for the AUTHOR; and Sold by A. MILLAR, in the *Strand*; J. SCOTT, in *Exchange-Alley*; T. JEFFERYS, at the Corner of *St Martin's Lane*, *Charing-cross*; Messrs. GREIG and CAMPBELL, at *Union-Stairs*, and by the Author, at his House at *Deptford*.

M,DCC,LIV.

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A

T H E A T T E N T I O N

S H O U L D B E C A L L E D U P O N

W H O W I S H T O B E I N F O R M E D

O F T H E P R O G R E S S O F T H E

A D V E R T I S E M E N T .

THE several Branches of Mathematicks treated of in this Book are expeditiously taught by the AUTHOR, at his House in *Deptford*; where may be had all Sorts of Sliding Rules and Scales: As also Sectors for delineating Ships, Diagonal Scales, &c. on Brass, Wood, or Paste-board. Attendance from six to eight every Evening, except *Wednesdays* and *Saturdays*.

To the right Honourable the
LORDS COMMISSIONERS,
For executing the Office of
Lord HIGH ADMIRAL
OF
GREAT BRITAIN and *IRELAND*,
And of his Majesty's
PLANTATIONS ABROAD, &c.
THIS TREATISE
ON
SHIP-BUILDING AND NAVIGATION

Is with the utmost Submission, inscribed

By their Lordships

Most dutiful,

Most humble, and

Most obedient Servant

MUNGO MURRAY.

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T H E
P R E F A C E.

THOUGH the art of Ship-building is of the utmost consequence to the trade and security of this nation, and a competent knowledge of the theory of it necessary for every Shipwright, yet I cannot think of a subject which has been so little treated of in our language.

This consideration induced me to offer the following sheets to the publick, which are calculated for the instruction and improvement of shipwrights in the art of delineating or drawing of ships, being fully persuaded that any thing which may be conducive to this end must be of great service to the publick in general. I therefore flatter myself that this attempt will meet with a favourable reception, though it were to be wished it had been undertaken by some person of abilities greatly superior to mine.

In order to make this treatise as useful as possible, I have briefly explained the nature of proportion, the principles of geometry, the invention of logarithms, and their use in the construction of the line of numbers, by which, I presume, any person that is acquainted with common arithmetick, may, with a little application, be able to construct the lines himself, and have a clear idea of all the operations by the sliding rule, in measuring surfaces and solids; a method so expedient and useful, that it is universally practised in measuring all the
plank

plank and timber received into his majesty's yards, and used not only by shipwrights, but by many other artificers, as joiners, painters, &c.

The divisions on the line of numbers, in plate No. II. are taken from the scale of equal parts in the same plate, which is divided in the exactest manner, and may be of great use to the reader, not only in comparing the distances measured on the scale with the table of logarithms, in order to examine how the lines have been graduated, but also in the construction of geometrical figures, as any given or required distances may be measured by it to a very great accuracy.

Tho' it must be allowed that the principles of geometry, trigonometry, logarithms, and their various uses, have been sufficiently explained by many eminent authors who have wrote on these subjects, yet I thought it requisite to treat concisely of them here, as introductory to the main design, which is to instruct the Shipwright to form all his work by mathematical rules; and, as he is furnished with every thing that is necessary in this treatise, I have no occasion to refer him to other books, with which perhaps he might not be furnished if I did.

In the second part, I have endeavoured to explain the method of representing solids upon a plane, with the application thereof to the delineating of ships, in which I have given definitions of all the terms and lines made use of in drawing; I have also shewn the methods that are generally practised, and the difficulties and inconveniences attending them, which I have endeavoured to facilitate by a sector of my own invention, constructed for that purpose; a method entirely new, and, though deduced from mathematical principles, does not restrain the artist from displaying all the skill and judg-

judgment he is master of in varying the form of the curves so as to be most suitable for the service for which the ship is designed.

The third part contains land surveying, geography and navigation, with an analemma divided in so curious and accurate a manner, that by a nonnius division the pole may be set to any latitude, even to three minutes, and all the problems that are usually solved by the globe may be performed with greater exactness by this instrument. I have also shewn the method of constructing the plain and *Mercator's* charts, the manner of keeping a reckoning, and of finding the latitude and variation of the compass by celestial observation; to which I have added tables of the sun's declination, of difference of latitude and departure, of meridional parts, of logarithms, and of artificial sines, tangents, and secants.

Tho' it may be the opinion of many that this 3d part, with the tables, might have been omitted, as having no necessary connection with the theory of Ship-building, yet, as I was desirous of making this treatise universally useful, I thought it requisite it should contain them, though it would have been my interest to have done otherwise.

Some time after the first, and most of the second part had been in the press, a treatise in *French* on the same subject, by the ingenious *M. du Hamel du Monceau*, Member of the Royal Academy of Sciences at *Paris*, and Fellow of the Royal Society at *London*, fell into my hands, and, as I imagined that every body would be desirous of seeing what has been wrote on Ship-building, by a foreign author of such distinction, I have added, by way of appendix, an abridgment of that work, which I doubt not will be agreeable to such as are not acquainted with the *French* language, or have no opportunity of perusing the original.

I flat-

I flatter myself, from the high repute Mons. *Du Hamel's* writings have every where justly acquir'd, that the additional price of 2s. 6d. for the *English* abridgment of his book on Ship-building, annexed to mine, will not be thought much of, when it is considered that the original sells for 18s.

I cannot conclude this preface, without acknowledging the great obligation I am under to the principal officers and gentlemen in his majesty's service, not only in the yard where I have the happiness to be employed, but in several others, as well as in the navy, for their kindness in encouraging this work, several of them persons, whose abilities are such, that it would be the greatest vanity in me to imagine they would countenance the undertaking on account of any information they could expect to derive from it themselves; their true motives were doubtless a consciousness of the important service of such a piece to young shipwrights, and a generous disposition to encourage industry.

I believe it will appear very obvious to every body, that I have spared no pains or cost in endeavouring to render this work as compleat as possible, to which end I have submitted the mathematical part thereof to the perusal and amendment of a gentleman whose abilities in these matters would be indisputable with the publick, were I permitted to name him.

The success of my labour I rest entirely on the judgment and candour of my readers, by which I must stand or fall; whatever may be the event, I shall always have the secret satisfaction of reflecting, that I have sincerely aimed at what is useful, and very much wanted in the *English* language.

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THE THEORY OF SHIPBUILDING and NAVIGATION.

PART I.

CHAP. I. SECT. I.

James Child

Of Involution and Evolution of Quantities.

IF a number be multiplied by itself, and the product by the same number, and the new product again by the same, and so on, it is said to be involved into itself so many times, and the several products are distinguish'd into powers of different denominations, of all which the original number is called the root; the first product being its second power or square; the second the third power, or cube; the third the fourth power, or biquadrate, &c. These powers are usually denoted by a small figure annexed above the last digit of the root. Thus 12^2 signifies the second power or square of the root 12, being equal to 144; 12^3 the third power, or cube thereof, equal to 1728, &c. and so the several powers may be orderly expressed thus, $12, 12^2, 12^3, 12^4$, &c. that is, 12 root, $12 \times 12 = 144$ square, $12 \times 12 \times 12 = 1728$ cube, $12 \times 12 \times 12 \times 12 = 20736$, biquadrate, &c.

Square and cube numbers have borrowed their denominations from geometrical figures or extensions, the root being represented by a right line, which has but one dimension, viz. length; the square by a plane, or right lined figure of two dimensions, having equal length and breadth; and the cube by a right lined solid of three dimensions, having equal length, breadth, and thickness.

The nature of space admits of no other modes of extension, than length, breadth, and thickness, neither is it possible to conceive any body otherwise to exist than under these limitations.

That the reader may have a distinct idea of the method of extracting

the square and cube roots, we think it necessary to make the following remarks on multiplication, and to shew how any root, by being divided into two parts (or made a binomial) may be raised to the second or third power, &c.

Since multiplication may be considered as a manifold addition, it is certain that if either or both multiplicand and multiplier be divided into parts, and all the parts of the one be multiplied by all the parts of the other, the sum of all the products will be equal to the product of the whole multiplicand multiplied by the whole multiplier, and this method is what is in effect performed by the common rule, when either or both consist of more than one significant figure, as will appear by the following examples, *viz.*

E X A M P L E I.

Let the number 24 be divided into two parts, *viz.* 20 and 4, and multiplied by 4.

$$\begin{array}{r} 20 + 4 = 24 \\ \underline{4 \quad 4} \\ 80 + 16 = 96 \end{array}$$

By the common rule

$$\begin{array}{r} 4 \times 4 = 16 \\ 4 \times 20 = 80 \\ \hline 96 \end{array}$$

E X A M P L E II.

Let the number 248 be divided into three parts, *viz.* 200, 40, and 8, and multiplied by 24 or 20+4.

$$\begin{array}{r} 200 + 40 + 8 = 248 \\ \underline{20 + 4} = 24 \\ 4 \times 200 + 4 \times 40 + 4 \times 8 \quad 992 \\ \underline{20 \times 200 + 20 \times 40 + 20 \times 8} \quad 496 \\ 20 \times 200 + 20 \times 40 + 20 \times 8 + 4 \times 200 + 4 \times 40 + 4 \times 8 = 5952 \end{array}$$

By the common rule.

$$\begin{array}{r} 4 \times 8 = 32 \\ 4 \times 40 = 160 \\ 4 \times 200 = 800 \\ 20 \times 8 = 160 \\ 20 \times 40 = 800 \\ 20 \times 200 = 4000 \\ \hline 5952 \end{array}$$

Though there is no occasion to set down the several products, as in the above examples, because the excess above the tens may be retained in the memory, and added to the next place, which is always the method

thod observed in practice; yet this will very much assist us in perceiving the reason of the rules for extracting the square and cube roots of any numbers, by carefully observing the steps by which any root is raised to those powers, as in the following example, *viz.*

Let the root be $24 = 20 + 4$.

$$\begin{array}{r} 20 + 4 \\ \hline 20 + 4 \\ \hline 4 \times 20 + 4 \times 4 \\ \hline 20 \times 20 + 4 \times 20 \\ \hline \text{Square, or second power, } 20 \times 20 + 4 \times 20 + 4 \times 20 + 4 \times 4 \} = 576 \\ \text{or } 20^2 + 2 \times 4 \times 20 + 4^2 \end{array}$$

$$\begin{array}{r} 24 \\ \hline 24 \\ \hline 4 \times 4 = 16 \\ 4 \times 20 = 80 \\ 4 \times 20 = 80 \\ 20 \times 20 = 400 \\ \hline 576 \end{array} \quad \begin{array}{l} 4 \times 4 = 16 \\ 4 \times 40 = 160 \\ 20 \times 20 = 400 \\ \hline 576 \end{array}$$

It is evident from this operation, that any square number whose root is divided into two parts, is equal to the sum of the squares of those parts, and double their product added together.

This observation will hold equally true when the root consists of more than two figures, by repeating the process for every significant figure, as in the following example, *viz.*

Let the root be 2435, or $2000 + 400 + 30 + 5$

$$\begin{array}{l} 1^{\text{st}}, \text{ suppose the root } 2400 \\ \text{binomial parts } 2000 + 400 \\ 2^{\text{d}}, \text{ root } 2430 \\ \text{binomial parts } 2400 + 30 \\ 3^{\text{d}}, \text{ root } 2435 \\ \text{binomial parts } 2430 + 5 \end{array} \quad \begin{array}{r} 2000^2 = 4000000 \\ 2 \times 2000 \times 400 = 1600000 \\ \hline 400^2 = 160000 \\ \hline 2400^2 = 5760000 \\ 2 \times 2400 \times 30 = 144000 \\ \hline 30^2 = 900 \\ \hline 2430^2 = 5904900 \\ 2 \times 2430 \times 5 = 24300 \\ \hline 5^2 = 25 \\ \hline 2435^2 = 5929225 \end{array}$$

By this method it is plain, that we find a square for every figure in the root, and that in finding the square of the first figures, we suppose all the rest to be cyphers, and so on till the whole square is composed by the addition of one significant figure after each operation.

Note, That for every figure that is annexed to the first figure in the root, there will be two in the square; thus, 2^2 is 4, 20^2 is 400, 200^2 is 40000, 2000^2 is 4000000, &c.

In like manner the cube of any number may be found, by dividing the root into parts, as in the following example, *viz.*

Let the root be 24, or $20 + 4$, as before.

Then the square will be $20^2 + 2 \times 20 \times 4 + 4^2$	$\begin{array}{rcl} 1^{st}, & 20^2 & = 8000 \\ 2^{d}, & 3 \times 20^2 \times 4 & = 4800 \\ 3^{d}, & 3 \times 20 \times 4^2 & = 560 \\ 4^{th}, & 4^2 & = 64 \end{array}$
$20 + 4$ root	
$20^2 \times 4 + 2 \times 20 \times 4^2 + 4^3$	
$20^2 + 2 \times 20^2 \times 4 + 20 \times 4^2$	

Cube, or third power $20^3 + 3 \times 20^2 \times 4 + 3 \times 20 \times 4^2 + 4^3 =$ cube of 24 = 13824

Hence it appears, that if any root be divided into two parts, the cube will be composed of the four following sums, *viz.*

1st, The cube of the first part.

2^d, Three times the square of the first part multiplied, by the second part.

3^d, Three times the first part, multiplied by the square of the second part.

4th, The cube of the second part.

If the root consists of more than two figures, the same method may be observed as in composing the square, by taking the two first figures for one part, and the next figure for the other part, and so on till all the significant figures are brought in. This is so plain, that we think it needless to give any example here; and as the only use we shall make of it will be to shew how to extract the cube root, we shall just observe under this head, that for every figure annexed to the first figure in the root, there will be three in the cube, besides the cube of the first figure; thus, 2^3 is 8, 20^3 is 8000, 200^3 is 8000000, &c.

S E C T. II.

Evolution of Quantities.

Evolution is the unfolding, or resolving of any number into the parts of which it is composed, and is called the extraction of the root of any given power; by means of which, we find a number, that being multiplied by itself as many times less one, as the index of the power contains units, will produce the given number.

In

In order to this, the method already observed, and the steps taken in the involution of the binomial root must be carefully attended to, in which it will not be difficult to discern how each part of the root is concerned in the power.

To Extract the SQUARE ROOT.

As the square was composed by multiplication and addition, the root must be found by division and subtraction.

When the root of any number is required, the first thing to be done is to prepare it, by points set over such places as the index of the power directs, always beginning at unity, and proceeding towards the left hand, if the given number (which we shall call the resolvend) be integers, and towards the right hand, in decimal parts; now the index of the square being 2, there must be a point over every second figure, as in the following example, *viz.*

Let the given number be $592922\dot{5}$

Having pointed the given resolvend as above, to find the first figure take the greatest root that is contained in the first period, which in this case is 2000, the square of which is 4000000. Subtract this from the resolvend 5929225, and there will remain 1929225, which is called the first dividend.

Now this number contains double the product of the first figure multiplied by the second figure, and the square of the second figure, as is evident from the method we used in composing the square.

Divide this dividend, therefore, by double the first figure, and the quotient will be the second figure, provided that when it is multiplied by double the first figure, and the product added to the square of the second figure, the sum does not exceed the dividend. In this example 4000 is double the first figure in the root, and when this is made a divisor to the dividend, the quotient will be 400.

In the next place, multiply this second figure by double the first, or which is the same thing, by the divisor, and add the square thereof to the product, and their sum will be 1760000; subtract this from the dividend, the remainder will be 169225, the second dividend.

And now we have in effect subtracted the square of the first two figures; for in the first step we subtracted 2000^2 , and in the next $2 \times 2000 \times 400 + 400^2$, all which make 2400^2 . The second dividend will therefore contain double the product of the first two figures multiplied by the third, and the square of the same third figure. Therefore,

To find the third figure, double the first two figures for a divisor to
this

this dividend, and the quotient will be 30; multiply this by the divisor, and add the square of the quotient to the product; the sum will be 144900. When this is subtracted from the dividend, the remainder will be 24325, the third dividend; to which there must be a new divisor found by doubling the figures already found in the root, and the quotient will be 5, the fourth figure. And by observing the same method as before in the operation, there will be no remainder, so that 2435 will be the true root required.

$$\begin{array}{r}
 \text{Refolvend } 5929225 \\
 2000^2 = 4000000 \\
 \hline
 1^{\text{st}} \text{ divisor } 2000 \times 2 = 4000 \quad 1929225 \quad 1^{\text{st}} \text{ dividend} \quad \left(\begin{array}{l} 2000 \\ 400 \\ 30 \\ 5 \end{array} \right) = 2435 \\
 4000 \times 400 = 1600000 \\
 400^2 = 160000 \\
 \hline
 1760000 \\
 2^{\text{d}} \text{ divisor } 2400 \times 2 = 4800 \quad 169225 \quad 2^{\text{d}} \text{ dividend} \\
 4800 \times 30 = 144000 \\
 30^2 = 900 \\
 \hline
 144900 \\
 3^{\text{d}} \text{ divisor } 2430 \times 2 = 4860 \quad 24325 \quad 3^{\text{d}} \text{ dividend} \\
 4860 \times 5 = 24300 \\
 5^2 = 25 \\
 \hline
 24325 \\
 \hline
 \text{.....}
 \end{array}$$

By comparing this operation with that by which the square number 5929225 was composed as above, the reader will observe, that the same numbers that were there added together to make up that sum, are hereby regularly subtracted from it. So that this method of discovering the root is only the reverse of that by which it was raised.

We shall conclude this head with observing, that there is no necessity in practice for annexing all the cyphers to the divisors and dividends, nor of subtracting the square of the first figure from the whole resolvent; but the several periods which it consists of, may be brought down one at a time, as in the following example, *viz.*

Re-

$$\begin{array}{r}
 \text{Resolvend } 179409850624(423568 \text{ root}) \\
 \underline{16} = \text{greatest square in } 17 \\
 \begin{array}{r}
 1^{\text{st}} \text{ divisor } 82) \quad 194 \\
 \quad \quad \quad 2 \quad 164 = 82 \times 2 \\
 \hline
 2^{\text{d}} \text{ divisor } 843) \quad 3009 \\
 \quad \quad \quad 3 \quad 2529 = 843 \times 3 \\
 \hline
 3^{\text{d}} \text{ divisor } 8465) \quad 48085 \\
 \quad \quad \quad 5 \quad 42325 = 8465 \times 5 \\
 \hline
 4^{\text{th}} \text{ divisor } 84706) \quad 576006 \\
 \quad \quad \quad 6 \quad 508236 = 84706 \times 6 \\
 \hline
 5^{\text{th}} \text{ divisor } 847128) \quad 6777024 \\
 \quad \quad \quad 8 \quad 6777024 = 847128 \times 8 \\
 \quad \quad \quad \dots\dots\dots
 \end{array}
 \end{array}$$

Observe always, that after having doubled the root in the quotient for a divisor, we are to enquire how oft it may be had in the dividend; so as when the quotient figure is annexed to the divisor, and that increased divisor multiplied by the same quotient figure, the product may be the greatest number that can be had in the dividend: And so proceed from period to period till the whole is finished.

By pursuing this method in extracting the root of the square number 5929225, the reader will observe that the operation is exactly the same as before, omitting the cyphers.

To Extract the CUBE ROOT.

We shall observe the same method in extracting the cube root, as we have done already in the square root; that is, by considering it as divided into two parts in different operations, till we have discovered all the significant figures thereof.

Let the number, or resolvend, whose cube root is required be 13824.

When it is properly pointed as above, it appears that the root will consist of two figures.

The first figure in the root will be the greatest root that can be had in the first period 13000, which is 20; the cube of which 8000, must be subtracted from the resolvend, and the remainder will be 5824, for a dividend.

As we have already subtracted the cube of the first part 20, this number must contain three times the square of that first part multiplied by the second part, three times the first part multiplied by the square of the second, and the cube of the second part, added together.

ther. Therefore, if this dividend be divided by three times the square of the first part = 1200, the quotient will be 4, the second figure required: Then 3 times $20^2 \times 4$, 3 times 20×4^2 , and 4^3 must be added together, and the sum subtracted from the dividend, as in the following example, *viz.*

$$\begin{array}{r}
 \text{Refolvend } 13824 (24 \text{ root} \\
 20^2 = 8000 \\
 \hline
 \text{Divisor } 3 \times 20^2 = 1200 \quad 5824 \\
 3 \times 20^2 \times 4 = 4800 \\
 3 \times 20 \times 4^2 = 960 \\
 4^3 = 64 \\
 \hline
 5824 \\
 \dots
 \end{array}$$

By comparing this example with that which we have given before, where the binomial root $20 + 4$ is cubed, the reader will observe, that the very same numbers which compose the cube 13824, and are there added together, are here regularly subtracted from it. This method of extracting the cube root, is, therefore, only the reverse of that by which it was raised.

We shall give one example more, without annexing cyphers to the divisors and dividends.

Let it be required to extract the cube root of 94818816

$$\begin{array}{r}
 94818816 (456 \\
 64 \\
 \hline
 1 \text{ divisor } 4^2 \times 3 = 48 \quad 30818 \text{ dividend} \\
 3 \times 4^2 \times 5 = 240.. \\
 3 \times 4 \times 5^2 = 300.. \\
 5^3 = 125 \\
 \hline
 27125 \\
 2 \text{ divisor } 45^2 \times 3 = 6075 \quad 3693816 \text{ dividend} \\
 3 \times 45^2 \times 6 = 36450.. \\
 3 \times 45 \times 6^2 = 4860.. \\
 6^3 = 216 \\
 \hline
 3693816 \\
 \dots
 \end{array}$$

It is here to be noted, that the two last figures in the dividend must be excluded, before we enquire how often the divisor (which wants two cyphers) is contained in it, because when we proceed to find the second figure in the root, the first must be considered in the place of tens; and when

when we are to find the third figure, it must be considered in the place of hundreds, this will appear very plain by annexing the cyphers in the operation.

As the divisor found by this rule, multiplied by the new figure in the root, is not all that is to be subtracted from the dividend, but also three times the first part multiplied by the square of the second, and the cube of the second, as in the foregoing example; it will sometimes happen that the quotient must not be taken for the next figure in the root, thus, if the dividend 30818 be divided by 4800, the quotient will be 6, but $3 \times 40^2 \times 6 + 3 \times 40 \times 6^2 + 6^3 = 33336$ which exceeds the dividend, therefore it will be necessary sometimes to try how much the number to be subtracted will amount to by the foregoing rule, before we can determine upon the new figure in the root.

If, as it often happens, a number has not a root that can be expressed by a rational number, place as many pairs of cyphers in the square, and ternaries of cyphers in the cube, on the right hand of the remainder, as you would have decimal places in the root, and work as before, distinguishing them from the integers by a comma between; and thus you may approach infinitely near the exact root.

Mathematicians have proceeded further in the involution and evolution of quantities, viz. to the 4th, 5th, 6th, 7th, 8th, and 9th powers, called biquadrat, sursolid, square cubed, second sursolid, and biquadrat squared, but as we shall not have occasion to apply these in practice, it is sufficient barely to mention them.

How to raise a given root to any power, or to extract the root out of any given power, by the help of logarithms, shall be shewn when we come to treat of logarithms, and their various uses.

CHAP. II. SECT. I.

Of P R O P O R T I O N.

THE whole body of the mathematicks is chiefly concerned in comparing quantities one with another; their mutual relation is what is called proportion, which is either arithmetical or geometrical; and as quantities may be represented by numbers, or lines, we shall first consider proportion with respect to numbers.

Arithmetical proportion is, when in comparing two or more numbers, the lesser is subtracted from the greater, the remainder is called the difference; and when several differences are equal, those numbers are said to be in an arithmetical proportion to one another. As if we compare 2 to 4, and 4 to 6, and 6 to 8; the difference between 2 and 4 is 2, equal to the difference between 4 and 6, and to that of 6 and 8; therefore, 2, 4, 6, 8, or 8, 6, 4, 2, or 2, 5, 8, 11, 14, or 14, 11, 8, 5, 2, are all ranks of numbers in arithmetical proportion, or in arithmetical progression continued. 2, 5, 7, 10, are also proportionals, for the difference between 5 and 2 is the same as between 10 and 7, but then, because it is not the same with the difference betwixt 7 and 5, these four numbers are said to be in a discontinued, as the former ranks are said to be in a continued arithmetical proportion. Hence the following inferences.

1. If three quantities are in arithmetical proportion continued, the sum of the extremes is equal to the double of the mean, as in this, 8, 10, 12, where 20, the sum of the extremes 8 and 12, is equal to double of the mean 10.

2. If four quantities are so, the sum of the extremes is equal to the sum of the means.

as 3, 5, 7, 9, here, $3 + 9 = 12$ and $5 + 7 = 12$.

3. If never so many quantities are so proportional, the sum of the extremes is always equal to the double of the middle term, if the number of the terms be odd, or to the sum of any two terms equally distant from the extremes, as in the following series.

$$\begin{aligned} &2, 4, 6, 8, 10, 12, 14. \\ &2 + 14 = 8 \times 2 = 16. \\ &\text{or } 4 + 12 = 8 \times 2 = 16, \text{ \&c.} \end{aligned}$$

And this must always hold good, because the last term comprehends the

the first, together with the common difference superadded, as often as the number of its place is distant from the first term: But the first term has no addition of the difference at all; and as the second term has one difference or ratio more than the first; the third one more than the second, &c. so the last but one has one less than the last of all; the last but two one less than the last but one, &c. whence the sum of any two of these equally distant from the extremes must be equal to the sum of the extremes, because one increases as much as the other decreases.

Therefore, the sum of any number of terms in such a progression may be had, by multiplying the sum of the extremes by half the number of the terms.

To find the sum of never so many quantities in this progression, it is only necessary that the extremes and the number of terms be given: so that if by having the first term and the common excess you would find the last, it might be done with great dispatch, by multiplying the number of terms, lessened by unity, into the common excess, and then adding the first term to the product.

Thus, if the last term of a progression of 73 places were required, and the common difference were 4, and the first term 3; you need only multiply 72 by 4, and to the product 288 add 3, and you have 291 for the last term in the progression.

So that if the progression begins with a cypher, which is the most natural and simple of all, then the sum of all the terms will be equal to the sum of the extremes multiplied by half the number of the terms. Thus suppose

0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39.

The last term 39, multiplied by 14, the whole number of terms, gives 546; the half of which 273, is the sum of all the terms.

From whence it will follow, that the sum of all the terms in any such progression beginning from 0, is half the sum of so many terms, all equal to the greatest.

S E C T. II.

Of GEOMETRICAL PROPORTION.

Geometrical proportion is when in comparing two or more numbers, one is divided by the other, the quotient is called the ratio; and when several ratios are equal, the numbers are said to be in a geometrical

proportion; thus, $2 : 6 :: 5 : 15$ are proportionals; for 6 divided by 2 is 3, and 15 divided by 5 is 3, and so the ratio's are equal. When two numbers are compared, the former is called the antecedent, and the latter the consequent; but when more than two are compared, they are called terms, of which the first and last are called extremes, and all the intermediate ones, means. In order to know whether numbers be proportionals, it is only finding the ratio of each pair, and here it will be indifferent whether the antecedents or consequents of every pair be made divisors, as in the preceding numbers $2 : 6 :: 5 : 15$, if 2 the antecedent be divided by 6, the consequent, the quotient is $\frac{1}{3}$, and if 5 be divided by 15, the quotient is $\frac{1}{3}$, so the ratio's are equal, as before when the antecedents were made divisors.

When in a rank of numbers they increase in a geometrical proportion, the ratio will be a common multiplier, and is found by dividing any one of the consequent terms by its antecedent, for so the quotient will be the ratio.

As $\left\{ \begin{array}{l} 3, 9, 27, 81, 243, \&c. \text{ Here the common multiplier is } 3. \\ 2, 4, 8, 16, 32, \&c. \text{ Here the common multiplier is } 2. \end{array} \right.$

It is plain that if either of these antecedent terms be multiplied by the ratio, the product will be its consequent.

When a in rank of numbers they decrease in a geometrical proportion, the ratio will be a common divisor, and is found by dividing any one of the antecedent terms by its consequent.

As $\left\{ \begin{array}{l} 243, 81, 27, 9, 3, \&c. \text{ Here the common divisor is } 3. \\ 32, 16, 8, 4, 2, \&c. \text{ Here the common divisor is } 2. \end{array} \right.$

In like manner, if either of these terms is divided by the ratio, the quotient will be the next term in the progression.

If in comparing several numbers together, we find the ratio of the first and second, to be the same with the ratio of the second and third, third and fourth, fourth and fifth, and so on; those numbers are in a geometrical proportion continued.

If in comparing four numbers together, we find the ratio of the first and second to be the same as the ratio of the third and fourth, but that the ratio of the second and third is not the same; those numbers are called discontinued proportionals, as $2 : 4 :: 6 : 12$, for the progression stops here at 4.

The manner of expressing continued proportionals is by separating the terms by two points, as $2 : 4 : 8 : 16 : 32, \&c.$ but in discontinued proportionals the terms where the progression stops are separated by four points, as

$5 : 10 :: 6 : 12 :: 7 : 14, \text{ or } 14 : 7 :: 12 : 6 :: 10 : 5.$

If

PROPOSITION I.

If three numbers are in a geometrical proportion, the product of the extremes will be equal to the product of the middle term multiplied into itself, or which is the same thing, to the square of the middle term.

As in these numbers 2, 6, 18.

$$2 \times 18 = 6 \times 6 = 36.$$

PROPOSITION II.

If four numbers are in geometrical proportion, (whether continued or discontinued) the product of the extremes will be equal to the product of the 2 means.

Let the numbers be $5 : 10 :: 6 : 12$

$$5 \times 12 = 10 \times 6 = 60$$

$$\text{or } 5 : 10 :: 20 : 40$$

$$5 \times 40 = 10 \times 20 = 200$$

From these two propositions the following inferences may be drawn, *viz.*

1st, If the product of any two numbers is equal to the square of a third, those three numbers are in geometrical proportion continued.

2^d, If the product of any two numbers is equal to the product of any other two, those four numbers are proportionals, and the numbers multiplied into each other will be either 2 means, or 2 extremes, as in the following examples, *viz.*

$$2 \times 16 = 4 \times 8 = 32$$

$$16 : 4 :: 8 : 2$$

$$4 : 16 :: 2 : 8, \text{ \&c.}$$

$$\text{or } 8 \times 12 = 6 \times 16 = 96$$

$$8 : 6 :: 16 : 12$$

$$6 : 12 :: 8 : 16, \text{ \&c.}$$

PROBLEM I.

To find a mean proportional between any two given numbers.

Note, By a mean proportional we are to understand such a number as if multiplied by itself, the product will be equal to the product of the two given numbers.

Rule, Multiply the given numbers by one another, and extract the square root of the product, that root will be the mean required.

Ex-

Example, Let the given numbers be 3 and 27.

$3 \times 27 = 81$ the square root of which is 9 the mean required.

P R O B L E M II.

To find a fourth proportional to the three given numbers, so that the ratio of the third and fourth may be equal to the ratio of the first and second.

Rule, Multiply the second number by the third, and divide the product by the first, the quotient will be the fourth number required.

Example, Let the given numbers be 2 : 6 :: 7.

$6 \times 7 = 42$, and $42 \div 2 = 21$, the fourth number required.

The reason of both these rules is evident from the two foregoing propositions, for where there are three numbers given to find a fourth, though the fourth is not known, we know that the product of it when multiplied by the first will be equal to the product of the second and third numbers, (*per prop. II.*) therefore, if that product is divided by the first term, the quotient will be the fourth number required. This is the foundation of the rule of three, which we suppose the reader acquainted with already.

In finding a fourth proportional where three numbers are given, the two first give the ratio, and the question as to the fourth proportional concerns the third number.

Direct proportion is, when the greater the term is by which the question is made, the fourth term will be also the greater; and the lesser that term is, the fourth will also be the lesser.

Reciprocal, or inverse proportion is, when the greater the term is by which the question is made, the fourth will be lesser, and the lesser that term is, the greater the fourth.

Continual proportion, thus expressed, $\div\div$, is, when all the terms between the first and the last are both antecedents and consequents in the same proportion.

Example, 8, 12, 18, 27, are $\div\div$; for $8 : 12 :: 12 : 18 :: 18 : 27$.

Wherefore in such series, the last term subtracted from the sum of all the terms will give the sum of all the antecedents, and the first term subtracted from the said sum will give the sum of all the consequents.

If four quantities be proportional, they will also be so alternately, inversely, in composition, in division, conversely, and mixtly.

Ex-

Example		$A : B :: C : D$
1, directly	$A : B :: C : D$ in numbers	$12 : 9 :: 8 : 6$
2, alternately	$A : C :: B : D$	$12 : 8 :: 9 : 6$
3, inverfely	$B : C :: D : A$	$9 : 8 :: 6 : 12$
4, in compo- fition	$\left\{ \begin{array}{l} A + B : B :: C + D : D \\ A + C : C :: B + D : D \end{array} \right.$	$\left\{ \begin{array}{l} 21 : 9 :: 14 : 6 \\ 20 : 8 :: 15 : 6 \end{array} \right.$
5, in divi- fion	$\left\{ \begin{array}{l} A - B : B :: C - D : D \\ A - C : C :: B - D : D \end{array} \right.$	$\left\{ \begin{array}{l} 3 : 9 :: 2 : 6 \\ 4 : 8 :: 3 : 6 \end{array} \right.$
6, converfe- ly	$\left\{ \begin{array}{l} A : A + B :: C : C + D \\ A : A + C :: B : B + D \end{array} \right.$	$\left\{ \begin{array}{l} 12 : 21 :: 8 : 14 \\ 12 : 3 :: 8 : 2 \end{array} \right.$
7, mixt- ly	$\left\{ \begin{array}{l} A + B : A - B :: C + D : C - D \\ A + C : A - C :: B + D : B - D \end{array} \right.$	$\left\{ \begin{array}{l} 21 : 3 :: 14 : 2 \\ 20 : 4 :: 15 : 3 \end{array} \right.$

All these are evidently proportionals, the product of the extremes being equal to that of the means, excepting the inverted, wherein the product of the first and second term is equal to that of the third and fourth, which is a property peculiar to that kind of proportion.

P R O P O S I T I O N III.

If the product of any two numbers is divided by a third, the quotient will be a fourth proportional, the divisor will be the first, and the numbers multiplied into each other the second and third terms in the progression.

Example, Let the given numbers be 5 and 8, and their product be divided by 10, a third number:

$$5 \times 8 = 40 \div 10 = 4.$$

Therefore, $10 : 5 :: 8 : 4$ (by inference 2d) for the product of the means is equal to the product of the extremes.

P R O P O S I T I O N IV.

When two numbers are multiplied into one another, they may be made mean proportionals to other two numbers, of which 1 must be the first, and the product of the two numbers will be the last or fourth term.

Example, Let the given numbers be 6 and 8.

It will be $1 : 6 :: 8 : 48 = 6 \times 8 = 1 \times 48$. Hence in multiplication,

As

As 1 is to the multiplier, so is the multiplicand to the product; and in division, as the divisor is to 1, so is the dividend to the quotient.

P R O P O S I T I O N . V.

If any two numbers be multiplied, or divided by any same third number, the products or quotients will be proportional to the numbers so multiplied or divided. Let the numbers be 2 and 4, each to be multiplied by 3; then $3 \times 2 = 6$, and $3 \times 4 = 12$, and $2 : 4 :: 6 : 12$; or if 18 and 15 be divided each by 3, the quotients will be 6 and 5, and $18 : 15 :: 6 : 5$.

The powers or roots of proportionals will likewise be proportionals.

$$\begin{array}{l} 2 : 4 :: 3 : 6 \text{ root} \\ 4 : 16 :: 9 : 36 \text{ square} \\ 8 : 64 :: 27 : 216 \text{ cube} \end{array}$$

Hitherto we have considered the doctrine of proportion, only with respect to numbers; but as any quantity may be represented by numbers, all that has been said with regard to them may likewise be applied to any thing that can be augmented or diminished. A line of 2 feet long has the same proportion to a line of 6 feet long that the number 2 has to 6, but the method of finding the proportions of lines, &c. to one another requires the knowledge of the principles of geometry, which shall be the subject of the next chapter.

C H A P. III. S E C T. I.

Of GEOMETRY.

AS in arithmetick we treat of numbers by comparing them to one another, without considering their relation to any particular quantity, so in geometry we shew how to compare quantities to one another, and find their proportions without arithmetical calculation.

Geometry may therefore be fitly called, The science of extension abstractedly considered, without any regard to matter.

GEOMETRICAL DEFINITIONS,

Def. 1. Quantity is any thing that can be augmented or diminished, and may comprehend extension, weight, motion, &c. for one may be taken as greater or lesser, heavier, or lighter, swifter, or slower, in relation to another, of things of the same kind; but there can be no comparison between quantities of different kinds; as hours and miles; for an hour is neither greater nor less, heavier nor lighter, &c. than a mile.

Def. 2. All things that are capable of extension are to be considered either as lines, surfaces, or solids.

Def. 3. A line is a quantity of one dimension, where the length only is considered.

Def. 4. A surface is a quantity considered under two dimensions, *viz.* length and breadth.

Def. 5. A solid is that which has three dimensions, *viz.* length, breadth, and thickness, these two last are sometimes called height and depth.

Def. 6. A point, in the mathematical sense, and in respect of continual quantity, is that wherein neither of the foregoing dimensions are considered. It therefore consists of no parts; for then it would be a solid, surface, or line. It is analogous to an instant in time, which partakes neither of the past or the future. The centers of circles, &c. in diagrams are not mathematical points, but sensible objects whereby the understanding, considering them abstractedly, is assisted in mathematical speculations.

PLATE

Def. 7. Parallel lines are such as are every where equally distant from one another, as AB , and CD ; for if the lines AC , and BD , are equal, and are the shortest that can be drawn from the points A and B , to the line CD , the right lines AB , and CD , if infinitely produced, would be always equidistant, and consequently would never meet; but if BD were shorter or longer than AC , the lines if produced would meet.

Def. 8. An angle is the inclination of two lines that meet in a point, so as not to constitute one straight line, and this will be the case of all straight lines that are not parallel, as the lines AB , and CD , if they are produced; they will meet in the point O , which is called the angular point; the lines that form the angle are called the legs of the angle.

The quantity of any angle is not determined by the length of the legs that form it, but by the distance of any two points, one of them in one leg, and the other in the other, both equally remote from the angular point; now the greater this distance is, the greater will be the angle, and if there are two angles, as AOC , and aoc , and if OB , OD , ob , od , are all equal to one another, then the angle AOC will be greater than the angle aoc , because the line bd is shorter than the line BD , and when these distances are equal, the angles will be equal.

All straight lines are measured by a rule, staff, or scale of equal parts, as inches, feet, yards, &c. but angles are measured by arches of circles.

Def. 9. Angles that are form'd by any line drawn from any point obliquely to another line, are called oblique angles; and those angles that are form'd by drawing a line from any point directly the shortest way till it meets any right line, are called right angles.

Def. 10. A perpendicular line is the shortest that can be drawn from a given point to any given line, that is, when the right line AC , standing upon the right one BD , leans unto neither part, and makes the angles ACD , ACB on both sides equal; those angles are called right ones, and the line AC is perpendicular to DB .

Hence it is evident, that if in the line DB any two points, s , t , be taken equally distant from C , and any point, f , in the line AC , be equally distant from these two points, s , t , then will the line AC be perpendicular to DB .

Def.

Def. 11. A circle is a plain surface contained within one continued curve line, called the periphery, or circumference; and is drawn by fixing the point of one leg of a pair of compasses in any assignable part of a surface, and carrying the point of the other leg round till it arrives again at the place from whence it began to move: The fixed point is called the center, and is equally distant from all points in the circumference described by the other leg. PLATE I.
Fig. 5.

Def. 12. The radius or semi-diameter is a strait line drawn from the center to any part of the circumference, of which there may be an infinite number all equal to one another.

Def. 13. The diameter is a line drawn through the center till it meets the circumference both ways, and is therefore always double the radius; there may be an infinite number of diameters all equal, they all intersect one another in one point, which is the center; so that when the intersection of any two diameters is found, the center of the circle is likewise found. The diameter is the longest strait line that can be drawn within the circle.

Def. 14. A chord is a strait line less than a diameter, drawn within the circle from any one point to another, both in the circumference, and cuts the circle into two unequal parts, called segments.

Def. 15. An arch is any part of the circumference.

If a circle be cut by a diameter, the segments are equal, and are called semicircles; and if a radius be drawn perpendicular to a diameter, it will divide the semicircle into two equal parts, called quadrants, each containing one quarter part of the whole circle. The space contained between two radii and an arch, is called a sector.

What an arch wants of a semicircle is called the supplement of that arch, and what it wants of a quadrant is called the complement thereof.

Def. 16. When an arch is the 360th part of the circumference it is called a degree; the 60th part of a degree is called a minute; the 60th part of a minute is called a second; and the 60th part of a second is called a third, &c.

It is by these degrees and minutes that all angles or arches are measured, in order to which it must be observed, that the circumference of every circle, whether great or small, is supposed

PLATE to be divided into 360 degrees; but the length of the degrees
 I. of a small circle will always be less than the degrees of a large
 Fig. 55. one in proportion as the radius of the one is less than the radius of the other.

To illustrate this, draw four semicircles from the center C; divide the outer one into 180 equal parts; now it is very plain that if strait lines are drawn from each of these divisions to meet all at the center, they will pass through all the inner semicircles, and so divide them into the same number of degrees.

A circle may be conceived to be formed by the motion of a strait line, for if one end be fastened at the center, the other end carried round till it returns to the same point where the motion began will describe the circumference; and if several points are taken in the line, as at *a, b, d, f*, they will each describe circumferences of circles, and *C a, C b, C d, C f*, will be the several semidiameters: this line is represented by a thread fastened at the center; draw the diameter *A C B*, this thread by its motion from the point *A* to the point 180 will form all the various angles that can be made by two strait lines, and will always make two angles with the diameter; from this description the following inferences may be deduced, *viz.*

Inf. 1. The greatest angle that can be made by two right lines will be less than 180 degrees, for if the thread be drawn through the point *A*, it will then lie upon the diameter *A B*, and make no angle with it, but if carried to 20, the angle from *A* will be 20 degrees, and the angle from *B* will be so much less than 180 degrees, *viz.* $180 - 20 = 160$; and as it is carried through the points 30, 40, 50, 60, 70, 80, the angle from *A* increases, and that from *B* decreases; when it comes to 90, both angles will be equal to 90 degrees, and in this position the thread will be perpendicular to the diameter; after it passes 90, the angle from *A* will still be increasing, and the other decreasing till the thread comes to the point *B*, where it again falls in with the diameter, and makes no angle; and as the arch from *A* to *B* is a semicircle, and contains 180 degrees, it is impossible to make an angle of 180 degrees by two strait lines.

Inf. 2. A right angle contains 90 degrees, and the two lines that form it are perpendicular to each other; if these lines are produced they will make 4 angles, each 90 degrees,

Inf. 3.

Inf. 3. If a right line stands upon another right line, and makes two angles with it, those two angles will be equal to two right ones, or 180 degrees: For if the line is perpendicular, they will be equal, by the preceding; if the line is oblique, as from any point in the circumference to the center, except the point 90, suppose from 40; then from A to 40 is the measure of one angle, and from B to 40 is the measure of the other angle, but the sum of these two is 180; the angle from A to 40 is as much less as that from B is more than 90 degrees; the first is called an acute, and the last an obtuse angle.

Inf. 4. If one of the two angles formed by the meeting of two lines is known, the other may be found by subtracting the known angle from 180 degrees, the remainder being the other angle.

Inf. 5. If several right lines are drawn so as to meet in one point in another right line, the sum of all the angles will be 180 degrees, and the sum of all the angles that can be made round a point will be 360 degrees.

Inf. 6. If there are several equal chords in the same circle, the arches subtended by these chords will be equal.

Def. 17. A triangle is a space contained within three lines, which by their meeting form three angles: When the three lines are strait, it is called a rectilineal triangle; when they are arches of the same circle, it is called a spherical triangle; when the three sides are equal, the triangle is called equilateral; when only two are equal, it is called isosceles; and when all the three sides are unequal, it is called scalene.

Def. 18. Quadrilateral, or four sided figures are such as are limited by four lines forming four angles. Such as are rectilineal are either parallelograms or trapezia.

Def. 19. A parallelogram is a figure whose opposite sides are equal and parallel, of which there are four kinds, *viz.* a square, a rectangle, a rhombus, and rhomboids.

Def. 20. A square is a figure limited by four equal sides, all perpendicular one to another, as A B C D; that is, a quadrilateral figure, whose sides and angles are all equal, is called a geometrical square.

Def. 21. A rhombus is a figure that hath four equal sides, but no right angle, the opposite angles being equal, *viz.* $\angle EGH = \angle EFH$, and $\angle GEF = \angle GHF$, but all oblique.

Def. 22.

PLATE I. *Def.* 22. A rectangle, or a right angled parallelogram, hath four right angles, and its opposite sides equal and parallel, *viz.*
 Fig. 11. $IM = KL$, and $IK = ML$; this figure is often called an oblong, or long square.

Fig. 12. *Def.* 23. A rhomboides is an oblique-angled parallelogram, and the sides that form the angles are unequal.

Fig. 13. *Def.* 24. Every quadrilateral figure that has neither opposite sides, nor opposite angles equal, is called a trapezium.

A right line drawn from any angle (as D) in a four sided figure to its opposite angle (B), is called a diagonal, and divides the figure into two triangles (ABD and BCD).

Def. 25. A polygon is a figure that hath more than four sides, and may be either regular or irregular.

Fig. 14. *Def.* 26. A regular polygon has all its sides and angles equal, and may be inscribed in a circle, and all the angular points (*a, b, c, d, e, f*) will touch the circumference.

Regular polygons derive their names from the number of their sides or angles at the center of the circle they are inscribed in; thus a polygon of 5 sides is called a pentagon, of 6 sides a hexagon, of 7 sides a heptagon, of 8 sides an octagon, &c.

Fig. 15. *Def.* 27. An irregular polygon has many unequal sides, standing at unequal angles, as ABCDEFG.

All irregular polygons may be reduced to regular figures, by drawing diagonal lines in them; thus the polygon ABCDEFG, the diagonals GB, BF, FC, and CE being drawn, will be reduced to five triangles, *viz.* ABG, GBF, BFC, FCE, and CDE.

S E C T. II.

GEOMETRICAL PROPOSITIONS:

PROPOSITION I. PROBLEM.

Fig. 16. **A**T a given point in a right line, (as *o*), to make an angle equal to a given one, (ABC.)

Open your compasses to any convenient distance, and from B
 as

as a center, describe an arch, as st ; with the same extent de-
scribe the arch fg , from the center o ; then take the chord st in
your compasses, and set it off from f to k in the arch fg , and
draw the line ok , and so the angles ABC , and kof , will be
equal. PLATE I.

Hence an angle of any number of degrees may be made, for
if with the radius of any divided circle we describe an arch (as
 lm) from the point C , as center, in a right line, as CB , and
then take the quantity of the given angle (suppose 20 degrees)
from the divided circumference of that circle by whose radius the
arch was described, and set that off upon the arch from l to m ,
and draw the line Cm . mCl will be the angle required; this
in effect is making one angle equal to another given one, for
any angle may be found in the divided circle by drawing two se-
miameters to two points of division, including the measure
thereof in the circumference.

If the angle is already made, describe an arch as before to
meet both sides of the angle, the extent of this arch measured
upon the divided circumference will give the quantity of the
angle.

PROPOSITION II. THEOREM.

If two right lines intersect or cut one another, the opposite
angles will be equal; the contiguous angles, as a and c , taken to-
gether will make 180 degrees (by *Inf. 3, Def. 16*), but if instead of
 c we take d , the sum of the angles a and d will likewise be 180
degrees, therefore c and d must be equal; for the same reason, as
the angles c or d , added to the angles a or b , will make 180 de-
grees, it will appear that those opposite angles are likewise equal. Fig. 17.

To illustrate this by numbers, let the angle a be 30 degrees,
this subtracted from 180 will leave 150, for the angle c , and the
angle d will likewise by the same method be found 150, then
150 subtracted from 180 will leave 30 for the angle b .

PROPOSITION III. THEOREM.

If a right line GH intersects two parallels, AB and CD , the
opposite angles $GE B$, $CF H$, will be equal. Fig. 18.

To prove this, as the lines AB , and CD , are parallel by con-
struction, they may be considered as one broad line, crossed by
the

PLATE I. the line GH , then the Angles $GE B$, and $CF H$, will be equal by the preceding, and if the angles are equal, the lines will be parallel; for supposing AB not parallel to CD , let the line LM be drawn, and supposed parallel to CD , then will the angle GEM be equal to $CF H$; but this was supposed equal to $GE B$, therefore $GEM = GE B$, which is impossible.

Fig. 19. *Inf. 1.* If a right line, GH , cuts several parallels, AB , CD , EF , IK , it will make all the inward angles on the same side of the line equal, that is, the angles 1, 2, 3, will all be equal to the angle GRB , and therefore will be equal to one another; this will likewise hold true with regard to the angles 4, 5, 6, for they will all be equal to the angle ARG .

Fig. 18. *Inf. 2.* The alternate opposite angles will likewise be equal, that is, the angle AEH will be equal to DFG , which may be thus proved, $GE B$ is equal to $CF H$, by this proposition; it is also equal to AEH , by *prop. II.* therefore AEH is equal to $CF H$, and $CF H$ equal to DFG .

PROPOSITION IV. THEOREM.

Fig. 20. If the diameter or radius of a circle cuts any chord into two equal parts, it will be perpendicular thereto, and a perpendicular cutting a chord into two equal parts will pass through the center.

Let b be the middle of the chord AB , through which draw the radius CD , draw also the dotted lines CA and CB , those lines (by *Def. 12.*) will be equal, the points A and B are equally remote from b by supposition, and therefore the line Cb is perpendicular to AB (by *Def. 10.*)

Let b be the middle of the chord, and Cb perpendicular to it; I say it must pass through the center, for suppose it passes through the point f , then the lines fA , and fB , would not be equal, therefore the line fB would not be perpendicular to AB , which is contrary to the supposition.

Inf. 1. The perpendicular that divides the chord into two equal parts, also divides the arch into two equal parts, for the line Db being supposed perpendicular to AB , the chords DA and DB will be equal, and therefore the arches will be equal.

Inf. 2. If two chords are bisected by two perpendiculars, those perpendiculars will intersect one another in the center of the circle.

P R O P.

P R O P. V. THEOREM.

PLATE
I.

If a right line AD be drawn to touch the circumference in the point B , and from that point the chord BF be drawn; the angle ABF , made by the chord and tangent AD , is measured by half the arch $FE B$, of which BF is the chord. Fig. 21.

To demonstrate this, draw the radius CE perpendicular to the chord BF ; this will bisect the chord and arch (by the preceding); so BE is half the arch $FE B$, and is the measure of the angle BCE ; all that remains to be proved then is, that the angles ABF and BCE are equal.

Draw the radius CB to the point B , which will be perpendicular to the line AD , because the radius is the shortest line that can be drawn from the center to the circumference; therefore the angle ABF , together with the angle $FB C$, will be 90 degrees: Draw the diameter LG parallel to the chord FB , which will be perpendicular to EC ; therefore the angles ECB , and BCL , taken together, will also make 90 ; but the angles BCL and $FB C$ are equal, being alternate to the parallels BF and GL ; and if each be subtracted from 90 degrees, the remainders ECB and ABF will be equal.

To illustrate this by numbers, let the arch $FE B$ be 80 degrees; then the arch EB will be 40 degrees = the angle at the center (r); and because the angle $EC L$ is 90 degrees, the angle a must be 50 degrees; again, the lines FB and GL being parallels, and the line CB crossing them, the angles a and d are alternate; the angle d must therefore be 50 degrees = a ; now as the angle ABC is 90 degrees, the angle ABF must be 40 , which is half the arch $FE B$.

In like manner the angle FBD will be measured by half the arch $FGHLB$; for the line FB makes two angles with the line AD , therefore (by *Inf. 3. Def. 16.*) the angle FBD will be 140 degrees, which is half the arch $FGHLB$, for the arch $FE B$ is 80 degrees by supposition, which subtracted from 360 , the whole number of degrees in a circle, there remains 280 for the arch $FGHLB$, the half of which is 140 .

This proposition ought to be well attended to, for the following propositions will be clearly demonstrated by it.

E

PROP.

PROP. VI. THEOREM.

PLATE I. An angle at the circumference of a circle is measured by half the opposite arch; that is, the angle BAC is measured by half the arch BDC .

Fig. 22.

To demonstrate this, draw the line GF to touch the circle at the point A , then the three angles GAB , BAC , CAF , taken together will make 180 degrees (*by Inf. 5. Def. 16.*); the three lines BA , AC , CB , divide the whole circumference into three arches, BEA , AIC , CDB ; therefore the halves of those arches added together will make 180 degrees; but BAG is measured by half the arch BEA , and the angle FAC is measured by half the arch AIC (*by the preced.*) and what remains to complete the 180 degrees is half of the arch CDB , which must therefore be the measure of the angle BAC .

Inf. 1. The three angles of any rectilineal triangle added together will make 180 degrees; for being inscribed in a circle, the sides will be chords, and will cut the whole circumference into three arches, the halves of which arches are the measures of their opposite angles; the angle at A is measured by half the arch BDC , the angle at B by half the arch AIC , and the angle at C by half the arch AEB .

Inf. 2. The angle at the center is double the angle at the circumference, if they stand upon the same arch; the angle at the center O is measured by the whole arch BEA , and is therefore double the angle at the circumference at C , which is measured by half the same arch.

Fig. 23. *Inf. 3.* If several angles are made at different points of the circumference, and all stand upon the same arch, they will all be equal; and if the arch is a semicircle, they will all be right angles.

The angles at the points qpm are all equal, because they stand upon the same arch $AEBDC$; and the angles at the points $3, 4, 5, 6$, are all 90 degrees, because they all stand upon the diameter AD , which divides the circumference into two equal parts, and therefore 180 degrees each.

Fig. 7. *Inf. 4.* In an isosceles triangle, the angles at the base are equal, that is, the angle at B is equal to the angle at C .

N.B. By the base of an isosceles triangle is meant the side which joins the two equal sides.

Inf.

In an equilateral triangle all the three angles are equal. PLATE I.

Inf. 5. If in two triangles two sides of the one be equal to two sides of the other, each to each respectively; and if the angle formed by those two sides be also equal, their bases and the other two angles will likewise be equal. Fig. 24.

In the triangles ABC , and DEF , let the side AC be equal to DF , the side AB equal to DE , and the angle at A equal to the angle at D ; then the angle at E will be equal to the angle at B , and the angle at F equal to the angle at C , and the side FE equal to the side CB ; for let each be inscribed in a circle; because the angles at D and A are equal, the arches on which they stand will be equal, of which FE and CB are the chords, and therefore they must be equal; and the chords DF and AC , being equal, the angles E and B must be equal; and the same may be said of the angles at F and C .

This demonstration supposes the circles to be equal, which may be thus proved: The arch subtended by the chord BC , and that subtended by the chord EF , contain the same number of degrees, because the angles at A and D are equal; the other two arches subtended by the chords AB and AC , will contain as many degrees as those subtended by the chords DE and DF ; but these chords are equal; therefore, the arches, and of consequence the circles, are equal.

Inf. 6. If in two triangles two angles of the one be equal to two angles of the other, each to each respectively, and a side of each opposite to the same angle be also equal, the triangles will be equal in all respects; and if all the respective sides of any two triangles are equal, their angles, and consequently all their parts, will be equal.

P R O P. VII. THEOREM.

If between two parallel lines AB and CD , any two lines, as *Fig. 1.* lm , rs , are parallel to each other, those lines will be equal, and the distances between the points of intersection rl , fm , will likewise be equal.

To demonstrate this, draw the perpendiculars ln , rt , which (by *Def. 7.*) will be equal; the angles at n and t being both right, are likewise equal; the angles at f and m are also equal (by *Inf. 1. Prop. 3.*) therefore the triangles rst , and lmn , will be equal in every respect, (by *Inf. 6. of the preceding*) the sides,

PLATE therefore, lm and rs will be equal; and if to the equal sides

I. mn, st , be added the line ns, sm and tn will be equal; but Fig. 25. tn is equal to lr ; therefore lr and sm will be equal.

P R O P. VIII. THEOREM.

In any triangle as ABC , if the side AB be produced to D , and AD is double of AB ; if the line DE is drawn parallel to BC , then will AE be equal to twice AC , and DE equal to twice BC .

To demonstrate this, draw the line CF parallel to AD ; then CF will be equal to BD (*by Prop. 7.*) $= AB$ by supposition; the angle CFE is equal to the angle ABC ; for the angle at D is equal to either of them (*by Inf. 1. Prop. 3.*) and the angle at A is equal to that at C ; therefore the triangles ABC and CFE will be equal in all respects, *viz.* $CE = AC$, $FE = BC = DF$, and $CF = AB$.

After the same manner it may be proved, that if AD contain AB any number of times, AE will contain AC , and DE will contain BC the same number of times.

Inf. 1. In any triangle, as ABC , if a line DE is drawn parallel to any side, as BC ; then AB will be to AD as AC is to AE , and as BC to DE .

To demonstrate this, let the line AD be $\frac{3}{4}$ of AB , then DE will be $\frac{3}{4}$ of BC , and AE $\frac{3}{4}$ of AC .

Divide the line AB into four equal parts in the points 1, 2, D , through which draw lines parallel to AC to meet the line CB in the points f, e, b ; through these draw lines parallel to AB to meet the line AC in the points $E, 7, 8$; these lines will form 10 equal triangles; for the lines $E, f, 7, e, 8, b$, being parallels, the angles $B, b, D, b, e, c, e, f, a, f, C, E$, will be equal, and the angles $C, f, e, f, e, a, e, b, c, b, B, D$, will likewise be equal; the bases DB, cb, ae, Ef , being between the same parallels, are also equal; therefore the triangles are equal in all respects, and C, f, f, e, e, b , and b, B , will be all equal; but C, f is equal to $E, a = a, c = c, D$; now CB contains C, f four times, and DE contains its equal E, a only three times; therefore DE is $\frac{3}{4}$ of BC ; again, AC contains C, E four times, and AE contains its equal $E, 7$ three times; therefore AE is $\frac{3}{4}$ of AC .

To illustrate this by numbers, let the side AC be 40, the side
AB

A B 32, and the side B C 52, and let the line D E be drawn PLATE parallel to B C; if any one of the sides of the triangle E A D I. be given, the other two may be found by the rule of three. Fig. 25.
For A D (being $\frac{3}{4}$ of A B) = 24 by supposition, it will be

$$\begin{aligned} & \left\{ \begin{array}{l} A B : A D :: A C : A E \\ 32 : 24 :: 40 : 30 \end{array} \right. \\ & \left\{ \begin{array}{l} A B : A D :: B C : D E \\ 32 : 24 :: 52 : 39 \end{array} \right. \end{aligned}$$

If the side D E (= $\frac{3}{4}$ B C) = 39 by supposition be given,

$$\begin{aligned} & \text{Then } \left\{ \begin{array}{l} B C : B A :: D E : D A \\ 52 : 32 :: 39 : 24 \end{array} \right. \\ & \text{And } \left\{ \begin{array}{l} B C : C A :: D E : E A \\ 52 : 40 :: 39 : 30 \end{array} \right. \end{aligned}$$

If the side A E (= $\frac{3}{4}$ A C) = 30 by supposition be given

$$\begin{aligned} & \text{Then } \left\{ \begin{array}{l} A C : A B :: A E : A D \\ 40 : 32 :: 30 : 24 \end{array} \right. \\ & \text{And } \left\{ \begin{array}{l} A C : C B :: A E : E D \\ 40 : 52 :: 30 : 39 \end{array} \right. \end{aligned}$$

Inf. 2. As the radius A D is to the radius A B, so is the chord D E to the chord B C; for the lines D E and B C are parallel, Fig. 26. which may be thus proved.

In the triangle A B C let the angle at A be 40 degrees, then the angles at B and C will be 70 degrees each; and as the angle at A is common to both triangles, the angles at D and E will likewise be 70 degrees each; therefore the lines B C and D E will make equal angles with the line A B, and consequently will be parallel.

Hence, as the radius of any circle is to the radius of another circle, so is a chord of the first circle to the chord of the same number of degrees of the second circle; and if two triangles are similar, that is, the angles of one equal to those of the other respectively; then the sides about or opposite to the equal angles will be proportional; for being inscribed in a circle, their respective sides will be chords of arches of an equal number of degrees.

PROP.

P R O P. IX. PROBLEM.

PLATE To make a triangle whose sides shall be three given lines (of which any two put together must be greater than the third.)

I. Fig. 27. Let the given lines be $A B$, $C D$, $E F$.

Make the line $G H$ equal to any of the given lines, suppose $E F$; then take either of the other two lines, as $C D$, and from the point G , as a center with that extent, describe an arch, and with the extent $A B$ from the point H , as a center, describe another arch to cut the former in L ; draw the lines $G L$ and $H L$ and the triangle $G H L$ will be the triangle required.

P R O P. X. PROBLEM.

Fig. 6. To make an equilateral triangle upon any given line, as $A B$.

Take the line $A B$ in the compasses, with that extent describe an arch from the point A as a center, and with the same extent from the point B , as a center, describe another arch to cut the former, in the point C ; draw the lines $A C$, and $B C$; then will the triangle $A B C$ be equilateral, and each side equal to the given line $A B$.

The demonstration of both these propositions is manifest from the construction of the triangles.

P R O P. XI. PROBLEM.

Fig. 28. To raise a perpendicular from any given point D , of a given line $A B$.

Take any point C , at pleasure; from this as a center describe a circle to pass through the point D , and to cut the line $A B$ in E ; through the center C , from E , draw a line to cut the arch in F , and $E F$, will be the diameter of the semicircle $E D F$; draw the line $D F$ and it will be the perpendicular required.

P R O P. XII. PROBLEM.

From any given point F to let fall a perpendicular to a given line, as $A B$.

Draw a line at pleasure from F to cut the line $A B$ any where, as at E ; bisect the line $F E$; upon C , the middle of the line, as a center, with the extent $C E = C F$, describe an arch to cut the line

line A B in D; draw the line F D and it will be the perpendicular required. PLATE
1.

This is only the reverse of *Prop. 11.*

All that is necessary to illustrate these two propositions is to prove that in the triangle E D F the angle at D is a right one; this is evident from *Inf. 3. Prop. 6.* for the line E F being a diameter, and the angle at D being at the circumference, it is therefore a right one.

In practice there is no occasion to describe the whole circle; it will suffice to intersect the line A B in E, and draw a small arch, as *lg*, opposite to the points E and C.

When the point D is not near the end of the line, the perpendicular may be raised thus; from D set off two points L M equally remote from D; from M as a center, and with any convenient extent of the compasses exceeding M D, describe an arch, and with the same extent from L, as a center, describe another arch to cut the former in F; the line F D being drawn, will be the perpendicular required.

But if the point F be given; from F, as a center, describe an arch to cut the line A B in L and M; from which points, as centers, describe two arches to intersect each other in N; draw the line F N, which will cut the line B A in D; then will F D be perpendicular to A B.

P R O P. XIII. PROBLEM.

To divide any given right line (A B) into two equal parts. Fig. 29.

From the points A and B, as centers, describe two arches to intersect each other in D; and with the same extent describe two arches to intersect each other in C; the line D C being drawn will cut the line A B into two equal parts in G.

P R O P. XIV. PROBLEM.

Thro' any three given points, as A, B, D, not lying in a right line, to draw a circle. Fig. 30.

Join the points A B, B D with right lines; divide those lines into two equal parts; draw the perpendiculars F m, G n, and let them be produced till they intersect each other; the point of intersection C will be the center of the circle required; for as the circumference is to pass thro' the points A, B, D, the lines A B, B D must

PLATE must be chords; and if any two chords are bisected by two perpendiculars, they will, if produced, meet at the center, by *Inf.*
 I. 2. *Prop.* 4.

By this problem it is evident, that if any three points are taken in the circumference of a given circle, the center may be easily found; and if the segment of any circle is given, the circle may be completed, by taking any three points in the given segment, and then proceeding as before.

P R O P. XV. PROBLEM.

Fig. 31. Thro' a given point F to draw a line parallel to a given line A B.

Draw the line F G at pleasure; at the point F make the angle D F G equal to F G B, the line D F being drawn, will be parallel to the line A B; because the alternate angles are equal, by *Prop.* 3.

This may be done without drawing the line F G, thus; open the compasses till with one foot fixed in F, the other may cut the line any where, as in B; then with the same extent, from any other point in the line, as G, describe an arch; and with the extent G B from F, as a center, describe an arch to cut the former in D; the line F D will be the parallel required.

D E M O N S T R A T I O N.

In the triangles D G F and B G F, the side G B is = D F, and F B = D G, and F G common to both. Therefore the triangles are similar, and the alternate angles G F B and F G D are equal; and therefore the lines G B and D F are parallels, by *Prop.* 3.

P R O P. XVI. PROBLEM.

Fig. 32. To find a line which shall be a fourth proportional to three given lines as A B, C D, E F.

Upon the line L M take L g = A B, and draw the line g b, making any angle at pleasure with the line L M, and make g b = D C. Set off E F from L to N, and from the point N draw a line parallel to b g; thro' the point b draw the line L O, then will O N be the fourth proportional required.

For

For in the triangle LNO , gb is parallel to NO by construc- PLATE
 tion, therefore $Lg (= AB) : gb (= CD) :: LN (= EF) : L$
 $N O$. (by *Inf.* 1. *Prop.* 8.).

If it was required to find a third proportional to two given lines
 (as AB and CD), proceed in the same manner, only take $LI =$
 gb , and draw the line IK parallel to gb , then it will be

$Lg : gb :: LI : IK$ the third proportional required.

P R O P. XVII. PROBLEM.

To find a mean proportional betwixt two given lines as AB , Fig. 33
 CD .

Upon the line LM set off $LN = AB$ and $NO = CD$, bisect the line LO in C , upon which, as a center, with the extent $CO = CL$, describe a semicircle from the point N , &c.

From the point N erect a perpendicular, this will intersect the semicircle in the point E ; the line EN being drawn, will be a proportional to the lines $LN = AB$, and $NO = CD$.

D E M O N S T R A T I O N.

The triangles LEO and LEN are similar, for the angle LEO is a right one (by *Inf.* 3. *Prop.* 6), the angle $LE N$ is also right by construction; and the angle at L is common to both, therefore the angle $LE N$ is equal to the angle at O , the triangles $LE N$ and ENO are also similar, for they have each a right angle at N , and the angle at O in the one is equal to the angle $LE N$ in the other; therefore the angle at L will be equal to the angle NEO ; therefore $LN : NE :: NE : NO$ by *Inf.* 2. *Prop.* 8.

P R O P. XVIII. THEOREM.

The radius of any circle, is equal to the chord of 60 degrees of the same circle.

D E M O N S T R A T I O N.

Upon the radius AC make an equilateral triangle ABC , then Fig. 34
 (by *Inf.* 4. *Prop.* 6.) the angles at A , B , and C will be all equal,
 consequently 60 degrees each, but AB is a chord of the arch that mea-

F

mea-

PLATE measures the angle at C, which is 60 degrees; therefore the chord

I. A B of 60 degrees, is equal to the radius A C or B C.

Fig. 35.

P R O P. XIX. THEOREM.

Parallelograms or triangles, having the same or equal bases, and being between the same parallels, are equal.

D E M O N S T R A T I O N.

Let the parallelograms be A B C D, and C E D F.
Fig. 35. Because the sides C E and D F are equal, and parallel by construction; the angles A E C and B F D are equal (by *Prop. 3.*), and because A B and E F are also equal, if B E is added to each, then A E must be equal to B F, and as A C is equal to B D, therefore the triangles A E C and B F D are equal in every respect, and if the common triangle B E O is taken away from each, the planes A B C O and E F D O will remain equal; to each of which planes, if we add the triangle C O D, the parallelograms A B C D, and C E D F become equal.

Hence it appears that any two triangles (as C D A, C D E) standing upon the same base (C D), and between the same parallels (A F, C G) are equal; for they are halves of parallelograms of the same base and altitude (as A B C D and C D E F).

It is evident from this proposition, that we may make a right angled triangle equal to any given oblique one, for if we draw a line, as A F parallel, to either of the given sides, as C D, through the opposite angular point E, and erect a perpendicular at either end of the same side C or D, to meet the parallel in A or B; then will the right angled triangle C B D = C A D, be equal to the oblique one C D E.

Fig. 10. It is likewise evident from hence, that the rhombus E F G H,
12. is equal to the rectangle E F x z, and that the rhomboides N O P Q, is equal to the rectangle N n P p; and consequently that, by the same method we may make a rectangle equal to any given parallelogram.

P R O P. XX. THEOREM.

Fig. 36. In a right angled triangle A B C, the square of the longest side A C (called the hypotenuse) is equal to the squares of both the

the other sides AB , BC (called the base and perpendicular) taken together.

DEMONSTRATION.

The square $ACHI$, whose side is the hypotenuse AC , is equal to the rectangles $AHFG$, and $FGIC$ taken together; therefore all that we have to prove is, that the square $BCDE$ is equal to the rectangle $FGIC$, and the square $ABKL$ to the rectangle $AHFG$.

First it is evident, that the triangle BCI is one half of the rectangle $FGIC$, it is also equal to the triangle BCD , because it stands upon the same base BC , and between the same parallels BC , ED ; now the triangle $BCD = BCI$, is one half of the square $BCDE$, therefore the whole square $BCDE$, is equal to the whole rectangle $FGIC$, each being double the equal triangles BCI , BCD .

In like manner, the triangle $ABH =$ half the rectangle $AHFG$, standing upon the same base AB and between the same parallels AB , LH , is equal to the triangle AKB , which is one half of the square $ABKL$; therefore the square $ABKL$, and the rectangle $AHFG$ being double of the equal triangles ABK , ABH , are equal betwixt themselves.

C H A P. IV.

Of the Construction and Mensuration of Geometrical Figures.

HAVING explained the principles of geometry, we come now to shew their use in the construction, and mensuration of all plain figures, which we shall reduce to three classes, *viz.*

1. Those that are limited by three right lines, called triangles.
2. Those that are limited by four right lines, called quadrilaterals.
3. Those that are limited by many right lines, called polygons.

Mathematicians suppose all plain figures that inclose any part of space, to be comprehended under one of these denominations, for curvilinear figures are supposed to be limited by an infinite number of right lines; we shall begin with triangles, as no figure or space bounded by right lines, can have fewer than three sides.

S E C T. I.

Rectilineal Trigonometry, or the Construction of Right lined Triangles.

TRIGONOMETRY is that science by which we learn to measure the sides and angles of triangles, and is one of the most useful branches of the mathematicks; for as every triangle consists of six parts, *viz.* Three sides and three angles, and it often happens that some of them are unknown, yet if any three of them, provided one be a side, be known, the other three may be found by this art, either geometrically by scale and compasses, or by an arithmetical calculation. It is either rectilineal or spherical, we shall only treat of the first.

Fig. 5. In order to delineate any triangle, it is absolutely necessary to have a scale of equal parts to measure the sides by, and a line of chords for measuring the angles, these and the lines of sines, tangents, and secants, and several others, are laid down upon the plain scale, we shall first explain these terms, and then shew how to construct the lines.

DE-

DEFINITIONS.

1. The **RIGHT SINE** of any arch, is a perpendicular line drawn from one end of it, to a radius or diameter drawn to the other end; AD is the right sine of the arch AB , or it is half the chord of double that arch; for taking the arch BJ equal to AB , AJ will be the chord of the arch ABJ ; of which the sine AD is one half. Fig. 37.

2. The **SINE COMPLEMENT** of an arch, is that part of the radius intercepted betwixt the right sine and the center; thus, CD is the sine-complement of the arch AB , for it is equal to FA ; the right sine of the arch EA , which is the complement of the arch AB .

3. The **VERSED SINE**, is that part of the radius, intercepted between the right sine and the circumference; DB is the versed sine of the arch AB .

4. A **TANGENT** to a circle, is any right line so drawn as to touch the circumference in any point (B), and if to this point be drawn a radius, it will be perpendicular to the tangent; thus BM is a tangent to the circle in the point B , and the tangent of any arch, as AB , is that part of the line BM , intercepted between the point B ; at that end of the arch to which the radius is drawn, and the point H , where a line drawn from the center C , thro' A at the other end of the arch, intersects the tangent line BM ; therefore HB is the tangent of the arch AB .

5. The **SECANT** of an arch, is a streight line drawn from the center thro' one end of the arch, produced till it meet a tangent drawn from the point B at the other end of the arch; so HC is the secant of the arch AB .

The sine, tangent, and secant of the complement of an arch, are called the sine-complement, tangent-complement, secant-complement, or co-sine, co-tangent, co-secant; CR is the co-secant; ER is the co-tangent; AF the co-sine of the arch AB , and the sine, tangent, and secant of the supplement of an arch, are the same with those of the arch, for being drawn according to the definitions, there results the same line.

Hence the sine of 90 degrees is equal to the radius, for it is half the diameter by the definition of a sine, and is the greatest of all the sines; the sine of 30 degrees is half the radius, for it is half the chord of 60 degrees, which was proved to be equal to the

- PLATE II. the radius; the tangent of 45 degrees is equal to the radius, for let the angle NCE , or NCK be 45 degrees, NK the tangent, and EC the radius, are parallel, being both perpendicular to the diameter KB ; but NE and KC are also parallel, being both perpendicular to the diameter EC ; and therefore the tangents NK and NE are equal to the radius.

Construction of the Lines on the plain Scale.

I. THE LINE OF EQUAL PARTS.

- Fig. 1. This may be made of any length, as AB , and divided first into ten equal parts distinguished by the figures 1, 2, 3, &c. Then each of those divisions must be subdivided into 10 equal parts, so the whole line will be divided into 100 equal parts; the figures 1, 2, 3, &c. denoting 10, 20, 30, &c. of those parts, there will be no necessity for numbering the small divisions betwixt the figures; but for the better illustration of the scale, we shall distinguish those between A and 1, by the letters of the alphabet. Any number less than 100 may be readily found on this line, for if it be less than 10, count as many small divisions beyond the beginning of the line or point A , as there are units in the number, as, if 6 were required, it will be at the point f ; if 60, at the figure 6; if 66, count 6 small divisions beyond the figure 6, and you will have the point required.

If the number exceed 100, the small divisions must be subdivided again each into 10, but the length of our line will not admit of these; we shall therefore make use of diagonals, by which means any number under 10,000 may be had with as great certainty, as if the line AB were actually divided into 10,000 equal parts.

To perform this, let there be 50 lines at equal distances from one another, drawn parallel to AB , and having compleated the parallelogram $ABCD$, let the lower line CD be divided exactly like the upper line AB , into 100 equal parts, and numbered 1, 2, 3, &c. and draw the diagonal Aa .

Now the portions of the parallels, 10, 20, 30, &c. intercepted betwixt this diagonal and the perpendicular AC , will be 1 tenth, 2 tenths, 3 tenths, &c. of the line Ca ; for in the triangle ACa , let Ca be the base, then the portions of the parallels 10, 20, &c. will be bases of as many triangles, similar to the triangle ACa ; therefore $AC : Ca :: A_{10}$: the portion of the parallel 10, inter-

intercepted betwixt the diagonal Aa and perpendicular AC , but PLATE
 $A10$ is the tenth part of AC by construction, therefore the por- II.
tion of the parallel 10 , is the tenth part of Ca , or the thou- Fig. 1.
sandth part of the whole line CD or AB ; the figures $1, 2, 3,$
 $\&c.$ denote $100, 200, 300, \&c.$ and $a, b, c, 10, 20, 30, \&c.$ of
those parts.

Again, if there were 9 parallels drawn betwixt the line A B and the parallel 10, it is plain the portion of that next to the line A B, intercepted betwixt the diagonal C a, and perpendicular A C, would be the tenth part of the portion of the parallel 10, intercepted betwixt the same lines, or the 10000th part of the whole line C D or A B. In the plate we have only four intermediate parallels numbered 2, 4, 6, 8; so their portions intercepted betwixt the diagonal C a, and perpendicular A C, will be 2, 4, 6, 8, of those parts.

By these means the lines A B and C D, are in effect, divided into 10,000 equal parts; the figures 1, 2, 3, &c. denote 1000, 2000, 3000, &c. the small divisions on the lines A B and C D, betwixt those figures will be hundreds; the tens are not transferred to the lines A B and C D, but may be had at the intersections of the parallels 10, 20, 30, &c. with the diagonals. If the units Fig. 6. are even, they may be found at the intersection of the diagonals, 7-8. and the intermediate parallels, but if they are odd, as 1, 3, 5, &c. half the distance in the diagonal, must be taken between those parallels. To compleat the scale, there must be diagonals drawn from all the divisions in the line A B to the line C D, parallel to that drawn from the point A in the line A B, to the point *a* in the line C D.

It will be very easy to find any number under 10,000 upon this scale, as in the following example, *viz.*

For units, look for the parallel below the line A B, betwixt the point A and the parallel 10; thus, to find 6, look for the third parallel, its intersection with the diagonal A a (which is the first of all the diagonals) will be the point required; this parallel in Fig. 9. the plate is numbered 6, but the odd numbers must be found betwixt the parallels; for instance, the number 7 will be in the same diagonal A a, between the parallel 6 and the parallel 8.

If 10, 20, 30, &c. be required, look for the intersections of Fig. 10. the parallels so numbered, with the first diagonal A a, and you will have the point required. If the given number is 12, first, and

PLATE

II.

Fig. 1.

find 10, and the intersection of the parallel next below that, with the same diagonal will be 12; the number 25 will be in the middle of that part of the diagonal betwixt the second and third parallel below the parallel 20; so that if the number be less than 100, it will be in the first diagonal; if it exceeds 100, and is less than 200, it will be in the second diagonal; and if less than 1000, it will be in some of the diagonals, drawn from the several points *a, b, c, d, &c.* between A and 1, in the line A B to the line C D, thus the number 600 will be the sixth small division from A towards 1; but if 606 is required, it will be found at the intersection of the third parallel (numbered 6) with the diagonal *fg*; and where the same diagonal intersects the parallel, 60 will be the point for 660; and the middle of that part of the same diagonal, betwixt the third and fourth parallel below that of 60, will be the point for 667.

The number 6000 will be at Fig. 6. in the line A B; 6700 at the seventh small division beyond Fig. 6. in the same line; the number 6750 will be at the point where the diagonal drawn from the point 6700 in the line A B, intersects the parallel 50; and where the same diagonal cuts the third parallel below the parallel 50, will be the point for 6756.

The diagonals upon *Gunter's* scales, consist only of eleven parallels; the upper and lower are graduated, the one into inches, numbered 1, 2, 3, &c. to 9; the other into half inches, numbered 1, 2, 3, &c. to 18; so there are no small divisions betwixt the figures, but there is an inch before the beginning of one line, and half an inch before the beginning of the other line, divided into ten equal parts in the upper and lower lines; and diagonals drawn as in that, in the plate, which will divide the inch and half inch each into 100 equal parts; so that if the spaces betwixt the figures be 100, those betwixt the diagonals will be tens, and the units will be at the intersection of the diagonals and intermediate parallels; one example will suffice to shew how to take off any number by this scale: suppose 748, first find the figure 7, and observe where the perpendicular, from that point, intersects the eighth parallel from it, there fix one foot of the compasses, extend the other to the intersection of that parallel with the fourth diagonal, and you will have the extent for 748.

There are several other lines of equal parts which are graduated, and numbered exactly as the upper or lower line of the diagonal

gonal scale, and the divisions and subdivisions are in tens; but as PLATE II. one inch is the twelfth part of a foot, the diagonal scales for feet and inches, consist of seven parallels, the upper and lower lines Fig. 4. are numbered as in the other diagonals; the spaces betwixt the figures are to represent feet, which suppose one inch in the upper line *ST*, and half an inch in the lower line *NO*: They are numbered 1, 2, &c. the one contrary to the other, as upon most of the shipwrights rules. Set off half an inch to the right from *O*, at the beginning of the lower line to *o*; and one inch to the left from *S*, at the beginning of the upper line to *m*; then draw two diagonals, to be distant from one another one inch on the upper line, and to meet in the lower line at the middle of the inch; and at the other end draw two diagonals, to be at the distance of half an inch from one another in the lower, and to meet at the middle of the half inch in the upper line. It is evident the interfections of these diagonals will represent inches.

The other lines on the scale are taken from the equal divisions Fig. 5. of a circle, and are constructed in the following manner, *viz.*

1. With the radius you intend for your scale at *C* as a center, describe a semicircle, *ADB*; then upon the center *C* erect the perpendicular *CS*, which will divide the semicircle into two quadrants *AD*, *DB*, and draw the right lines *AD*, *DB*.

2. Divide the quadrant *BD* into 9 equal parts; each of those divisions will contain 10 degrees, as a quadrant or fourth part of a circle contains 90 degrees; place one foot of the compasses in *B*, and with the other transfer the several divisions from the circumference to the right line *BD*, then will *BD* be a line of **CHORDS**.

3. Upon the point *B* erect the perpendicular *BT*; then draw lines from the center *C* through the several divisions 10, 20, 30, &c. of the arch *BD* to meet the line *BT*, and number the several points of intersection 10, 20, 30, &c. the same as the arches; then will *BT* be a line of **TANGENTS**.

4. From the points 10, 20, 30, &c. in the arch *BD*, let fall perpendiculars to the radius *CB*; those several perpendiculars will Fig. 30. be the sines of those arches, and will divide the radius *CB* into a line of **SINES**, which must be numbered 10, 20, &c. from *C* towards *B*; for since the perpendicular let fall from the point 80, to the radius *CB*, is the sine of 80 degrees; *C 10* will be the sine of 10 degrees (by *Def. 2.* of this.)

PLATE

II.

Fig. 5.

5. If the same line be numbered from B towards C, it will become a line of *VERSED SINES*.

6. With one foot of the compasses in C, extend the other to the several divisions of the tangent line B T, and transfer those extents to the line C S; then will C S be a line of *SECANTS*.

As the shortest secant that can be drawn, is longer than the radius, and the longest sine is equal to the radius; the line of secants on the scale, is on the same line with the sines.

7. As the angle, which the diameter A B makes with any line drawn from the point A, to any point in the circumference, is one half of the angle, that the radius B C makes with a line drawn from the center C to the same point (by *Prop. 6. Chap. 3.*). Right lines drawn from the point A to the several points 10, 20, &c. in the arch B D, will divide the radius C D into a line of *SEMI-TANGENTS*, or tangents of half those arches.

8. Divide the quadrant A D into 8 equal parts, and with one foot of your compasses in A, transfer the several divisions to the right line A D, then will A D be a line of *rumbs* hereafter to be spoken of.

Fig. 6.

All these lines are transferred from the semicircle to the scale where they are drawn parallel to one another, and distinguished by the initial letters of their proper names, being what is called a plain scale.

The line of equal parts, and the line of chords, will be sufficient for the construction of any rectilineal triangle, but we think it necessary here to shew the uses of the other lines.

P R O B. I.

PLATE

I.

Fig. 39.

To make or measure any Angle by the Line of Chords

1. From the point C, let it be required to draw a line that shall make an angle of 40 degrees with the line C G.

With the extent of 60 degrees taken from the line of chords (which is always equal to the radius by *Prop. 18. Chap. 3.*) describe an arch; then take the extent of 40 degrees from the same line, and set it off upon the arch from A to D, and the angle A C D will be 40 degrees.

2. When an angle is given to be measured with the same extent, *viz.* 60 degrees of chords; from the angular point describe an arch as before, to intersect the lines that form the angle; the length

length of the chord of that arch intercepted between the lines, PLATE I.
 measured on the line of chords, will give the number of degrees Fig. 37.
 that the angle contains.

If the given angle is greater than 90 degrees, it may be made or measured at twice. Thus, suppose the angle 130 degrees; first take the chord of 90, and set it off from K to E, then take the chord of 40 degrees, (the remainder) and set it off from E to A, then will the angle A C K be 130 degrees: Any other two chords whose sum is 130 degrees, would have answered the same purpose. But as every semicircle contains 180 degrees; when the given angle exceeds 90 degrees, it may be very readily made, by setting off its complement to 180 degrees, from the extremity of the diameter: Thus, if we set off 50 degrees of chords, from B to A, and draw the line A C, we have the angle A C K = 130 degrees as before. When such an angle is given to be measured, if that extent is taken, and subtracted from 180 degrees, the remainder will be the quantity of the arch.

P R O B. II.

To find the Sine, Tangent, or Secant of any Arch; the quantity of the Arch, and the length of the Radius being given.

Fig. 38.

Let the angle be 40 degrees, and the radius 100.

Now as this is the length of the radius, by which those lines on the scale were constructed; take 40 from the line of sines, that extent measured on the line of equal parts, into which the radius is divided, will give the length in numbers, of the sine of 40 degrees; and by the same method, we have the tangents and secants of any arch; but when the radius is not the same, observe the following method.

Suppose the angle 40 degrees, as before, and the radius 88 inches.

Make D C A an angle of 40 degrees by the preceding, and produce the lines C D and C A; then take 88 from any scale of equal parts, and with that extent from the angular point C, describe an arch, as L B; from L let fall the perpendicular L M, which will be the sine; the perpendicular B T will be the tangent, and C T the secant of the arch L B; each of which measured upon the same scale of equal parts as the radius was taken from, will give the sine, tangent, and secant of the arch in numbers.

P R O B. III.

PLATE

I. *The Length of the Sine, Tangent, or Secant; and the Quantity of an Arch given to find the Length of the Radius.*
Fig. 38.

Let the angle be 40 degrees as before.

If the sine is given; at any point as M, of a right line, as CN; erect a perpendicular, and make ML equal to the given sine; then at L make an angle of 50 degrees, the complement of the given angle, and draw the line LC, which will be the radius required.

If the tangent is given, make BT equal to it, and make the complement angle 50 at T; so the line BC will be the radius.

If the secant is given, make CT equal to it, and make an angle of 40 degrees at C, and one of 50 at T, the line TB will intersect the line CN in B; then will CB be the radius required.

P R O B. IV.

The Length of the Sine, Tangent, or Secant; and the Length of the Radius given; to find the Quantity of an Arch.

Let the radius be 88.

If the sine is given, at any point, as M, of a right line, as CN, erect a perpendicular, make ML equal to the given sine; with the given radius 88, from L, as a center, describe an arch to intersect the line CN in C, and draw the line CL; then will LCM be the angle required.

If the tangent is given, erect a perpendicular at any point, as B, of the line CN; make TB equal to the given tangent; from B, as a center with the given radius, describe an arch to intersect the line CN in C; draw the line CT; then will TCB be the angle required.

If the secant is given, set off the given radius from C to B, at which point erect a perpendicular; then with the extent of the given secant, from C as a center, describe an arch to intersect the perpendicular in T; draw the line CT; then will TCB be the angle required.

DEMONSTRATION.

Draw the perpendiculars DE and GA, the one will be the
sine,

fine, and the other the tangent of the arch A D, (by *Def. 1* and 4. PLATE 1.) and C G the secant of the same arch; for C A = C D. 1.
is the radius of the circle from which those lines are constructed, Fig. 33.

CD : DE :: CL : LM, CA : AG :: CB : TB, and

CA : CG :: CB : CT by *Prop. 8. Chap. 3.*

But C D and C A, are radii of the arch A D; and D E the fine, G A the tangent, and G C the secant of the same arch; therefore L M will be the fine, T B the tangent, and C T the secant of the arch B L, of which C B is the radius.

PROB. V.

To construct a right lined Triangle, three things being known, one of which must always be a Side.

This will admit of four different Cases.

1st. When the three sides are given, the triangle may be constructed, as already shewn *Chap. 3. Prop. 9.*

2^d. Given two sides, (C A and C F) and the angle (C) included by those sides, to delineate a triangle (A F C).

First make the given angle at C; then set off C A and C F the given sides from the angular point, and draw the line A F. Fig. 49.

3^d. Given two sides A F and C F, and the angle (C) opposite to one of them.

Make the angle; as before, then from the angular point C set off one side C F, from F as a center; with the extent F A, the other given side, describe an arch to intersect the line C G, which will be in A or B, and C F A or C F B, will be the triangle required. So that as this case will admit of two solutions; we must know whether the angle opposite to the other given side is acute or obtuse, before the answer can be determined; for if obtuse, the least, if acute, the greatest will be the required triangle.

4th. Given one side (A C), and two angles (A and C).

Draw the given side A C; make one of the given angles at A, and the other at C, and produce the lines that form those angles, till they meet in F; then will A F C be the triangle required.

After the triangles are thus constructed, the unknown sides may be measured by the same line of equal parts, by which they were constructed, and the angles by the same line of chords; and although all triangles may be measured and delineated, as we have shewn in the solution of this problem, yet it is usual to divide them

PLATE

I. them into two classes, *viz.* right and oblique angled triangles, from whence arises the division of trigonometry, into two parts, *viz.* right and oblique; the last of which contains only the foregoing four cases.

P R O B. VI.

To construct a right angled Triangle.

Fig. 42. This will admit of six cases, occasioned by the different names, by which the sides are distinguished, *viz.* The hypotenuse, which is the side opposite to the right angle, as (AC). The perpendicular (CF) is one side, and the base (AF) the other side, meeting at the right angle. To distinguish one from the other, we shall draw the perpendicular up and down, and the base across the paper; the angle opposite to the base we shall call (simply) the angle, and that opposite to the perpendicular we shall call the complement angle.

Hence, as in a right angled triangle, one angle is always the same, being equal to 90 degrees; it may be said to consist of four variable parts, *viz.* the three sides, and an oblique angle; any two of which being given, by them the triangle may be constructed.

Fig. 41. *Case 1.* Given the hypotenuse AC, and the angle at C 50 degrees, to find the base and perpendicular.

Draw the line CG up and down the paper; then with 60 degrees of chords from the point C, describe an arch; make the angle at C 50 degrees, and set off the given hypotenuse from C to A; from the point A let fall the perpendicular AF, then will AF be the base, and CF the perpendicular of the triangle AFC, right angled at F.

Hence, if the hypotenuse of a right angled triangle be made the radius of a circle, by which the angles are to be measured, the base and perpendicular will become sines of their opposite angles; AF is the sine of the angle at C, and CF the sine of the angle at A, by the definition of a sine; and because the sine of an angle is the same with the sine of that arch, which is the measure of the angle; its length cannot be determined till the length of the radius is known; for which reason the same angle may have sines, differing infinitely in length from one another in proportion to the radii by which they are measured; so that this case

is

is the very same with *Prob. 2.* of this; for making the hypothe- PLATE 1
nuse the radius of a circle, we can find the sine of any arch in I.
the same manner as we have shewn in that problem.

Case 2. Given the base AF , and the angle at C 40 degrees, Fig. 42.
required the perpendicular CF , and the hypothenuse AC .

This may be done by *Prob. 2.* of this; for if the base of a right angled triangle be made the radius of a circle, the hypothenuse will be the secant, and the perpendicular the tangent of the angle at the base A , which being the complement of the given angle will be 50 degrees; this is also the same as *Case 4. Prob. 5.* of this for here is one side, and the angles given. Therefore draw the line AF across the paper, and make a right angle at F ; from F to A set off the given base at A ; make an angle of 50 degrees, and draw the line AC ; then will ACF be the triangle required.

Case 3. Given the perpendicular CF , and the angle at C 50 Fig. 43.
degrees.

This in effect, is the same with the preceding, and may be constructed as in *Prob. 2.* or *Case 4. Prob. 5.* of this; for if the perpendicular be made the radius of a circle, the base will be the tangent, and the hypothenuse the secant of the angle C . Therefore having set off the given perpendicular CF , make a right angle at F ; at C make the given angle 50 degrees; draw the line AC ; then will ACF be the triangle required.

Case 4. Given the base AF , and the hypothenuse AC , to find the perpendicular and the angles.

This and the two following cases may be done as in *Prob. 4.* of this; for either of the given sides may be made the radius of a circle, and the other sides will be sines, tangents, or secants; they may likewise be done as in *Case 2.* or *3.* of *Prob. 5.* of this; for we have two sides, and either the included angle, or the angle opposite to one of the sides given; in this case the opposite angle is given. Therefore draw a line across the paper, and set off the given base from A to F ; at the point F erect a perpendicular, then take the given hypothenuse AC with a pair of compasses, and from the point A , as a center, describe an arch to intersect the perpendicular in C ; draw the line AC , and the triangle will be constructed.

Case 5. Given the perpendicular CF , and the hypothenuse AC , to find the base AF , and the angles.

This is exactly the same with the preceding, only calling what was then the base, now the perpendicular; for making the right angle

PLATE angle at F as before, if we set off the perpendicular from F to C, and with the extent of the given hypotenuse A C, upon C as a center, describe an arch; it will intersect the line drawn perpendicular to C F in the point A, then will A C F be the triangle required.

I.
Fig. 43.

Case 6. Given the base A F, and the perpendicular C F, to find the hypotenuse and the angles.

Here we have two sides, and the included angle given; therefore by *Case 2. Prob. 5.* make a right angle as at F; then set off the given base from F to A, and perpendicular from F to C; draw the line A C, and the triangle will be completed.

It may now be presumed, that the reader, by a due attention to what has been said, may be able to construct any triangle when one side, and any other two parts are given; and it would be needless to give any more examples, as we shall have occasion to shew in another place, how to perform what has been done here geometrically, by arithmetical calculation: We shall therefore under this head only subjoin the following theorem, which should be well understood, and carefully attended to, *viz.*

T H E O R E M.

In a right angled triangle, any side may be made the radius of a circle, and the other sides may be found, by being considered as sines, tangents, or secants of the oblique angles.

Fig. 41. *Case 1.* If the hypotenuse A C is made radius, the base A F will be the sine, and the perpendicular C F the sine complement of the angle at C.

Fig. 42. *Case 2.* If the base A F is made radius, the hypotenuse will be the secant complement; and the perpendicular the tangent complement of the angle at C.

Fig. 43. *Case 3.* If the perpendicular C F is made radius, the hypotenuse will be the secant, and the base the tangent of the angle at C.

S E C T. II.

Construction of Quadrilateral Figures, &c.

P R O B. I.

To make a Square whose Side shall be the given Line C D. PLATE
I.

UPON one end of the line as D, erect a perpendicular; set off the given side C D from D to B; upon B and C, as centers; with the same extent, describe two arches, intersecting each other in A; and draw the lines A C, A B; then will A B C D be a geometrical square, all the sides and angles being equal. Fig. 9.

P R O B. II.

To make a Rhombus whose Side shall be the given Line E F, and the Angle E 50 Degrees.

First make the angle at E 50 degrees; make E G and E F equal; then with the extent E F, describe an arch from the center F, and another from the center G, intersecting the former in H, and draw the lines G H, H F; then will E F H G be the rhombus required. Fig. 10.

P R O B. III.

With two given Sides, as I K and K L, to make a Rectangle.

Erect a perpendicular at K, and set off the two given sides to I and L; from I, as a center, with the extent K L, describe an arch; and with the extent I K, from L as a center, describe another arch to intersect the former in M; draw the lines I M, L M, and the rectangle will be completed. Fig. 11.

P R O B. IV.

To Make a Rhomboides whose Sides N O, O Q, and the Angle at O are given.

Make the angle at O, and set off the given sides to N and Q; Fig. 12.
H with

PLATE with the radius OQ, from N as a center, describe an arch; then
 I. with the extent ON, from Q as a center, describe another arch
 Fig. 12. to intersect the former in P; draw the lines NP, PQ, and the
 rhomboides will be made; and if from the points N and P, be
 let fall perpendiculars to the points *n* and *p*; the rectangle
N n P p will be equal to the rhomboides N O P Q.

P R O B. V.

To describe a regular Polygon, having the Length and Number of Sides given.

This is in effect, the same with *Case 4. Prob. 5.* of the preceding section, for every regular polygon may be inscribed in a circle, and if there be a radius drawn to every angle of the polygon, it will contain as many triangles as it has sides; so that if 360, the number of degrees in a circle, be divided by the number of sides, the quotient will be the angle at the center; and if that is subtracted from 180 degrees, the remainder will be the sum of both the angles at the base, which is the side of the polygon; and because the triangles have two of their sides equal, the angles at the base will also be equal; so that in every triangle we have one side, and all the angles given.

Fig. 14. Let *a b* be the side of a hexagon, then $360 \div 6 = 60$, and $180 - 60 = 120$, the sum of the angles at *a* and *b*, therefore each angle will be 60 degrees: Make therefore at *a* and at *b* an angle of 60 degrees, and produce the lines till they meet; from the point of their intersection describe a circle thro' *a* and *b*; the distance from *a* to *b* will divide the circumference into six equal parts, and when the chords are drawn to these points, we shall have the hexagon required. Here the angles at the base are the same with the angle at the center; consequently a polygon of 6 sides consists of as many equilateral triangles; but as all other polygons consist of as many isosceles triangles as they have sides; and the angles at the center, and circumference may be easily found by the foregoing method; there can be no difficulty in constructing them, as there is no more required, but to find the center of a circle which shall pass thro' the extremities of the given side.

PROB. VI

To describe an Ellipsis.

An ellipsis is a figure contained under one curve line, called the periphery, but differs from a circle; for if there be two diameters drawn in a circle at right angles to one another, they must be of equal lengths, and all points in the circumference are equally distant from the center; but the ellipsis has two diameters of different lengths, at right angles to each other, TS the longest, is called the transverse diameter, and CE the shortest, the conjugate diameter; so that the circle is, as it were, flattened one way, and the radius Tx reduced to $x C = x E$; and if all the sines in the semicircle are shortened in the same proportion, they will give the points through which the semi-periphery of the ellipsis must pass: In order to perform this, make the radius, or half the transverse diameter $Tx = x E$, a side, and $x E$ half the conjugate diameter $= FG$ the base of a triangle $x F G$; from the circumference of the circumscribing circle, draw several lines, as m, s, n, t , perpendicular to the transverse diameter; transfer those sines to the side $x F$ of the triangle, viz. make $xz = nt$, $xy = ms$, &c. draw zb and yg parallel to the base FG , transfer the extent zb from n to p , and the extent yg from m to o , then will p and o in the lines nt, ms , be two points thro' which the semi-periphery must pass; and if more lines are transferred from the semicircle to the side $x F$, we may by the foregoing method find a sufficient number of points, thro' which the semi-periphery may easily be drawn.

If with the extent $Tx = Sx$, half the transverse diameter, and from the centers C or E , the extremities of the conjugate diameter, two arches are described, they will intersect the transverse diameter in the points F, f , each of which is called the focus of the ellipsis; and if from any point in the periphery a line be drawn to each of these points, the sum of those lines will always be equal to the transverse diameter, viz. $Fa + fa = Fb + fb = Fc + fc = FD + fd = Fe + fe = TS$; and this being the peculiar property of the ellipsis, any number of points may be found by the following method.

Having found the two focal points F, f , as before, with any extent of the compasses, so it be less than the transverse diameter,

PLATE as Ti , from the focus F describe an arch; and with the extent
 I. Si , the remaining part of the transverse diameter, describe ano-
 Fig. 45. ther arch to intersect the former in the point a , which will be in
 the periphery of the ellipsis; in the same manner you may find as
 many points as you please.

From what has been said it is plain, that if there be a thread
 equal in length to the transverse diameter, fastened to two pins
 one in each focus; and if a pencil, or any other convenient in-
 strument be moved round within the thread, so as to keep it at
 its full extent, it will describe the true periphery of the ellipsis;
 and if both ends of the thread be fastened at the center, it will
 describe a circle; so that this figure may properly be said to have
 two centers, whereas a circle has but one.

P R O B. VII.

To make a Circle equal to a given Ellipsis.

Find a mean proportional between the transverse, and conjugate
 diameters by *Prop. 17. Chap. 3.* and this will be the diameter of
 the circle required.

S E C T. III.

Of Mensuration of plain Surfaces.

HAVNIG in the preceding sections shewn the construc-
 tion of plain geometrical figures, and the method of re-
 ducing those that are oblique angled, to right angled ones of e-
 qual surfaces; we shall now briefly shew how to measure those
 surfaces, or the whole spaces contained within the circumscrib-
 ing sides.

These spaces are always estimated, in squares of some assigned
 dimensions, as inches, feet, yards, &c. and the number of such
 squares contained in any figure, is called its area or superficial
 content.

Now as a square has four right angles it will necessarily fol-
 low, that all figures must be reduced to right angled ones be-
 fore they can be measured geometrically, so that the whole of
 super-

superficial measure may be said to consist in finding the area of PLATE
 a square or rectangle; in order to which, let us suppose a rect- I.
 angle as $abcd$, to be one inch broad; it is plain it will con- Fig. 44-
 tain as many square inches as it is inches in length, which in
 this case is 12, and the rectangle may be cut in 12 square inches,
 but if it were any broader, and of the same length, it would contain
 12 times as many square inches as there are inches in breadth;
 so the square $ABCD$, of 12 inches long, and 12 inches broad,
 will contain 144 square inches, which is the area of a superficial
 foot, or square whose side is one foot; hence the following rules
 will appear very plain.

P R O B. I.

*To find the Area of a Square or Rectangle, the Length and Breadth
 being given.*

Rule. Multiply the length by the breadth, the product is the
 area; this will admit of two cases.

Case 1. If the length and breadth are taken by one kind of
 measure; as if there is a rectangle 48 inches long, and 24 inches
 broad; then $48 \times 24 = 1152$ the area in square inches. If the
 length and breadth are taken in feet, the area will likewise be in
 feet; the like may be said of yards, rods, &c.

Case 2. If the length is taken in one kind, and the breadth in
 another kind of measure; as suppose a plank 20 feet long, and
 9 inches broad whose area is required.

Rule. Multiply the length by the breadth, and divide the pro-
 duct by 12; the quotient will be the area. $20 \times 9 = 180$; and
 $180 \div 12 = 15$ feet the area required.

The reason of dividing the product by 12 is obvious, for if
 the breadth were 9 feet, the area would be 180 feet; in this case
 the multiplier (9) is feet, whereas in the other it is but the 12th
 part of 9 feet, and of consequence the first product will be but the
 12th part of the last, and therefore must be divided by 12, and
 the quotient will be the true product, which in this case will be
 less than the multiplicand, because the multiplier is a fraction,
viz. $\frac{9}{12}$, and $20 \times \frac{9}{12} = \frac{180}{12} = 15$. We have been the longer
 upon this head, because we shall have occasion in another place
 to mention the measuring of plank; for it is always accounted
 as an oblong square, tho' it generally tapers, the breadth is taken
 in

PLATE in the middle, which will be exactly true when both edges of the plank are streight, as in *Fig 11. Plate 1.* the trapezium *n o p t* Fig. 11. = the parallelogram I K M L, for the triangle *r n M* = triangle *r I o*; hence in measuring plank.

1	:	length in feet	:	:	breadth in feet	:	} area in feet.
12	:	length in feet	:	:	breadth in inches	:	
144	:	length in inches	:	:	breadth in inches	:	

This may be easily applied to all artificers works, as wainscotting, paving, &c. 9 : length in feet : : breadth in feet : area in yards.

P R O B. II.

To find the Area of a Triangle.

Rule. Multiply the base by half the perpendicular, (let fall Fig. 35. from one of the angles to the opposite side) the product will be the area required, as in the triangle D E F, let D F be 40 feet, and the perpendicular E M be 18 feet; the half of which is 9, the $40 \times 9 = 360$ the area in feet = 20×18 , and $360 \div 9 = 40$ the area in yards.

P R O B. III

To find the Area of a Trapezium.

Fig. 45. 1. If the trapezium has two sides parallel; as *m n, s t*, let fall the perpendicular *t r*, and multiply it by half the sum of both the parallel sides; the product will be the area.

Note. We suppose the arch *s t* to be so small, that it may be accounted a right line; then the parallelogram *m n r t* + triangle *s r t* = trapezium *m n s t*, and the parallelogram's area = $r t \times (n t)$ half the sum of $n t + m r$, and the triangular area = $r t \times \frac{1}{2} r s$.

Fig. 13. 2. If none of the sides are parallels, as B C, D A, divide the given trapezium into two triangles, by drawing the diagonal B D, and draw the perpendiculars A x, and C z; then multiply the diagonal by half the sum of the two perpendiculars; the product will be the area required

P R O B. IV.

To find the Area of an irregular Polygon.

Fig. 15. *Rule.* Reduce the polygon (as A B C D E F G) into triangles (as

(as ABG, BGF, BFC, FCE, and CDE); find the areas of those triangles, and add them together; the sum will be the area of the whole trapezium.

P R O B. V.

To find the Area of a regular Polygon.

It is evident that the area of every regular polygon is equal to the sum of the areas, of as many isosceles triangles as it contains sides, and that a perpendicular let fall from the center to any side, would be the radius of the inscrib'd circle; the area of the whole polygon may therefore be found by the following rule, *viz.*

Multiply the sum of all the sides by half the radius of the inscribed circle, or the radius by half the sum of the sides; the product will be the area required.

By this rule, it will be easy to find the area of any regular polygon, when the side, and the radius of the inscribed circle are both given: But as it sometimes happens that only the side, and sometimes only the radius of the circle can be practically measured, we shall lay down the proportions they bear to one another in most of the figures that occur in practice, which are calculated to a sufficient exactness in most cases, *viz.*

The Proportion that the Sides bear to the Radii of the inscribed Circles.

Equilateral triangle		as 1 to	0 . 288
Pentagon or polygon of 5 sides	5		0 . 688
Hexagon	6		0 . 866
Heptagon	7		1 . 038
Octagon	8		1 . 207
Nonagon	9		1 . 378
Decagon	10		1 . 538
Undecagon	11		1 . 703
Dodecagon	12		1 . 866

We shall give one example under this head, which we think will be sufficient.

Required the Area of an Octagon whose Side is 24.

As 1 : 1 . 207 :: 24 : 28,968, the radius of the inscribed circle.

24 × 8 = 192 = sum of the sides.

Then

PLATE
I.

Then multiply 28.968 by 96 = half the sum of the sides;
and the product will be 1780, 928 the area required.

P R O B. VI.

To find the Area of a Circle.

As mathematicians consider a circle as a polygon of an infinite number of equal sides, the circumference will be equal to the sum of all the sides: If this then be multiplied by half the radius, the product will be the area, the same as if it were a polygon; but before this can be done, the circumference must be found, by investigating the proportion it bears to the diameter, which has been calculated to great exactness by several eminent mathematicians, and is universally allowed to be as 1 is to 1.141592, &c. in practice, as 1 to 3.1416; hence the area of a circle may be found by the following rule, *viz.*

Multiply the diameter by 3.1416, and the product will be the circumference; this again multiplied by $\frac{1}{4}$ of the diameter (which is the same as half the radius) will give the area, which will be the same as if the diameter be squared, and the $\frac{1}{4}$ of the product multiplied by 3.1416, as will appear by the following:

E X A M P L E.

What is the Area of a Circle whose Diameter is 128.

$128 \times 3.1416 = 402.1248$; then $\frac{1}{4}$ diameter $32 \times 402.1248 = 12867.9936$; the area required: $= 128 \times 32 \times 3.1416$; for when several numbers are to be multiplied into one another, it will be indifferent in what order the operations are performed; but if instead of 32 we take $1\frac{3}{4}$, which is equal to it,

and multiply the diameter by this fraction, it will be $\frac{128 \times 128}{4}$

the square of the diameter 16384 divided by $4 = 4096$, also $128 \times 32 = 4096$, and $4096 \times 3.1416 = 12867.9936$, the area as before; now because 4 will always be a divisor, and 3.1416 a multiplier; we may take $\frac{1}{4}$ of 3.1416 , which is 7854 , for a multiplier, and then there will be no occasion for a divisor; and the area of any circle will be found by multiplying the square of the

the diameter by .7854, and the product will be the area 128×128 . PLATE
 $= 16384$ the square of the diameter, and $16384 \times .7854 =$ I.
 12867.9936 the area as before: Hence the areas of circles will
 be in the same proportion as the squares of their diameters: And
 because the square of 1 is 1, the area of a circle whose diame-
 will be .7854; or, if the circumference, which is 3.1416, be
 multiplied by a $\frac{1}{4}$ of the diameter (1) or .25, it will give .7854
 the area as before.

P R O B. VII.

To find the Area of a Sector.

Rule. Find the area of the whole circle; then say, as 360, the
 degrees contained in the whole circumference; are to the area of
 the whole circle; so is the number of degrees contained in any Fig. 20.
 arch, as ADB to the area of the sector ACBD.

P R O B. VIII.

To find the Area of the Segment of a Circle, as A b B D.

Rule. Find the area of the triangle made by the radii AC, BC,
 and the chord AB, which subtract from the area of the sector
 ACBD; the remainder will be the area of the segment A b B D.

But it often happens that we have the segment of so large a
 circle, that a small part of the circumference may be taken for a
 right line; and when the plane will not contain the radius, it
 will be difficult to know whether the curve be an arch of a circle,
 or of an ellipsis, or it may be neither; for as the periphery of the
 ellipsis falls within that of the circle, so there may several curves
 fall between them, or within the ellipsis, which mathematicians
 have given no rules to investigate. In such cases, we presume the
 following method may do for practice, and be pretty near the
 truth in most cases.

Draw several lines perpendicular to the chord, or right line, as
 TS, at equal distances from one another; suppose 1 foot. These
 will divide the whole surface, whether it be the segment of a cir- Fig. 45.
 cle, or of any other curve, into as many trapezia, less one, as
 there are perpendiculars, and two right angled triangles besides;
 now the areas of all these added together, will give the area of

PLATE I. the whole segment; and as every one of the trapezia have two parallel sides, and every perpendicular is a side to two trapezia, or to a trapezium and a triangle; the sum of all the perpendiculars multiplied by the distance between each, will be the area of the whole; and if the distance be 1 foot, the sum of all the perpendiculars measured in feet, will be the area in feet of the whole space contained within the right line and curve.

Fig. 45.

But as one side of every trapezium is a curve; if this side should differ perceptibly from a right line, draw a chord to it, which will cut off a small segment; in which, if there be drawn other two chords to meet at the middle of the arch, we shall have a triangle, the area of which must be added to that of the trapezium; these segments will be greatest at each end, if it be an arch of a circle, which may be reduced to two or more triangles; but there are some curves that approach so near a right line at each end, that a small part may be taken for such, without any sensible error.

P R O B. IX.

To find the Area of an Ellipsis.

As every ellipsis is equal to a circle, whose diameter is a mean proportional betwixt the transverse and conjugate diameters, and this is found by multiplying the transverse diameter by the conjugate, and extracting the square root of the product; the square of this mean diameter multiplied by .7854, will be the area of the ellipsis; hence the following rule:

Multiply the transverse diameter by the conjugate, and that product by .7854; the last product will be the area required.

E X A M P L E.

Let the transverse diameter be 48, and the conjugate 32.

Then $48 \times 32 = 1536$, and $1536 \times .7854$

$= 1206.3744$, area required.

It remains to be proved, that the mean proportional to the transverse, and conjugate diameters, will be the diameter of a circle equal to the ellipsis; for which take the following.

D E-

D E M O N S T R A T I O N.

Let the ellipsis T C S E, be inscribed in a circle whose diameter is T S; then the area of the semicircle will be equal to the sum of the areas of all the trapezia contained in it; now the ellipsis contains the same number, but smaller trapezia, and the sum of their areas is the area of the semi-ellipsis; therefore the area of a circle, is to the area of the ellipsis; as a trapezium in the semicircle, is to the same trapezium in the semi-ellipsis; but a trapezium in the semicircle is to a trapezium in the ellipsis, as the transverse diameter is to the conjugate; by the construction of the ellipsis; hence as the area of the circle : area ellipsis :: T S : C E, and multiplying the two last terms by T S, their products will still be proportional to the two first terms; that is, as the area of the circle : area of the ellipsis :: T S $\times$ T S : C E $\times$ T S, but T S $\times$ T S, is the square of the diameter of the circle, and C E $\times$ T S is the square of the mean proportional.

Fig. 45.

S E C T. IV.

Of Mensuration of Solids.

A Solid is that which has three dimensions, *viz.* length, breadth, and thickness; and as the areas of all surfaces are estimated in squares, so are the contents of any solid estimated in cubes.

A cube is a solid limited by 6 equal square surfaces, like a die. If the side of the square be 1 inch, foot, yard, &c. the side of the cube will be the same; and the solid is said to be a cubic inch, foot, or yard. Fig. 46;

A parallelopipedon is limited by two parallel and equal squares, called its bases, and four rectangles for its sides, which may be called the height or length, and if they are perpendicular to the bases it is a right one; but when the sides are oblique to the bases, then it is an oblique parallelopipedon; the like may be said of the following figures.

A prism is limited by two parallel and equal polygons for its bases, and as many rectangles as the polygon has sides; if the bases be triangles, it is called a triangular prism, but if squares, it is a parallelopipedon, &c. Fig. 47.

PLATE A pyramid is a prism, taper'd to a point at the top; it has but

I. one base, which is a polygon, and as many isosceles triangles as
Fig. 48. the polygon has sides, meeting at the top, which is called the vertex; it is exactly one third part of a prism of the same base and altitude.

A cylinder has two equal and parallel circles for its bases, and
Fig. 49. the space betwixt them is limited by one curve surface. If one side of a rectangle be fixed as an axis, the opposite side moved round, will describe the curve superficies, and the other two sides will describe the bases; of this form is a rolling stone.

A cone is, in respect of a cylinder, what a pyramid is in respect of
Fig. 50. a prism, and is exactly one third of a cylinder of the same base and altitude; it has but one base, which is a circle, and tapers to a point at the top, called its vertex, like a sugar loaf. If the perpendicular of a right angled triangle be fixed immoveably as an axis, the hypothenuse, turned round, will describe the curve surface, and the base of the triangle will describe the circular base of a cone.

A sphere or globe, is a solid contained under one curve surface, every part of which is equally distant from one point, called its center, and may be formed by the revolution of a semicircle round the diameter. It is exactly two thirds of a cylinder, whose altitude and diameter of its base are equal to that of the globe.

In order to find the contents of these regular solids, let us examine how they may be composed; and if we may not be allowed to say, that a great many plain surfaces of one inch square, and infinitely thin, laid upon one another, will constitute a cubic inch, because the sum of ever so many cyphers will not make one unit, yet it is very plain, that if several dies, or cubes of one inch, be laid upon one another, they will compose a parallelopipedon, containing as many cubic inches as it is inches in height; but if the height be only one inch, the solid will contain just as many cubic inches as the base contains superficial. If the side of the base be 12 inches, the area will be 144, and it is plain it will take 144 cubic inches to cover this base. It will require 12 such squares of one inch high to compleat the cube, so that a cubic foot will contain 1728 cubic inches, that is 144×12 ; hence the following rules may be deduced.

PROB.

P R O B. I.

To find the solid Content of a Cube, Parallelopipedon, Prism, or Cylinder.

Rule. Find the area of the base by the problems in the preceding section, which multiplied by the perpendicular distance betwixt the bases, gives the solid content in the same kind of measure as the dimensions are taken. Or multiply the length, breadth and thickness (all taken by one kind of measure) into one another; the product is the content in cubes of the same measure; hence 1 : area base :: length : content. or 1 : length :: depth $\times$ breadth : content.

P R O B. II.

To find the solid Content of a Pyramid or Cone.

Rule. Multiply the area of the base by $\frac{1}{3}$ of the height; the product is the content 3 : area base :: height : content.

P R O B. III.

To find the solid Contents of a Globe.

Rule 1. Find the area of a circle whose diameter is equal to that of the globe.

Fig. 53.

Rule 2. Multiply the area by double the diameter, and divide the product by 3; the quotient is the content of the globe. 3 : area circle $\times$ 2 :: diameter of the globe : content.

P R O B. IV.

To find the solid Content of the Frustum of a Pyramid or Cone.

Note. If either of these be cut, by a plane, parallel to the base, the top cut off, will be a pyramid or cone, and the remaining part its frustum, which will have two bases; the small one is that of the cone or pyramid cut off; the great one, is the base of the whole cone or pyramid before it is cut: Hence, if we can find the content of each, and subtract the lesser from the greater, the remainder is the content of the frustum; so all that is wanting is to find the height of each, and to do this:

1st. Sub-

PLATE I. 1st. Subtract $\frac{1}{2}$ the diameter of the least from $\frac{1}{2}$ the diameter

I. of the greatest base: then

2^{dly}. Multiply the perpendicular distance betwixt the bases by $\frac{1}{2}$ the diameter of the greatest base.

3^{dly}. Divide this product by the difference of half the bases, and the quotient is the height of the whole cone before it is cut, and subtracting the distance betwixt the bases from this, we have the height of the cone cut off.

DEMONSTRATION.

Fig. 50. Let $n m A C$ be the frustum; draw the perpendicular $s m$; the line $C m$ is the difference of half the bases, and the triangles $m C s$, and $t C x$ are similar; therefore $C s : m s :: C t : t x$, and $\frac{m s \times C s}{C s} = t x$, but $t x$ is the height of the cone; therefore $m s$ the height of the frustum.

PROB. V.

To find the solid Content of irregular Solids, which are limited by several curve and plain Surfaces.

To do this, let the solid be supposed to be cut by several planes, parallel to the base, and at one foot, or inch distance from one another. Every section will form two equal plain surfaces. Half the sum of all the areas, including the areas of both bases, will nearly be the content in feet or inches; but if the solid has no plain surfaces parallel to one another, let two small parts be cut off, which may be measured as parts of a globe, cone or pyramid.

In all the foregoing problems it will be convenient to take the length, breadth and thickness, in the same kind of measure in which the content is required; but very often it happens that the content is required in feet, and the length given in feet; but the breadth and thickness in inches, or partly in feet, and partly in inches; as in, timber, bales, casks, &c. The value of timber is estimated by the load of 50 feet, and the freight of bales, &c. is by the tun of 40 feet. Such of our readers as are not well acquainted in decimals or cross multiplication, may reduce the feet into inches, so first find the content in inches, this divided by 1728, gives the content in feet; and to render this method as useful and expeditious as possible, we have subjoined three tables: the

the first is for dividing a number by 1728; the other two for finding the value of a remainder, and may be of use in dividing by 144, or by 12; and tho' most of our readers may be presumed to have the last table by heart, yet as all may not, we choose to insert it.

TABLE 1.

1 -	1728
2 -	3456
3 -	5184
4 -	6912
5 -	8640
6 -	10368
7 -	12096
8 -	13824
9 -	15552

TABLE 2.

1 -	144
2 -	288
3 -	432
4 -	576
5 -	720
6 -	864
7 -	1008
8 -	1152
9 -	1296
10 -	1440
11 -	1584
12 -	1728

TABLE 3.

1 -	12
2 -	24
3 -	36
4 -	48
5 -	60
6 -	72
7 -	84
8 -	96
9 -	108
10 -	120
11 -	132
12 -	144

E X A M P L E.

Required the content of a case:

Length 7 feet 5 inches, or 89 inches; breadth 3 feet 5 inches, or 41 inches; 2 feet 5 inches, or 29 inches depth.

Now $89 \times 41 \times 29 = 105821$, and when this is divided by 1728, the quotient will be 61 feet, and 413 remaining; to find the value of this, look for it, or the number next less in the second table, which is 288; against which is 2, that is $\frac{2}{12}$ of a cubic foot, which are called inches; again there will be a remainder of 125; the next number less in the third table is 120, against which is 10, that is $\frac{10}{12}$ of an inch, and a remainder of 5, which is $\frac{5}{12}$ of $\frac{1}{12}$ of an inch; so the content is 61 feet, 2 inches, 10 primary parts, and 5 secondary parts; all concisely express'd thus, 61. 2. 10. 5.

It must be observed, that by one inch is understood 144 cubic inches, being the 12th part of a cubic foot; by one of the first parts 12 cubic inches, and by one in the last part is understood 1 cubic inch.

But when the length is given in feet without any odd inches, and the other two dimensions in inches, the operation may be performed without reducing the feet to inches; only dividing by 144.

E X -

E X A M P L E.

What is the content of a piece of timber 24 feet long, 18 inches broad, and 14 inches deep; $24 \times 18 \times 14 = 6048$, and $6048 \div 144 = 42$, the content in feet; after the same manner any other piece of square timber may be measured; but in practice it is not always required to find the exact contents of timber, for sometimes the computed is less, and sometimes more than the real content.

It would be very difficult to find the exact contents of a tree, but as it generally grows pretty near round and tapering, it will be somewhat like a frustum of a cone; notwithstanding which, it is measured as if it were a parallelopipedon, and to find the square base in some places, the circumference of the tree is taken by girding it with a line pretty near the middle, and $\frac{1}{4}$ of this is accounted the side of the square;—now it is plain that the area of such a square will be above $\frac{1}{4}$ less than the area of the circle, and the tree measures so much less than the true contents.

In other places the tree is hewed somewhat in the form of an irregular prism of four flat sides and four round; the base will be an octagon, contained under four equal chords, and four arches of circles, but in measuring the tree the chords are supposed to be produced till they meet, and form a square; the area of this, multiplied by the length, is accounted the content, tho' it is plain, the tree thus hewed, does not contain near so much, because there is wood wanting at the corners, these are called waness, and the flat sides are called squares; besides the tree may be hewed in such a manner as to make it contain more than the real contents of the tree, even if it were allowed to be a cylinder, so that there may be very great impositions on the purchasers; to prevent which, the government contract, that the tree shall be hewed in such a manner, that what is to be called the side of the square shall bear a certain proportion to the diameter of the tree, which may be easily discovered by the callipers; for if they be applied to the waness, we have the diameter of the tree, and if to the flats, the side of the square, or the thickness; now because the larger the waness are, so much more will the tree measure; it must be hewed so that two waness shall not exceed one square. What is meant by a wane should likewise be expressed, for it is generally allowed to be the round part

part of the tree where the wood is wanting to compleat the square, or the chord of it, which may be taken with a pair of compasses, as in *Fig* 53. BE is the wane, and is exactly half the square TB; but in some contracts the portions of the chords, which are produced without the circle to compleat the square, are called waness, as in *Fig* 52. $DT = \text{half } TE$.

It is very difficult to hew a tree exactly to this standard, and very often the waness are as big as the squares, as in *Fig* 54. where the squares divide the circumference into 8 equal parts; by which means the content of the tree, measured as a parallelopipedon, would be to the real content measured as a cylinder, nearly as 34142 to 31416; for which reason, before it is measured, it must be reduced to its proper thickness at the measuring place, which is nearly the middle of the tree: For tho' all trees taper, and consequently are greater at the butt than the top end, yet they are allowed to be cylinders, the diameters of which are taken at the middle. But there will be no occasion to hew the tree, as the proportion is known, which the thickness of the tree, when properly hewed, shall bear to the whole diameter. All that is necessary, is only to construct a line of equal parts, which shall have the same proportion to a line of inches, that the diameter of the tree has to this thickness. If the tree happens to be thicker one way than the other, a mean proportional must be found for the diameter.

The construction of a line of equal parts, that shall have the same proportion to a line of inches, that the diameter shall have to the thickness when the tree is hewed so that the flat shall be double the wane, will admit of two cases.

Case 1. When by the waness are understood the portions of the chords, produced without the circle to compleat the square.

1st. Erect a perpendicular at K, and from K to C set off any number of inches, and from K to O double the line KC; then draw the line CO, with which as radius, from the center O, describe a circle. Fig. 52.

2^d. Divide the line CO into the same number of equal parts as the line OK contains inches; then will $21 \frac{1}{2}$ of those divisions be equal to 24 inches very nearly; that is a tree whose diameter is 24 inches, will be $21 \frac{1}{2}$ inches thick when hewed.

Now if the chords be produced till they meet, they will form

K

a

PLATE a square : And it is very plain that C O is half the diameter of the tree, and K O half the thickness ; and because the wane C D, or

Fig. 52. its equal C K, is half the flat C B, the tree is properly hewed according to the contract. Hence it is evident, that when the tree is so hewed, the diameter will contain as many equal parts of the line C O, as the thickness taken upon the flat will contain inches.

Case 2. When by the wanes are understood the chords of the arches, or the round parts of the tree, where there is no wood taken off, as B E, D N, I G, F T ; and it be required to hew the tree ; so that the flats T B, F I, G N, D E be double those wanes.

1st. Make an angle of 45 degrees, at the point B, or which is the same thing, an angle of 90 degrees at M ; and taking the points B and E, equally remote from M ; draw the line B E, and make B A and A T each equal to B E ; so shall B T be double of B E.

2^d. Thro' the points T, B, E, describe a circle ; and thro' the center C draw C A perpendicular to T B ; so will the flat T B be double the wane B E, the line C B half the diameter of the tree, and the line C A half the thickness : And if the line C B be divided into the same number of equal parts, as the line C A contains inches ; it is plain the diameter, when measured on this line, will contain as many equal parts of this, as the thickness contains inches. The following example will suffice to illustrate what has been said on this head.

E X A M P L E.

Fig. 52. Let there be a piece of timber 20 feet long, and 24 inches diameter, to be hewed so as to make the flats, according to *Case 1*. and let us suppose that when the timber is served in for measurement, it is found, by applying the callipers to the flats, to be $22\frac{1}{2}$ inches thick. Now to know if it be properly hewed, measure the diameter by a line graduated, as C O, which will be found to be nearly $21\frac{1}{2}$; which shews there should be one inch more hewed off ; and therefore $21\frac{1}{2}$ must be taken for the side of the square base, which will make the content in feet 64 ; for the thickness is not quite $21\frac{1}{2}$, it being only 21.466.

Now 144 : square of the thickness in inches :: length in feet : content in feet : That is, 144 : 460.789156 :: 20 : 64.

The content may be found by measuring the diameter by a line

line of inches, for which the following proportion must be taken: PLATE I.
 ken; as 180 is to the square of the diameter in inches so is the length in feet to the contents in feet; for 144 is to 180 as the square of the thickness to the square of the diameter, which may be thus proved: $144 \div 4 = 36$, and $36 \times 5 = 180$, now $\frac{1}{4}$ of the square of the thickness taken upon the flat, multiplied by 5 will be the square of the diameter; for the squares of K O, and K C both together, are equal to the square of O C, by *Prop. 20. Chap. 3.* but the square of K C is $\frac{1}{4}$ of the square K O; therefore five times the square of K C will be equal to the square of O C. In this example the square of the thickness is 460.789156, which is the second term in the proportion, when 144 is the first: But if $\frac{1}{4}$ of the second term be multiplied by 5, and that product taken for the second term, it is certain, to preserve the same proportion; that $\frac{1}{4}$ of the first term must likewise be multiplied by 5, and the product made the first term. Now this is the very case here, when the square of the diameter is taken for the second term; $460.789156 \div 4 = 115.197289$; this $\times 5 = 575.986445$; and $24 \times 24 = 576$; then $180 : 576 :: 20 : 64$ the content as before; tho' it is plain this exceeds the real content, because of the wood that is wanting at the corners. It is even more than the whole tree would measure, allowing it to be a cylinder of 24 inches diameter; for the square of the diameter $576 \times .7854 = 452.3904$ the area of the base in inches. Again, $144 : 452.3904 :: 20 : 62.8$, the content in feet.

If it be contracted that the tree is to be hewed, as in *Case 2.* when the diameter 24 inches is applied to the line constructed for that purpose; it will measure 20.71. Then $144 : 20.71 \times 20.71 :: 20 : 59.5$ the content in feet. This may likewise be done by taking the square of the diameter in inches for the second term, if 193.3 be taken for the first: $193.3 : 24 \times 24 :: 20 : 59.5$, as before: And that 144 is to 193.3 :: as the square of the thickness taken on the flat is to the square of diameter, may be thus proved: Let the radius C B be 10000; then will the sine of the angle A B C be 8268; and $8268 \times 8268 : 144 :: 10000 \times 10000 : 193.3$. By this it appears that there will be 7 *per Cent.* difference in hewing the tree by this method.

The works of the several artificers relating to building, whether superficial or solid, may be measured by the preceding rules: But as all the operations require multiplication and division; this,

in some cases, is deemed too tedious for practice, on which account they make use of the sliding rule. But before the lines necessary for that purpose can be constructed, there must be some method found to multiply and divide natural numbers, by adding or subtracting artificial ones: This is most effectually done by the Logarithms; which shall be the subject of the next chapter.

C H A P. V.

Of LOGARITHMS.

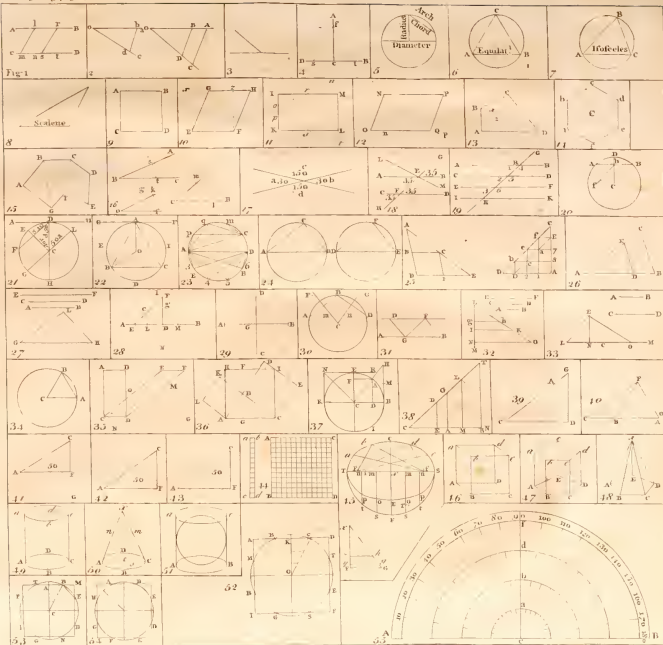
IT is not our business here to construct tables of these admirable numbers, they being already calculated to great exactness. The learned are obliged for this useful discovery to the indefatigable labour of the noble inventor, Lord *Neper*. We shall only explain so much of the nature of them, as is necessary for understanding the use and construction of the line of numbers.

LOGARITHMS are artificial numbers adapted to natural numbers, and so contrived, that by adding the logarithms of any two numbers, their sum will be the logarithm of the product of these two numbers, or by subtracting the less from the greater, the remainder will be the logarithm of the quotient of the one divided by the other. From this description, the following inferences will easily be deduced, *viz.*

1. Every natural number must have a proper logarithm, and therefore a table should be made to find it by inspection.
2. If the logarithm of any number be increased, the correspondent natural number will be increased likewise.
3. If the logarithm of any number be added to itself, (or which is the same thing, if it be doubled) the sum will be the logarithm of the square of the natural number.
4. If the logarithms of any two numbers are known, the logarithm of the product of those two numbers may with certainty be found: For, if the two known logarithms are added together, their sum will be the logarithm of the product.

By a careful attention to these inferences, we may easily make
loga-

[illegible]



logarithms to the following rank of natural numbers in a continued geometrical proportion, *viz.* 1 : 2 : 4 : 8 : 16 : 32 : 64 : 128, &c. which may be continued to any number of terms. Here 1 is the first term, and 2 the ratio; so every term is the product of the preceding term multiplied by 2, as will appear by bare inspection.

Unity (or 1) is the first natural number, and its logarithm must be a cypher (*by Inf. 2.*) for unity neither multiplies nor divides any number; so its logarithm must neither increase nor diminish any other logarithm.

The logarithm of 2 may be assumed at pleasure, but this will determine the logarithm of all the rest: Suppose it 10; the next natural number is 4. Now 4 is equal to 2×2 , therefore its logarithm will be $10 + 10 = 20$, which must be the logarithm of 4. The next natural number is 8, or 4×2 : Add therefore the logarithms of these two numbers, *viz.* 20 and 10, and their sum 30 will be the logarithm of 8 (*by Inf. 3 and 4.*).

It is easy to observe, that as the rank of natural numbers is formed by a continual multiplication of each preceding term by the ratio; so their logarithms are formed by a continual addition of the logarithm of the ratio: And as this logarithm may be assumed at pleasure, so there may be different sorts of logarithms, as in the following, *viz.*

1	:	2	:	4	:	8	:	16	:	32	:	64	:	128	:	256	:	512	:	1024	Numbers.
0	:	10	:	20	:	30	:	40	:	50	:	60	:	70	:	80	:	90	:	100	Logarithms.
0	:	15	:	30	:	45	:	60	:	75	:	90	:	105	:	120	:	135	:	150	Logarithms.

It is evident, that either of these ranks of logarithms will answer the proposed end: For if it were required to multiply 32 by 8, the logarithm of 32 is 50, the logarithm of 8 is 30, and $50 + 30 = 80$, which is the logarithm of 256 $= 32 \times 8$.

This may suffice to shew, that if there were a table of logarithms to all the natural numbers we should have occasion for, there would be no need of multiplication or division. The difficulty will be, to make such a table for all the intermediate numbers, which I presume the inventor might effect in the following manner.

Instead of assuming the logarithm of 2, he might chuse 1.0000000 for the logarithm of the natural number 10, the double of which would be 2.0000000; for the logarithm of 100, and

and the treble 3.0000000; for the logarithm of 1000, and so on, as in the following table.

Natural Numbers.	Logarithms.
1	0.0000000
10	1.0000000
100	2.0000000
1000	3.0000000
10000	4.0000000
100000	5.0000000

It is evident from the foregoing table, that the logarithms of all the natural numbers between 10 and 100 would begin with 1; between 100 and 1000 with 2; and between 1000 and 10000 with 3, &c. these initial figures are called characteristicks, and denote how many places the first figure of the natural number stands from unity: It is also evident, that the logarithm of any natural number under 10, would be less than 1, with 7 cyphers annexed, and therefore would begin with 2, 3, &c. with 6 figures more annexed. But to make it contain the same number of figures as the logarithms of the numbers above 10, he prefixed a cypher to it, which is the characteristick of all the natural numbers under 10.

Having thus assumed 1.000000 for the logarithm of 10, the half of it 0.5000000 would certainly be the logarithm of the square root of 10, which the inventor with great care and pains must have extracted to 7 decimal places. If this root were multiplied by 10, the logarithm of the product would be 1.5000000, the half of which 0.7500000, would certainly be the logarithm of the square root of that product. In this manner, I presume, he proceeded to find the proportionals between 1 and 10, till the root came to more than 9, and then found mean proportionals betwixt that and the next root less than 9, till at last, after a great number of trials, he came to the root, or absolute number 8.9999999, which is so very near 9, that it may be taken for the same, the logarithm of which he found, by the same number of additions and halvings, to be 0.9542420.

In the same manner he might proceed to find the logarithms of 5 and 7, and having found these, the logarithms of 2, 3, 4, 6, 8, might easily be found, for half the logarithm of 9 would be the logarithm of 3, and if the logarithm of 5 is subtracted from the
loga-

logarithm of 10, the remainder will be the logarithm of 2, the double of which is the logarithm of 4, and that doubled again will be the logarithm of 8; and if the logarithm of 2 is added to the logarithm of 3, the sum will be the logarithm of 6.

By these means, I presume, after a great deal of indefatigable pains, and an uncommon application, he at last finished his table, which was justly esteemed one of the most useful discoveries in the art of numbers, and has accordingly been universally received by all mathematicians, and the lord *Neper* is allowed the whole honour of the invention without any rival.

Other methods have been proposed by authors who have wrote on this subject, whereby the operations may be shortened in the construction of the table; but, as our design in this place is only to make the reader acquainted with the coherence of the logarithms and natural numbers, being the same with that of numbers in arithmetical and geometrical progression; I think the preceding method the most likely to answer that purpose, as being the most intelligible, and the fundamental principle, upon which those methods that have been found to shorten the work must be grounded. Tables being already calculated by the inventor, as well as by several succeeding mathematicians, to a great exactness, there is now no necessity for that trouble, we shall therefore, in the following propositions, shew the manner of finding logarithms in the tables, and some of their various uses in arithmetical operations.

P R O P. I.

To find the logarithm of any given number.

Rule, Look for the number in the first column, (under N^0) and if it consists of less than 3 figures, its logarithm will be found in the first page, with its proper characteristick. If it consists of 3 figures, it will be found in the following pages in the first column, and right against it in the column under o, you will find its logarithm, with a proper characteristick.

If the given number consists of 4 places, find the first three as before, and look for the last figure at top, and in the column under that, right against the three first figures, you will find the proper logarithm: Only you are to observe, that in this case the characteristick will be 3: For the absolute number must always contain

contain one place of integers more than the characteristick does of units. If one, or more, of the last figures are decimals, the logarithm will be the same: The difference will be only in the characteristick, which, as we have observed before, always denotes how many places the first figure of integers stands from unity; the following examples will be sufficient under this head.

Num.	Logar.	Num.	Logar.
8	0.903090	7569.	3.879038
88	1.944483	756.9	2.879038
699	2.844477	75.69	1.879038
5463	3.737431	7.569	0.879038

P R O B. II.

To find the absolute number corresponding to any given Logarithm.

Rule, Without regarding the characteristick, look for the given logarithm in the table; and right against it, in the first column, under N^o , you will find the three first figures, and at top, the fourth figure of the number required. But, if the number thus found should consist of fewer places than is expressed by the characteristick, the deficiency must be made up by annexing cyphers: And if it consists of more places, one or more of the last figures must be decimals, as in the following examples.

E X A M P L E I.

Let the given logarithm be 3.914872. Against .914872; in the table you will find 822, in the first column under N^o ; so that, as the characteristick is 3, and the logarithm is found in the column under 0 at top, the number sought will be 8220. But if the characteristick had been 2, the number would have been 822; and if it had been 1, the last figure would have been a decimal, and only the two first figures integers, *viz.* 82.2.

If the above logarithm .914872 had not been found under 0, in the table, the fourth figure would not have been a cypher, but one of those at top of the table under which it had been found.

E X A M P L E II.

Let the given logarithm without the characteristick be .018345. This will be found in the column which has the figure 6 at top; the

the absolute numbers must therefore be taken to 4 places of figures at least, the characteristick always denoting how many of those figures must be reckoned as integers, as in the following, *viz.*

Logarithms	Numbers
5.918345	828600
4.918345	82860
3.918345	8286
2.918345	828.6
1.918345	82.86
0.918345	8.286

If the given logarithm cannot be had exactly in the tables, we must take the nearest to it; suppose it 3.861080; the natural number corresponding thereto will be more than 7262, but less than 7263. But because the given logarithm is nearer to that of 7262, that may be taken for the required number: Those who incline to more exactness may find a figure of decimals by the following method.

From the given logarithm	3.861080	} difference
Subtract the next less —	3.861056	
From the next greater log.	3.861116	} difference
Subtract the next less —	3.861056	

Then say, as 60 (the difference betwixt the two nearest logarithms to the given one) is to 10 so is 24 (the difference betwixt the given and next less) to 4, the decimal required: So 7262.4 is the natural number corresponding to 3.861080.

That the natural number is by this method found to great exactness, may be proved by adding the logarithms of any two numbers together whose product is equal to it.

Thus, $605.2 \times 12 = 302.6 \times 24 = 7262.4$

Num.	Log.	Num.	Log.
605.2	2.781899	302.6	2.480869
12	1.079181	24	1.380211
7262.4			
	3.161080		3.861080

The reason of this is plain, for if to the logarithm of 7262, be added 60, the natural number will be increased a whole unit; but if only 1 tenth, 2 tenths, &c. of 60 be added to it, the natural number will be increased only 1 tenth, 2 tenths, &c. of an unit.

L

Hence,

Hence, by the reverse of this method we may find the logarithm of a number of five figures; for, after finding the logarithm of the first four figures, subtract that from the next greater logarithm in the tables; then say, as 10 is to the difference betwixt the logarithms so is the fifth figure in the natural number to the number to be added to the logarithm of the first four figures.

Let the number whose logarithm is required be 72624.

$$\begin{array}{rcl}
 & \text{Log.} & \\
 7263 & 3.861116 & \\
 7262 & 3.861056 & \\
 \hline
 & \text{difference } 60 &
 \end{array}
 \left\{ \begin{array}{l} 10 : 60 :: 4 : 24, \text{ and} \\ 24 + 3.861056 = 3.861080 \end{array} \right.$$

But because the natural number has five figures, the characteristick must be 4.

P R O P. III.

Multiplication and Division by Logarithms.

Rule, Add or subtract the logarithms of the natural numbers, their sums will be the logarithms of the products, and their remainders the logarithms of the quotients; and as the rule of Three requires both these operations, we shall refer thereto for examples.

P R O P. IV.

The Rule of Three, by Logarithms.

Rule, Add the logarithms of the second and third terms together, and from the sum subtract the logarithm of the first term; the remainder will be the logarithm of the fourth term required.

E X A M P L E. I.

If 64 give 21, what will 72 give?

$$\begin{array}{rcl}
 & \text{Log.} & \\
 \text{second term } 21 & 1.322219 & \\
 \text{third term } 72 & 1.857332 & \\
 \hline
 \text{product } 1512 & \text{sum } 3.179551 & \\
 \text{first term } 64 & 1.806180 & \\
 \hline
 \text{fourth term } 23.62 & 1.373371 &
 \end{array}$$

As

As the last logarithm cannot be had exactly in the tables, we must (as already observed) take the nearest to it, which is, 373280. against which, in the column under *number*, is 236, and the figure at the top is 2; so that 23.62 will be the nearest, for the characteristick being 1, the two last figures will be decimals.

We shall in the next examples show the use of the logarithms, when any of the terms are mixed numbers, or decimal fractions; and here we think it needless to perplex our readers with negative signs, as the whole business may be done by using the same process as if they were all integers; for then the characteristicks will all be positive, and denote how many places of figures are contained in the product, or quotient; and we may find how many are decimals, by the very same rule that is made use of when the operations are performed by multiplication and division of the natural numbers.

E X A M P L E II.

If 16.5 give 3.75, what will 49.5 give?

We shall work this as if the terms were all integers, and likewise as mixt numbers; the difference will be only in the characteristick.

3.75	Log. 2.574031	0.574031
49.5	2.694605	1.694605
185.625	5.268636	or 2.268636
16.5	2.217484	1.217484
11.25	3.051152	1.051152

In the first operation, when the logarithms of the second and third terms are added together, the characteristick is 5: This shews there will be six figures in the product: But then, because there is one place of decimals in the multiplicand, and two in the multiplier, there must be three places of decimals in the product, and only the first three figures are integers. And this is agreeable to the characteristick, in the second operation; which being 2, shews there will be three places of integers; but this does not determine how many places of decimals will be requisite to compleat the product. Again, when the logarithm of the first, is subtracted from the sum of the other two, in the first o-

operation the characteristick is 3, which shows there will be four figures in the quotient. But because there are three decimal places in the dividend, and but one in the divisor, there must be two decimals in the quotient. So the first two figures will be integers, and the last two decimals: It is the same by the second operation; where the characteristick is 1. The first operation seems to have the advantage of the second, because it discovers how many decimals will be in the product or quotient.

E X A M P L E III.

If 165 give ,375, what will ,495 give?

The figures in this being the same with the former, the operation will also be the same; the difference will be only in the value of the figures in the product, and quotient. The second and third terms being decimals, their product will likewise be decimals; and the characteristick being 5, it will consist of six places: But when this comes to be divided by the first term, which is 165, all integers, the dividend will contain six places of decimals more than the divisor, and therefore the quotient must likewise have six decimal places; whereas, by the preceding operation, the characteristick of the logarithm of the quotient is three, which shows it will contain only four significant figures; to which there must be two cyphers prefixed, to make up the deficiency, and then it will be the same as if the operation was performed by natural numbers; $.375 \times .495 = .185625$, and $.185625 \div 165 = 001125$: But if the divisor is a fraction, as ,165, and the dividend the same as before, then it will contain only three decimal places more than the divisor; so the quotient must have three decimal places, $.185625 \div ,165 = 1.125$.

P R O P. V.

Extraction of Roots, by Logarithms.

Rule, Divide the logarithm of the power by the index of the power, the quotient will be the logarithm of the root; but if the root be given, and the power required, multiply the logarithm of the root by the index of the power; the product will be the logarithm of the power.

NB. The index of the square is 2, of the cube 3, &c. See Chap. I. Sect. 1.

E X-

E X A M P L E I.

What is the square root of 576?

Log. Index Log.
Power 576 $2.760422 \div 2 = 1.380211$, the natural number
corresponding to which is 24, the root required.

E X A M P L E II.

What is the cube root of 13824?

This number, consisting of five figures, cannot be found in the tables, therefore we must make use of the method in *Example 2. Prop. 2.* of this Chap. *viz.* find the logarithms of 1382, and of 1383, their difference will be 314; then $10 : 314 :: 4 : 125.6$

Log.		Log.	
1383	140822	Dif { 1382	140508
1382	140508	} 314 {	Log. of 13824 is
			4.140633, $\frac{1}{3}$ of which is
$314 \times 4 = 1256 \div 10 = 125.6$			1.380211, and

the natural number corresponding to this last logarithm is 24, which is the cube root of 13824.

E X A M P L E III.

Admit two cylinders of equal length; the diameter of the one 32 inches, and its content 4096 cubic inches, the diameter of the other 16 inches, required the content in cubic inches?

Here, as the lengths are equal, the contents will have the same proportion to one another, as the areas of their bases, which being circles, it will be as the squares of their diameters; that is,

As the square of 32 (whose Log.	1.505150	$\times 2$ is =	3.010300)
Is to the contents 4096 (whose Logarithm is	=		3.612360)
So is the square of 16 (whose Log.	1.204120	$\times 2$ is =	2.408240)
			sum 6.020600

To the required contents 1025 (whose Logarithm is = 3.010300)

E X-

EXAMPLE IV.

Admit 93 feet to be the length of the keel of a ship of 508 tons, and a ship of 400 tons to be built, exactly similar to the other; required, the length of her keel?

In order to solve this it must be observed, that the contents of similar solids have the same proportion to one another, that the cubes of their similar sides have, therefore, the following will be a general proportion in all cases where the dimensions are similar, *viz.*

As the tonnage of any ship, or the solid contents of any body, is to the cube of the keel, or any other part; so is the tonnage of any other similar ship, or the contents of any other similar body, to the cube of her keel, or any other similar part. Hence, 508 : cube of 93 :: 400 : cube of the required keel, the cube root of the fourth term must be extracted, for the length of the keel: First, to cube 93, by the logarithms.

Log.		
93	$1.968483 \times 3 =$	5.905449 Log. of the cube of 93
400 its Log. is		2.602060
sum of 2d and 3d terms		8.507509
508 first term, its Log.		2.705864
Log. of the cube, divide by 3)	5.801636	
85.88 the required keel	1.933878	
(or 86 nearly)		

Here the usefulness of logarithms is very evident, for the cube of 93 would consist of 6 places, as appears by the characteristic; and this again being multiplied by 400, the product would consist of 9 places; and when this product is divided by 508, the quotient will have 6 places; and the cube root of this must be extracted to four places at least; for 85.88 is the length of the keel required, being the nearest natural number to the Log. 1.933878.

What has been already said, we presume, is sufficient to recommend the practice of these admirable numbers to our readers, though they may be extended to the solution of most questions which require an arithmetical calculation. But they do not stop here; for they discover a method of performing the foregoing o-
pera-

perations, even without the help of numbers: This is effected by the line of numbers invented by Mr *Gunter*, which we shall treat of in the next chapter, and shall only here remark, that numbers may be added or subtracted, by a scale of equal parts and a pair of compasses, as in the following examples, where we shall make use of the same scale of equal parts before described, as in *Plate 2. Fig. 1.*

E X A M P L E I.

Let the two given numbers be 36 and 48.

Rule, Extend from the point A (where the line A B begins) to either of the given numbers, suppose 36; set the same extent forwards from the other given number 48, and it will reach to 84, the sum required in the same line A B.

E X A M P L E II.

Let it be required to find the sum of 3010 and 4771. As these numbers cannot be had on the line A B, find them in the diagonals, and transfer them to the line A B, in the points x, z ; the extent from A to x will reach from z to y . Now, to find the value of y , take the distance of the point y from figure 7, in the line A B, and set it off from figure 7, in the line C D; then a ruler laid from this point to y , in the line A B, will intersect the diagonal next before y , in the required point, which will be found to be 7781.

As subtraction is only the reverse of addition, it will be needless to give any examples, this being not intended for practice.

The reason of the operation is so plain, as to require no demonstration: For if two rulers, one of ten inches, and another of fourteen, be laid so as to make one strait edge when joined to one another, they will make 24 inches; and if there be six inches cut off from a ruler of 24 inches, there will remain only 18 inches.

C H A P. VI. S E C T. I.

Construction of the Line of Numbers.

PLATE I. **T**HIS line may be of any length, but as there must be a particular scale adapted to it, we shall fix upon the line Fig. 1. A B, which being divided into 10,000 equal parts, will answer our purpose.

The intent of the line of numbers is only to add or subtract logarithms, so that all that is necessary to this end is to place the logarithms properly upon the line.

The logarithm of 10, by the table at the end of the book, is 1.00000. But because our scale contains only 10,000, we shall

1-0000

2-3010

3-4771

4-6020

5-9990

6-7781

7-8451

8-9030

9-9542

10-1000

fix upon that number for the logarithm of 10, and all those under 10 will be as in the margin. Find them all amongst the diagonals, and transfer them to the line A B in the points *x, z, t, p, j, s, r, n*.

Draw the line G H, parallel and equal to the line A B, and transfer the points *x, z, t, &c.* to this line from the line A B.

We have now the logarithms of all the numbers from 1 to 10 upon the line G H; and if to the end of it be joined another line, of the same length, and graduated and numbered properly, we shall have all the numbers from 1 to 100. But as our scale will not admit of these; draw the line E F parallel and equal to G H; and instead of doubling the line G H, take E N, (half the line E F) and make it the length of a line of numbers, by transferring the logarithms, as was done on the line G H, but there must be a scale of equal parts adapted to the line E N, which must contain 10000 equal parts, if we make use of the same table of logarithms as before. And by this means the line A B would contain 20000 equal parts, which would require double the number of parallels. Instead then of making a new scale, we may make the same diagonals answer our end: For it is only taking

taking half the logarithms. We shall therefore accommodate the logarithms to our scale, as in the margin; transfer all these from the diagonals to the line E N in the points 2, 3, 4, 5, 6, 7, 8, 9, 1; then graduate and number the line N F, the other

1-0000
2-1505
3-2385
4-3010
5-3495
6-3890
7-4225
8-4815
9-4771
10-5000

half of the line E F, exactly as the line E N. These last will be the logarithms of 20, 30, &c. for the logarithms of 20, 30, &c. are the logarithms of 2, 3, &c. added to the logarithm of 10; but E N is the logarithm of 10; E 2, E 3, the logarithms of 2, 3, &c. therefore, if N 2, N 3, &c. be made equal to E 2, E 3, &c. E N 2, E N 3, &c. must be the logarithms of 20, 30, &c. They may be transferred to the line N F from the diagonals, if to each of the logarithms in the margin we add 5000. So the logarithm of 20, will be 6505, the half what it is in the

tables. Now to find the units, or intermediate points betwixt 10 and 20, 20 and 30, &c. find in the tables, the logarithms of 11, 12, 13, &c. to 20, and the logarithms of 21, 22, &c. these must be divided into two equal parts, to accommodate them to our scale; and being found in the diagonals, they may from thence be transferred to the line N F.

We have now the whole line E F divided into 18 unequal parts; 1 at E, and the figures 2, 3, &c. denote so many units; 1 at N, and the figures 2, 3, &c. to F, denote so many tens; 1 at F 100: The intermediate divisions betwixt the figures in the line N F are units; so the 6th division betwixt the figures 2 and 3 is 26; the first betwixt figure 1 and 2 is 11, and so of all the rest.

If the spaces betwixt the figures in the line E N be graduated, as those in the line N F, they will be tenths of units: And because the difference betwixt the logarithms of 1 and 10, the logarithms of 10 and 100, and of 100 and 1000 are all equal; 1 at the point E may be accounted 10, and 1 at N 100, at F 1000. The figures in the line E N will now be tens, and those in the line N F hundreds. The intermediate divisions betwixt the figures in the line E N will now be units, and those in the line N F will be tens. So 3 in the line E N will be 30, and in the line N F 300. The divisions in the line E N betwixt 5 and 6, will be 51, 52, &c. in the line N F 510, 520, &c.

In order to find the points for 101, 102, 121, 122, &c. we
M must

PLATE II. must find the logarithms of those numbers, and transfer them from the diagonals as before directed; which would require the spaces betwixt the figures to be divided into 100 equal parts; but the length of our scale will not admit of this. The divisions betwixt the figures 1 and 2 are sub-divided into five, by transferring the logarithms of 102, 104, 122, 124, &c. and the divisions betwixt 2 and 3 are only sub-divided into two, by transferring the logarithms of 205, 215, 235, &c. The spaces betwixt the other figures are only divided into ten; so the units can only be had by taking $\frac{1}{10}$, $\frac{1}{10}$, $\frac{1}{10}$, &c. as near as the judgment can direct.

The line being thus constructed, it will be easy to find any number upon it: And that this may be done with all possible expedition, where the spaces are divided into ten parts betwixt the figures, every fifth is distinguished by longer strokes than the tens. Again, where the spaces can admit of being divided into more than ten parts, the sub-divisions are distinguished by shorter strokes than the tens. Now the value of these strokes are determined by the value of the figures, which being arbitrary, they must be determined before we can find any number upon the line. If the number be less than 100, 1 at E may be unity; then 1 at N will be ten, 1 at F 100. The strokes representing the tens in the line E N, will be tenths of units, and those in the line N F will be units. The short strokes betwixt the tens are estimated according to their number, for if there were 9 intermediates, each would be 100th part of an unit in the line E N, and tenths of units in the line N F. If there be only 4 intermediate strokes, each will be 200th parts, or two tenth parts of unity: And if there be but one stroke, it will be 500th, or 5 tenths.

If 2 were required, look for that figure in the line E N; if 20, it will be at 2 in the line N F; if $3\frac{1}{2}$, or 3.5, count five strokes beyond the figure 3 in the line E N; this, as was before observed, will always be longer than any of the others. If 400 were required, 1 at E must be accounted ten, 1 at N 100: So figure 4 in the line N F will be 400; 470 will be 7 strokes beyond figure 4; 475 will be in the middle betwixt the 7th and 8th stroke beyond figure 4; if 473, we must take little more than $\frac{1}{10}$ of the space betwixt the 7th and 8th stroke, but this cannot be had to a great nicety.

The line G H is called a single line of numbers, and the line E F a double one: This last containing double the numbers that the

the former does; and it is only two lines of numbers joined to one another, both graduated and numbered alike. PLATE II.

It is very plain that the figures 2, 3, &c. in the line G H Fig. 1, 2. are double the distance from one another, that the same figures are in the line E N or N F; so that we may, by inspection, find either the square or root of any number on these lines. A few examples will illustrate this.

Let it be required to find the square of 6. To do this, is to multiply it by itself, or to double its logarithm. Now this is at the point 6 on the line E N, extend therefore from E to figure 6, the same extent will reach from 6 to $36 = 6 \times 6$: But 36 is double the distance from E that 6 is; and 6 in the line G H is likewise double the distance from G that 6 in the line E N is from E: So we shall have no occasion for compasses; we need only look for the root on the line G H, and the square will be on the line E F right against the root: If the root be less than 10, 1 at G and at E may be units; but if it exceed 10, 1 at G must be 10, and 1 at E 100: If the square be given, and the root required; look for the square on the line E F, and the root will be against it on the line G H. Let the square root of 81 be required. 81 will be found in the line N F, and will be double the distance from E, that the root is from E. We must therefore find, by compasses, half the distance from E to 81, and whatsoever figure or point is at this middle point in the line E F; the figure of the same name, or point of the same value in the line G H, will be double that distance from G, and therefore must be against 81; and in this case 9 is the square root of 81.

As the square of any number is double the distance of the root from E; so is the cube of any number triple the distance of its root from the point E. And in order to find the cubes or roots of numbers by inspection; draw the line I K equal, and parallel to the lines E F, and G H. Divide it into three equal parts in the points L, M. Make each of these a line of numbers, either by adapting a scale of equal parts to it, or taking one third of the logarithms, and making use of the same diagonals as before. This line is called a triple line of numbers. Now if the cube root of any number be required; look for the cube on this line, and its root will be right against it on the line G H; for the same reasons, that the square is on the line E F. If the root be given, and

the cube required; look for the root on the line G H, and its cube will be on the line I K, right against the root.

Note. Because the lines G H, E F and I K, are at too great a distance from one another to find the cubes, squares and roots, without a pair of compasses; there is a double line drawn close to the single line, as in *Fig. 2.* and a single line drawn close to the triple line, as in *Fig. 3.* so that they may serve for a table of cubes and squares.

S E C T. II.

Of the Use of the double Line of Numbers:

IT is evident from the construction of the line of numbers, that the logarithm of any number may be had by a pair of compasses: Thus, extend from the beginning of the line to the number whose logarithm is sought; that extent measured on the scale of equal parts, will give the logarithm required.

E X A M P L E.

Let it be required to find the logarithm of 4.

Extend from the beginning of the line to 4; that extent measured on the scale of equal parts will give 6021, which is the logarithm of 4. The like may be said of any other number; which is very plain, being only the reverse of the method by which the line was constructed.

The intent of finding the logarithms of any numbers in this manner, is in order to add, or subtract them. But if this can be done by the line of numbers only, we shall have no occasion for the scale of equal parts.

We have already shewn how to add any two numbers by the scale of equal parts; therefore we may add the logarithms of any two numbers in the same manner; and the sum will be the logarithm of their product.

E X A M P L E.

Let it be required to add the logarithm of 3 to the logarithm of 4. This

This cannot be done by the scale of equal parts without having a table of logarithms; but this defect is supplied by the line of numbers. The logarithm of 3, by the table, is 4771, and the logarithm of 4 is 6021. Now the points 3 and 4 upon the line of numbers, are the same distance from the beginning of that line, that the numbers 4771 and 6021, are from the beginning of the scale of equal parts: So that it will be the same thing to extend from the beginning of the line of numbers to 3, that it would be to extend from the beginning of the scale of equal parts to 4771, and when this extent from 1 to 3 is set forward from 4, it will reach to 12, which point is 10792 equal parts from the beginning of the line of numbers; and that number being the sum of 4771 and 6021 (the logarithms of 3 and 4) it is the logarithm of 12, as appears by the table.

Multiplication by the Line of Numbers.

Rule. Extend from 1 to either of the given numbers, that extent will reach from the other given number to the product: A few examples will suffice to illustrate this.

E X A M P L E I.

Let it be required to multiply 8 by 6.

$$1 : 6 :: 8 : 48.$$

The extent from 1 to 6 will reach from 8 to 48.

E X A M P L E II.

Let it be required to multiply 98 by 8.

Here the distance from 1 to 8, when set forward from 98, will go beyond the end of the line; for, if 1 at the beginning of the line be unity, all the figures on the first part will be units, and those on the second, tens, and 98 will be within two divisions of the end of the line. In this case, 1 at the beginning of the line must be accounted 10, so 98 will be found on the first part of the line; and because the extent from 1 to 8 is the same as from 10 to 80; when this is set forward from 98, it will reach to 784, the product required.

$$\left. \begin{array}{l} 1 : 8 :: \\ 10 : 80 :: \end{array} \right\} 98 : 784.$$

A slider having a line of numbers upon it, exactly the same with that on the rule, will perform the office of a pair of compasses; and being readier for practice, we shall shew how to work by it.

As in the first example, let it be required to multiply 6 by 8.

Set 1 upon the slider, against 6 upon the rule; look for 8 upon the slider, and against it, is 48 upon the rule; and when the slider is thus set, we have the product of any number multiplied by 6; for against 2 is 12, against 6 is 36, against 10 is 60, against 16 is 96; but 17 on the slider goes beyond the end of the line upon the rule. In this, and in such like cases, the value of the figures must be alter'd, as observed before; and 1 at the beginning must be 10, and 17 will be found in the first part of the line upon the slider, and against it you will find 102 upon the rule. If the number had been 170, the operation would have been exactly the same; it would be only calling the 1, 100, and adding a cypher to the product 102, which would make it 1020.

Division by the Line of Numbers.

This is only the reverse of multiplication, for the extent from the divisor to the beginning of the line, set back from the dividend, will reach to the quotient. Or by the slider; set the divisor on the slider against 1 on the rule, and against the dividend on the slider, you will find the quotient on the rule.

E X A M P L E.

Let 48 be divided by 8.

$$8 : 1 :: 48 : 6.$$

Set 8 on the slider, against 1 on the rule; and against 48 on the slider is 6 on the rule; and, without moving the slider, we have the quotient of any number divided by 8, by inspection; as the reader will easily perceive upon examination.

Hence, to reduce a vulgar fraction into a decimal, add a cypher to each part of the fraction; and if the denominator upon the slider is set against 10 on the rule; then against the numerator you will find the decimal fraction upon the rule. Thus, the vulgar fraction $\frac{6}{8}$ or $\frac{3}{4}$ will be found .75 in decimals.

The

The Rule of Three by the Line of Numbers.

This is only to find a fourth proportional to three given numbers.

Rule. Place the numbers, or suppose them to be placed, as in the rule of three direct; then extend from the first to the second; that extent set the same way from the third, will reach to the fourth number required.

By the slider, set the first term upon the slider against the second term upon the rule; and against the third term upon the slider you will find the 4th term required upon the rule.

E X A M P L E.

Let the given numbers be $12 : 20 :: 27$, to which a fourth is required that shall bear the same proportion to 27, that 20 does to 12.

Set 12 upon the slider against 20 on the rule; and against 27 upon the slider, you will find 45 upon the rule, which is the fourth term required; for $12 : 20 :: 27 : 45$, the product of the extremes ($12 \times 45 = 540$) being equal to the product of the means ($20 \times 27 = 540$).

In order to demonstrate the reason of this rule, it will be proper to observe; that to perform the operation by figures, 20 must be multiplied by 27, and the product divided by 12.

By the method already shewn for multiplication by the line of numbers, the extent from 1 to 20, set forward from 27, will reach to 540, the product; but then, as this product is to be divided by 12, the extent from 12 to 1 must be set back from this product 540. Now it is very plain, that in effect, we only set the distance between 12 and 20 forward from 27: For as we are obliged after we have set forward the distance between 1 and 20; to set back the distance between 1 and 12; it is plain, that betwixt this last point and 27, there will be exactly the same distance as there is between 12 and 20.

It must be observed, that in extending, if the second term is greater than the first, the fourth term will be to the right hand of the third; but if it be less, it will be to the left hand of the third: When we use the slider, it is indifferent whether the first term be taken on the slider, or on the rule, provided the third term be taken on the same line as the first is. Neither is it material which
of

of the means is taken for the second term; for $12 : 20 :: 27 : 45$; and $12 : 27 :: 20 : 45$.

Having now fully explained the construction and use of the line of numbers, we shall give some examples in cases that most commonly occur to the shipwrights. And as the slider is most expeditious, we shall always make use of it.

E X A M P L E I.

Suppose it were required to know how much an artificer would gain in 30 days, at the rate of 3 shillings per day?

To reduce this to the rule of three, it will be $1 : 3 :: 30 : 90$, and when the slider is so set, that is 1 against 3, then will 90 be against 30. But as the answer to these, and such like questions, is sometimes required in pounds, this must again be divided by 20; and the operation by the pen would be $3 \times 30 \div 20 = 4.5$. Now, here are two numbers to be multiplied by one another, and divided by a third, therefore it will be $20 : 3 :: 30 : 4.5$. And instead of setting 1, set 20 against 3, and against 30 you will find 4, and 5 of the small divisions, which are tenths; each of which, in this case, must be reckoned 2s. so that, 4 and 5 tenths will be 4*l.* 10*s.* 0*d.*

E X A M P L E II.

What is the $\frac{3}{5}$ ths of 45?

As the $\frac{1}{5}$ ths of 5 is 3, say by the rule of three.

If 5 gives 3, what will 45 give? The answer will be 27, for $5 : 3 :: 45 : 27$.

From this example, take the following rule for finding the quarters of masts and yards, having the partners and slings given; and also the fraction, that the quarters must be of the partners, or slings.

Rule. Set the denominator of the fraction against the numerator, and against the slings; will be the quarter required.

E X A M P L E III.

If a yard is 25 inches at the slings, what will it be at the yard arm, the proportion being $\frac{2}{5}$ ths of the slings?

Set 5 against 2, and against 25 you will find 10. $5 : 2 :: 25 : 10$.

E X-

E X A M P L E IV.

If a ship of 69 feet by the keel, be 23 feet broad; how broad will a ship of 75 feet be, that is built in the same proportion?

Set 69 against 23, and against 75, you will find 25; and the slider being thus set, we have by inspection, the breadth of any ship, if the length of the keel is known, and the proportion, the same as above. If the keel is in feet and inches, it will be proper first to work for the feet, and then for the inches.

E X A M P L E V.

What will be the breadth of a ship whose keel is 75 feet 9 inches, the proportion being as 3 to 1?

Set 3 against 1, and against 75 you will find 25; and without moving the slider, against 9 (inches) you will find 3; so the breadth required is 25 feet 3 inches.

E X A M P L E VI.

Suppose a ship to be of the following dimensions, *viz.*

Length of the keel	—	—	93 feet 4 inches.
Extream breadth	—	—	32 0
Breadth at the tranfom	—	—	18 4
Breadth at the top timber line	—	—	26 0

And suppose several other ships are to be built in the same proportion, and the lengths of the keels are given, as follows; the other dimensions will, by the foregoing method, be found to be as in the columns, *viz.*

Lengths of the keels.		Extreme Breadths.		Breadth at the tran.		Bre. at the top. line.	
f.	in.	f.	in.	f.	in.	f.	in.
108	10	37	4	21	4 $\frac{3}{4}$	30	3
117	8	40	2	23	0	32	7
123	0	42	0	24	0 $\frac{1}{2}$	34	0

What has been said in *Example 4.* will suffice for finding all these dimensions.

N

E X-

E X A M P L E VII.

The length of the keel, and extreme breadth, being given; to find the tunnage.

The general method is to multiply the length of the keel by the breadth, and that product by the half breadth; then divide by 94; and the quotient will be the tunnage required: Or, which is the same thing, multiply the breadth by the $\frac{1}{2}$ breadth; then say as 94: is to this product :: so is the length of the keel: to the tunnage.

Let the length of the keel be 93 feet 4 inches, and the breadth 32 feet.

The operation by the pen will be $93 \text{ feet } 4 \text{ inches} \times 32 = 2986.8 \times 16 = 47786.8 \div 94 = 508 \frac{3}{4}$.

The first step by the slider will be, set 1 against 93.4 (or 93 feet $\frac{4}{12} = 93.33$), and against 32 will be 2986.8. Now as this product is to be multiplied by 16, and divided by 94, it will be $94 : 2986.8 :: 16 : 508 \frac{3}{4}$; therefore, if you move the slider 'till 94 is against 2986.8, the tunnage 508 nearly, will be against 16. In finding the first product, there is no occasion for estimating the number, only let it be marked, so that 94 may be moved to it; we shall shew in another place how this may be done at once by the slider.

By a careful attention to the manner of solving these questions, it will be easy to apply the slider to any other question in the rule of three, whether direct or inverse; or any thing else that is performed by multiplication or division.

We shall only add a few examples in measuring plank and timber.

Of measuring Plank.

We observed in *Chap. 4. Sect. 3.* that all plank is considered as an oblong square, and measured as a plain surface, without any regard to the thickness; and that all the varieties thereof may be reduced to the rule of three by the following proportions.

$$\left. \begin{array}{l} 1 : \text{length in feet} :: \text{breadth in feet} : \\ 12 : \text{length in feet} :: \text{breadth in inches} : \\ 144 : \text{length in inches} :: \text{breadth in inches} : \end{array} \right\} \text{area in feet.}$$

E X-

E X A M P L E S.

Let there be 5 planks of the following dimensions.

L.	B.	L.	B.	Area.	L.	B.	Area.	L.	B.	Area.	
f.	in.	f.	f.	f.	f.	in.	f.	in.	in.	f.	
20	9	1	20	75	15	or 12	20	9	15	or 144	240
40	6	1	40	5	20	or 12	40	6	20	or 144	480
36	12	1	36	1	36	or 12	36	12	36	or 144	432
30	15	1	30	1.25	37.5	or 12	30	15	37.5	or 144	360
15	18	1	15	1.5	22.5	or 12	15	18	22.5	or 144	180

Here every example is done 3 different ways; which method may be very useful for proving the examples, of which our readers may furnish themselves for practice with as many as they please, the above containing 15 different questions in the rule of three, which we presume sufficient for our purpose: Their solutions will be found as before directed: For if 1 at the beginning of the slider be accounted 1 tenth, 1 in the middle will be unity. If then this 1, in the middle of the slider, be set against 20, on the rule; then will 15 on the rule be against 75 on the slider: Or, if 12 be set against 20; then 15 will be against 9: And if 144 be set against 240, 15 will likewise be against 9. The like may be said of all the rest.

Of measuring Timber.

We have shewn before how this may be done by the pen, *viz.* by finding the superficial content, as if it was plank. This multiplied by the thickness in inches, and the product divided by 12, the quotient will be the content in feet. So that here there will be two operations: The proportions are,

1*f.* 12 : length in feet : breadth in inches : area in feet.

2*d.* 12 : area in feet : thickness in inches : contents in feet.

Or, 1 : breadth in inches : thickness in inches : a fourth number.

And 144 : fourth number : length in feet : contents in feet.

That is, multiply the breadth by the thickness, if both be inches, and this product by the length in feet; divide the last product by 144, the quotient will be the content in feet.

E X A M P L E.

Required the content of a piece of timber 20 feet long, 18 inches broad, and 15 inches thick.

$$\begin{array}{l} 1\text{ft. } 12 : 20 :: 15 : 25 \quad \text{Or,} \quad 1 : 18 :: 15 : 270 \\ \text{Then } 12 : 25 :: 18 : 37\frac{1}{2} \quad 144 : 270 :: 20 : 37\frac{1}{2} \end{array}$$

Here we must draw out the slider twice; first 12 against 20, then 25 will be against 15; secondly 12 against 25, and $37\frac{1}{2}$ will be against 20, or 1 against 18; 270 will be against 15, and if 144 be set against 270, $37\frac{1}{2}$ will be against 20 as before. There will be no occasion to estimate the value of the fourth proportional to the three first numbers; it will be sufficient to mark it so as that the slider may be moved till 12 or 144 be against this point: But as this will be attended with some inconveniency, it will be best to make use of the inverted line, which performs it at once without moving the slider twice.

Description and Use of the inverted Line.

The slider is fitted betwixt two double lines of numbers, of which the lower one is inverted, in such a manner, that 12 upon it, is exactly against 12 upon the upper line; so 20, 30, &c. upon the inverted line, are as much to the left hand of the point 12, as 20, 30, &c. are to the right hand of the point 12 upon the upper line. In reading the inverted line, we begin at the right; and because the distance betwixt 1 and 12 is more than that betwixt 12 and 100, the inverted line begins at 1.4, for 1 would extend beyond the end of the ruler.

Now the slider having two double lines of numbers, graduated exactly as the upper line, it will follow that whatever way the slider be moved, the point 12 upon the upper line on the rule, and the point 12 upon the lower line on the slider, will be both against the same number.

To measure Timber by this Line, when the Breadth is not the same with the Thickness, and both given in Inches, and the Length in Feet.

Rule. First find any of the three given numbers upon the inverted line; then as this number upon the inverted line: is to either of the two given numbers upon the slider :: so is the third given

given number upon the upper line : to the content in feet upon the slider ; observing that the upper line upon the slider, compares with the upper line upon the rule, and the lower line upon the slider with the inverted line.

As in the foregoing example ; suppose a piece of timber 20 feet long, 18 inches broad, and 15 inches thick ; set 15 on the inverted line, against 18 on the slider, and against 20 on the upper line, you'll find $37\frac{1}{2}$ upon the slider ; the content the same as by the two operations.

The reason of this will appear very plain, only by considering in what manner it is performed by two operations, and making use of the same slider, and the upper line with which it compares : For first, to find the fourth proportional to 12 ; 15 :: 18, we draw the slider out till 15 upon it is against 12 upon the upper line ; and when in this position, 12 upon the slider will be against 15 upon the inverted line. Now the fourth number will be on the slider against 18 upon the upper line, which will be $22\frac{2}{3}$: And because 18 on the upper line, is the same distance from 12 upon the same line, that 18 is from 12 upon the slider ; or which is the same thing, that 18 upon the slider is from 15 upon the inverted line ; when we draw the slider to the left hand, to bring $22\frac{2}{3}$, the fourth number, to 12 upon the upper line ; the point 18 upon the slider will likewise come as far to the left, and therefore will be against 15 upon the inverted line ; which is the very thing we are directed to do by the rule : and when it is thus set, 12 upon the upper line will be against the fourth number on the slider ; and the content, without moving the slider, will be found upon it against 20 upon the upper line.

Tho' all plank is measured, as a surface ; the value is estimated by the load, which is 50 solid feet : The following proportion will serve to find how many superficial feet of plank will make a load, *viz.* As the inches thick : is to 12 :: so is 50 the solid feet in a load : to the superficial feet.

E X A M P L E S.

How many superficial feet of 2, 3, 4, inches plank will make a load,

$$2 : 12 :: 50 : 300$$

$$3 : 12 :: 50 : 200$$

$$4 : 12 :: 50 : 150$$

S E C T. III.

Of the single Line commonly called the girt Line.

IT was observed before in the construction of this line, that the figures upon it are double the distance from one another, that the same figures are upon the double, and triple the distance that they are upon the triple line: Also that the cube and square roots were had by inspection. We shall now shew the use of it in measuring timber: And first,

Case 1. When the timber is square, that is, when the breadth and thickness are both alike, and given in inches, and the length in feet; to find the content in feet.

Rule. Set 12 upon the girt against the length on the double line, and the content will be on the double line, against the thickness on the girt.

E X A M P L E.

Required the content of a piece of timber 8 inches thick, and 9 feet long. Set 12 upon the girt, against 9 upon the double line; and against 8 upon the girt will be 4 upon the double line; the required content in feet. The reason of this will appear very plain. If we work it by the double line, the proportion will be $144 : (8 \times 8) 64 :: 9 : 4$; so the extent from 144 to 64 will reach from 9 to 4; and if we move the slider till 144 upon it, be against 9 upon the double line, then will 4 be against 64. Now, if instead of 144 and 64 upon the double line on the slider, we take 12 and 8, the roots of these numbers upon the girt line; they being the same distance from one another, they will perform the same office; and because 144 is always the first term, and the square of the thickness the second term, the rule will be general.

This might likewise be performed by the double line without moving the slider twice: The proportions will be $12 : 8 :: 9 : 6$. Now when 12 is set against 8, then will 6 be the fourth proportional to the three first numbers; and in the next three numbers, the first and second terms being the same as before, there will be no occasion to move the slider; only look for 6, the fourth number

ber before found upon the same line with the 12, and against it you'll find 4, the content, upon the same line with the 8.

Case 2. When the breadth and thickness are unequal, to find the contents by the girt line.

First find a mean proportional between the breadth and thickness, in inches; then set 12 upon the girt, against the length in feet upon the double line and against the mean upon the girt; will be the contents in feet upon the double line.

E X A M P L E.

Required the contents of a piece of timber 20 feet long, 18 inches broad, and 15 inches thick. Before this can be done we must find the mean thus.

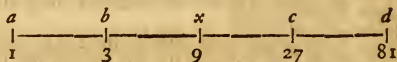
Look for either of the given numbers, suppose 18, upon the double line, and move the slider till this number is opposite to 18, the same number, upon the girt line; then look for 15, the other given number, upon the double line, and against it you'll find the mean required upon the girt line, which will be a little more than 16.4; and when 12 upon the girt is set against 20 on the double line; against the mean on the girt is $37\frac{1}{2}$, the content on the double line.

But it is plain that this method requires two operations; one to find the mean, and the other to set 12 to the length; so that it will be better to use the inverted line, as before directed.

To demonstrate the reason of this method of finding the mean, it must be observed, that to do it by the pen, the two numbers must first be multiplied into one another, and then the square root of the product will be the mean required. Let the two numbers be 3 and 27; their product is 81, the square root of which is 9, the mean required; for $3 \times 27 = 81 = 9 \times 9$.

Now to do this by the line of numbers, we must first extend from 1 to 3; that from 27 will reach to 81: And to find the square root of 81, we must find the middle point betwixt 81 and the beginning of the line; but this will be exactly in the middle betwixt 3 and 27; for let the extent from 1 to 3 upon the line of numbers, be represented by the line $a b$; and let c be the point 27; the extent $a b$ set forward from c , will reach to the product 81 at d . Now to find x , the middle of the line $a d$, divide $b c$ into two equal parts, which will give the point required: For
 $a b$,

a b , and c d , being equal by construction, they will be equally distant from the point x , the middle of the line.



The figures on the girt line, as was observed before, are double the distance from one another, that the same figures are upon the double: Therefore when 27 on the double line, is set against 27 on the girt, 3 on the double line, which then will be against 9 on the girt, must be the middle point betwixt 3 and 27 upon the girt, which may be proved by a pair of compasses; though these two numbers 3 and 27, are not to be found upon the girt line, which begins on most of the sliding rules at 4, and ends at 40: Now it is very plain, that if the line was produced to the left, of a sufficient length to begin with 1, that point would be as far to the left of 4, as 10 is to the left of 40; and 2 and 3 would likewise be as far to the left of 4, as 20 and 30 are to the left of 40; and if the extent from 40 to 30 be added to that betwixt 9 and 4, these two will be found equal to the extent betwixt 27 and 9.

The reason I presume, for calling the single line the girt line, is, because when the quarter of the circumference is taken for the side of the square, the tree is girted with a line, and the breadth and thickness being supposed equal, the content will be readily found by this line; and the reason for beginning at 4 will appear by the following examples.

E X A M P L E S.

Let there be 3 pieces of timber of 30 feet each, their breadth and thickness equal as below.

Long.	Thick.	Contents.
30	8	13.3
30	9	16.9
30	23	100

If the girt line begins at 1, when 12 upon it is set against 30 upon the double line, then 8 and 9 will be beyond the end of the double line; so the content cannot be had, unless we observe what point of the girt line is against 1 at the end of the double line, and then bring 1 at the beginning of the double line against the

the same point; whereas by beginning at 4, we can have the content for any thickness from 4 to 40.

But this line may be adapted to several other uses; We shall only mention the three following, *viz.*

1st. The length and breadth of a ship being given; to find the tonnage.

It was observed in *Exam. 7. Sect. 2.* that the method of doing this required two operations, *viz.* First to multiply half the breadth by the breadth; then, as 94 is to this product; so is the length of the keel to the tonnage. Now if we double the first and second terms, their products will be proportional to the third and fourth terms as before; the double of the second term is equal to the square of the breadth, and 188 is the double of 94; therefore 188 is to the square of the breadth; as the length of the keel, is to the tonnage. So that if 188 upon the double line of numbers on the slider, be set against the length of the keel upon the double line on the rule; then will the tonnage be upon the rule, against the square of the breadth upon the slider. But if, instead of the first and second terms upon the double line, we take their roots upon the girt line, because they are the same distance from one another; the tonnage may be found without moving the slider twice, as in the following example, where we shall take the same dimensions as before.

E X A M P L E.

Length of the keel 93 feet 4 inches, breadth 32 feet; required the tonnage.

Rule. Set the tonnage point upon the girt against 93 feet 4 inches, upon the double line; then against 32, the breadth upon the girt, is 508 upon the double line; the required tonnage.

Note. The tonnage point upon the girt may be found by setting 10 upon the girt, against 10 upon the double line; then against 188 upon the double line, make a mark upon the girt, which will be the tonnage point. Hence, if the length and tonnage be given, and the breadth required; set the tonnage point against the length upon the double line; then the breadth will be on the girt, against the tonnage on the double line. But if the tonnage and breadth are given, and the length required; set the tonnage upon the double line, against the breadth on the girt; then will the length be on the double line, against the tonnage point upon the girt line. As if it were required to find the breadth of a ship of 300 tons, the length of the keel being 78 feet: When the tonnage point is set against 78, then will 27, the required breadth upon the girt, be against 300 upon the double

O

line:

line : Or suppose the breadth 26 feet, and tonnage 280 ; required the length of the keel. Set 280 upon the double, against 26 on the girt line ; then against the tonnage point is 78 on the double line, the required length of the keel. So in this case, one foot in breadth will increase the tonnage 20 feet, which may be seen without moving the slider.

2d. To find the content of a tree by the girt line, the diameter and length being given, and supposing it to be hewed so as that the two wanes shall be equal to one square.

We observed before, that if by the wanes be understood, what the flat wants to compleat the side of the square, the proportion would be, as 180, is to the square of the diameter in inches ; so is the length in feet, to the contents in feet. Instead of the two first terms upon the double line, take their square roots upon the girt : The root of 180 will be found nearly 13.4, but there is no occasion to estimate the value, but only to find the point, which will be done by setting 10 on the girt against 10 on the double line, and the point will be upon the girt line against 180 upon double line.

E X A M P L E.

Suppose a tree 20 feet long, and 30 inches diameter : Required what the content will be when hewed, so as that the flat shall be equal to half the thickness ?

Set the point upon the girt, found as now directed, against 20 on the double line ; and against 30 on the girt, is 100 upon the double line ; the required content in feet.

But if by the wanes, be understood the round parts of the tree where there is no wood taken off ; the proportion, as was before observed, will be, as 193 nearly, is to the square of the diameter ; so is the length to the content. In this case we must make use of the square root of 193 ; for which purpose we may find a point upon the girt, in the same manner as the point for the root of 180 was found. Now if the point for 193 be set against 20 ; then against 30 upon the girt, will be 93 on the double line ; which is 7 *per Cent.* less than the former : A cylinder of such dimensions would measure only 98.17 ; so that by hewing the tree till the squares are half the thickness, it will measure near 2 *per Cent.* more than the full contents of the tree, if there had been no wood taken off.

3d. There are two points generally marked on this line ; one W G, the other A G, for wine and ale gallons, their use is in gauging ; for a wine gallon containing 231 cubic inches, and a circle whose area is 231, having for its diameter 17.14 inches : It is plain a cylinder of that dia-

meter

meter will contain as many wine gallons as it is inches in height; therefore if the length and mean diameter of any cask be given, the wine gallons that it will contain, may be found by the following proportion.

As the square of 17.14 is to the square of the mean diameter; so is the length, to the content in wine gallons.

Hence if the gauge point W G upon the girt line, (which is the square root of the first term) be set against the length on the double line; then the contents in wine gallons will be found on the double line, against the diameter of a cylinder, or the mean diameter of any cask upon the girt line; but if ale gallons be required, we must make use of the point A G, which should be exactly at 18.94, the diameter in inches of a circle, whose area is 282, the cubic inches in an ale gallon.

S E C T. IV.

Of the triple Line of Numbers.

IT was shewn in the construction of this line how to find the cubes, and their roots by inspection. We shall now shew how to find the dimensions of similar solids of different contents, as for instance: Suppose a ship of 508 tons to be 93 feet 4 inches by the keel, and 32 feet broad, and it be required to find the length of the keel, and extreme breadth of a ship of 400 tons.

This was performed by the logarithms in *Chap. 5. Ex. 4.* where it was observed that the proportion is, as the tonnage of one ship, or the content of any solid, is to the cube of the keel, or of any other part; so is the tonnage, or the content of any other similar solid, to the cube of the required keel, or any other similar part: Hence $508 : 400 :: \text{cube of } 93.4 : \text{the cube of the required keel}$; and the extent from 508 to 400 upon any line of numbers, will reach from the cube of 93.4 to the cube of 86, the length of the required keel. But there will be no need of finding the cube, (which is the fourth proportional to the three given numbers) if the root can be found: Now the cubes of any two numbers are three times the distance from one another that the roots are upon the same line; and because the two tonnages are the first and second terms, they will be the same distance from one another that the cubes of the keels are, which are the third and fourth terms, and therefore their roots will be one third of the distance from one another that the tonnages are; which in this case are 508 and 400. Let the distance then betwixt these two numbers

be divided into three equal parts, one of which being set back from 93.4, will reach to 86, the length of the keel required.

Now, as the roots are the same distance from one another upon the single line, that the cubes are upon the triple line; the keels will be the same distance from one another upon the single line, that the tonnages are upon the triple line. Therefore the following method may be used where there is a cube line adapted to the single line.

Rule. Set 94 feet 4 inches, or 93.33, the length of the given keel, upon the single line, against 508, the tonnage upon the triple line; then against 400 upon the triple line, is 86 upon the single line, which is the length of the keel required. And when the slider is thus set, we have, by inspection, the lengths of the keels of all ships that are similar to this, be the tonnage what it will: For if the tons are found on the triple line, the lengths of their corresponding keels will be against them on the single line.

The like method may be used in finding the other dimensions, as in the followings table, where the dimensions of a ship of 508 tons are supposed to be as in the columns in the upper line, and are pretty near to those of a ship of 20 guns; the other tonnages are nearly those of 40, 50, &c. guns. We have in each column set down the real dimensions in feet and inches, below those found by the rule, which are in decimals, that our readers may see that some dimensions are pretty near similar in all ships, and others arbitrary.

Guns.	Tons.	Keels.	Extreme Breadth.	Height of the Breadth	Transf. Breadth	Transf. Height.	Length in Ft. l'd.	Length of gun Deck.
20	508 real	93: 4	32: 0	13: 8	18:4	16:4	11: 0	113: 0
40	814 {rule	109, 0	37, 5	16, 0	21,5	19,3	12, 9	132, 0
	real	108:10	37: 6	16: 6	22:0	22:0	16: 0	133, 0
50	1052 {rule	118, 5	40,75	17, 5	23,5	21,2	14,16	144, 0
	real	117: 8 $\frac{1}{2}$	41: 0	18: 0	25:0	24:1	17, 8	144: 0
60	1191 {rule	123, 5	41,49	18, 2	24,4	22,0	14,65	150: 0
	real	123: 0 $\frac{1}{2}$	42: 8	19, 4	26:0	25:2	18: 6	150: 0
70	1414 {rule	131, 4	45, 0	19, 2	25,7	23,2	15, 4	159, 0
	real	131: 4	45, 0	20: 4	27:6	26:3	10: 4	160: 0
80	1585 {rule	136, 0	46,75	19,85	26,9	24,2	15, 9	165 0
	real	134:10 $\frac{1}{2}$	47: 0	21: 0	30:5	27:0	20: 0	165 0
90	1730 {rule	140, 3	48, 4	20, 0	27,6	24,8	16, 6	170, 0
	real	138: 4	48: 6	21: 9	31:5	27:9	20: 6	170, 0
100	2000 {rule	147, 0	50, 5	21, 6	29,0	26,1	17, 4	178, 0
	real	144: 6 $\frac{1}{2}$	51: 0	22: 9	33:0	29:0	21: 5	178: 0

It must be observed, that as every figure is three times placed upon the triple line, it will be indifferent in what part of the line the tonnages are taken. The only thing to be regarded is, that the slider must be so placed, that all the tonnages whose dimensions are required, be against some part of the single line.

Now, if the first 5 on the triple line be accounted 500; when 93.4 is set to 508; 40 on the single line, which is now 400, will be against 40000 on the triple line; and 64, which is at the beginning of the triple line, will be against 46.6 on the single. So that in this position, the numbers upon the triple line betwixt 64 and 40, will not be against any part of the single line; for there will be no numbers less than 64 upon the triple line, without altering the value of the figures; but 4 at the beginning of the single line, will always be the same distance from 64, the beginning of the triple line, that 40 at the end of the single line, is from 64000 at the end of the triple line. As the slider is now set, 4 at the beginning of the single line is accounted 40; and because the distance betwixt 64000 and 40000, is the same with that betwixt 64 and 40; the value of the figures may be alter'd, and 64000 at the end of the triple line may be called 64, and 40000 will be 40. But we must likewise alter the value of the figures on the single line; and 4 at the beginning of the line must be four units, and 40 on the triple, will be against 40 on the single line; 20 on the triple, against 31.7 upon the single, &c. as in the columns: To find the figures betwixt 64 and 40, let the second 5 upon the triple line be 500, and the slider set as before directed; then will 40 on the triple be against 4, which is at the beginning of the single line, but is now accounted 40; against 50 on the triple, is 43 on the single; against 60 on the triple, is 45.75 on the single, &c.

In the same manner the extreme breadths to any assigned tonnage may be found, supposing 32 feet to be the extreme breadth of a ship of 508 tons: Let the third 5 on the triple line be 500, and when 32 on the single is against 508 upon the triple, then we have all the numbers below 640 upon the triple line; and 1 upon the triple will be against 4 on the single. But if the second 5 be 500, and the slider properly set, 990 on the triple line, will be against 40 on the single; and as in this position we cannot find the dimensions corresponding to 1000, and the numbers above it; we must in these, and such like cases, observe what point of the triple line is against 40 at the end of the single line; and draw out the slider till 4 at the beginning of the single be against the same point, which in this example is 990: And as the value of the figures upon the triple line are not altered, 4 upon the single line must be accounted 40; and then
against

against 1000 is 40,1, against 2000 is 50,5, &c. the breadths corresponding to those tonnages. Again, let 5 be some given dimension of a ship of 508 ton; if the slider is properly set, 260 on the triple, is against 4, the beginning of the single line; and when 40 at the end of the single line is accounted 4, and brought against 260 on the triple; then against 100 on the triple, is 2,91 on the single, against 10 on the triple, is 1,35, against 1 on the triple, is 6,25 on the single. All the other dimensions in the columns are found by the same method, viz. by setting the given dimension to its proper tonnage; and to prove the work, after the length of the keels are found, we may use the double lines of numbers as before directed in *Ex. 6. Sect. 2.* of this *Chap.* Here the keel of a ship of 400 tons is found to be 86; when this is set against 93.4, the keel of 508 tons, against 32, the breadth of 508, is 29.6, the breadth of a ship of 400 tons: And as all the dimensions in the columns for a ship of 508 are known, look for them on the same line that her keel is taken, and against them will be found the corresponding dimensions for a ship of 400 tons.

The Tonnage of a Ship being given to find the Length of the keel, and extreme Breadth.

Before this can be done, the proportion that the length of the keel bears to the extreme breadth must be determined; which suppose as 3 to 1, and then six times the half breadth will be the required length of the keel. Now, because in finding the tonnage, when the length and breadth are given, we are directed to multiply the length by the breadth, and that product by the half breadth, and then divide this last product by 94, and the quotient will be the tonnage; it is certain, if the tonnage be multiplied by 94, the product will be the same as if the length, breadth, and half breadth, were multiplied into one another; and if any of these three be given, the others are found by the given proportion they bear to one another. We shall therefore work for the half breadth, which may be found by the following rule, viz.

First multiply the given tonnage by 94; then divide that product by 12, and lastly extract the square root of the quotient; and that root will be the half breadth required.

E X A M P L E.

Let the given tonnage be $127\frac{63}{4}$; then $127\frac{63}{4} \times 94 = 12000$, and $12000 \div 12 = 1000$, the cube root of which is 10, the half breadth; so 20 will be the extreme breadth, and 60 the length.

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The reason of dividing the product by 12, is because 12 times the cube of the half breadth, is always equal to the product of the height of the keel, breadth, and half breadth multiplied into one another, when the length is three times the breadth, as will appear by the following process.

The half breadth 10.

The breadth will be $10 \times 2 = 20$.

The length of the keel will be $10 \times 2 \times 3 = 60$.

Now, as in multiplication, when several numbers are to be multiplied into one another, it is indifferent in what order the operations are performed; so it is evident that $10 \times 10 \times 10 \times 2 \times 2 \times 3 = 10 \times 20 \times 60 = 12000$, but $10 \times 10 \times 10$, is the cube of the half breadth, and $2 \times 2 \times 3$ is 12; therefore 12 times the cube of the half breadth will be equal to the product of the length, breadth, and half breadth, multiplied into one another.

By the sliding rule, set 12 upon the double line on the slider, against 94 upon the double line; on the rule look for the tonnage 127 $\frac{62}{74}$, or 127,66 upon the slider, against which is 1000. Then to find the cube root of this, set the slider so that 64 upon the triple, is against 4 on the single; then against 1000 on the triple, is 10 on the single; the half breadth.

But as the length does not bear any constant proportion to the breadth, after finding the dimensions by this rule, the breadth may be altered, and the length of the keel can be had by the tonnage point. There is no invariable rule to determine the breadth, but in ships under 500 tons, this method will always bring it to less than a foot, and therefore may be very useful in determining proper dimensions for a ship of any number of tons.

C H A P. VII. S E C T. I.

Of the Construction and Use of several Lines on the Shipwright's Rule.

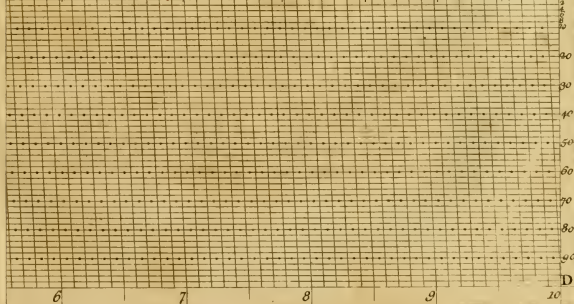
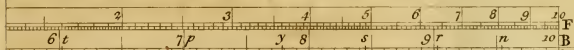
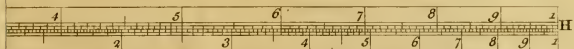
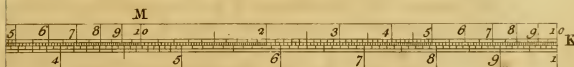
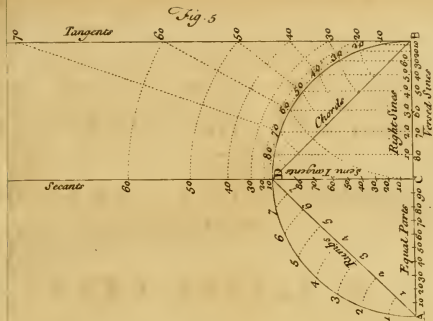
O f S E C T O R L I N E S.

THE instrument called a sector, is only a rule with a good joint, containing several different lines, each divided into some given proportion; as lines, chords, equal parts, &c. Every line upon one leg has a corresponding one upon the other leg, both exactly of the same length, and divided in the same manner; and all the lines on both legs meet at the center of the joint.

It is very useful in all the practical parts of the mathematicks, especially in dividing a line into any number of equal parts, or into any given proportion. As for instance, if it was required to divide a line, as A B, into any number of equal parts, suppose 9, (See *Plate 3. Fig. 1, 5.*) it is only opening the rule till the distance betwixt 9 and 9, in the line of equal parts, be equal to the line to be divided; and then the extent from 1 to 1, from 2 to 2, &c. in the same lines will be one ninth, two ninths, &c. parts of the line A B; for by opening the rule, the point 9, and every other point in the lines E, P, in effect, describe arches of circles; and if chords be drawn to these arches, they will form so many isosceles triangles, whose bases will be parallels to one another: Therefore C 9 is to 99, as C 1 is to 11, but C 1 is the ninth part of the line C 9; therefore 11 is the ninth part of the line 99.

In like manner, if it were required to make the line D F a line of numbers, when the rule is opened till the distance betwixt the extremities of the lines of numbers be equal to the line D F; then if the several distances from 1 to 1, from 2 to 2, &c. be set off from D towards F, the line D F will be a line of numbers.

Another method of dividing a line, as B D (See *Plate 3. Fig. 4.*) in the same proportion as the line A B is this. At the point B, the end of the divided line, make an angle, and set off the line to be divided from B to D; then draw the line A C parallel to A B, making D 3 equal A 3,
D 2



D 2 equal to A 2; D 1 equal to A 1: Draw the lines AD, 33, 22, 11: Then the triangles B 1 a, B 2 b, B 3 c, B A D, being similar, B A : B D :: B 3 : B c :: B 2 : B b :: B 1 : B a.

But before any of these methods can be used, the lines on the rule must be divided by some proportion given in numbers, or from the equal divisions of the arch of a circle.

The sector lines on the shipwrights rules, are for making masts and yards, of which there are four on each leg, divided into the same proportion, that the diameters at the several quarters bear to that at the slings; which being given in numbers, and expressed by the fraction, each quarter is of the slings, they may be constructed in the following manner:

Make an equilateral triangle A B C. (*Plate 3. Fig. 6.*) From the point A set off, on the lines A B, and A C, the equal parts expressed by the several denominators of the fractions, and draw lines across at these divisions. Then set off, on these lines, the equal parts expressed by the respective numerators of the fractions, and draw lines from A thro' these points to intersect the line B C. So if the side of the triangle be supposed to be the diameter at the slings, the several divisions of the line B C, from the point B, will be the diameters at the quarters. It will be proper to raise the fractions, so their denominators do not exceed 100.

E X A M P L E.

Let it be required to construct the line for yards, the quarters being the following fractions of the slings.

1st. $\frac{3}{4}$, or $\frac{81}{100}$. 2^d. $\frac{9}{10}$, or $\frac{81}{100}$. 3^d. $\frac{7}{8}$, or $\frac{70}{100}$. Yard arm $\frac{3}{4}$, or $\frac{75}{100}$.

Having constructed the triangle A B C, set off 84 equal parts, the denominator of the first quarter, from A to a; and from A to b draw the line a b, which will be parallel to B C. Upon the line a b set off 81 equal parts, the numerator of the first quarter, from a to c; and draw the line A c to intersect the line B C in the first quarter. Again, for the second quarter, its denominator is 100; therefore set off 100 equal parts from A to B, and from A to C, which in this case is already done, because the side of the equilateral triangle was made 100 equal parts: It remains only to set 90 equal parts from B to the second quarter, and draw a line from A to this point. The denominators of the third quarter and yard arm, are likewise 100; the numerator for the third quarter is 70, which set off from B to the third quarter, and draw a line from A to this point:

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The numerator for the yard arm is 40, which set off from B to y A, and draw a line from A to this point; by this means, if B C be supposed the diameter of a yard at the slings, B y A will be that at the yard arm; B 3 qr. that at the third quarter; B 2 qr. that at the second; and B 1 qr. that at the first quarter; they being by construction $\frac{2}{3}$, $\frac{7}{10}$, $\frac{9}{16}$, $\frac{3}{4}$, of the line B C.

The line being thus divided, may be transferred to each leg of the rule: But if the rule will not contain the length of the line, take such a length as may suit the rule, and with a pair of compasses, set off that length from A to G, and from A to F; and draw the line G F, which being parallel to B C, will be divided in the same proportion, and may be transferred to the rule. In the plate (*Fig. 7.*) the lines are only drawn to y A, but if continued, would meet in the center of the joint.

The line being thus constructed, and transferred to the rule, we shall shew the use of it in making a yard; (*Plate 3. Fig. 8.*) which suppose 71 feet long, of which A B is one half, and S S, the diameter at the slings 17 inches: Having divided the line A B into four equal parts at the points 1 qr. 2 qr. 3 qr. open the rule till the distance betwixt S and S at the extremities of the lines on the rule, be equal to 17 inches, the diameter of the yard at the slings; then the extent from the dots on the one leg, to those corresponding on the other leg, will give the diameters at those quarters, and at the yard arm, from which they may be set off upon the yard. But it must be observed, that only the half of each of those diameters thus found, must be taken, and that set off on each side of the middle line upon the yard; for which reason, in practice, it will do better to take half the diameter at the slings, and set the rule by that; and then we have half the diameters at the quarters by the rule:

The lines A B, and A C, (*Fig. 6.*) and the lines drawn from the point A for the quarters, may be produced to any length, and drawn upon a board. If then the lines A B, and A C, be divided into inches, halves, and quarters, there will be no occasion to transfer them to the rule: For suppose the diameter at the slings 21 inches, the half is $10\frac{1}{2}$; lay a ruler, or strait edged batten across the board, from $10\frac{1}{2}$ in the line A B, to $10\frac{1}{2}$ in the line A C, and make a mark upon the batten at each intersection with the lines drawn from A for the quarters. By this means we have, upon the batten, half the diameter at each quarter and yard arm; and these being set off on each side of the middle line upon the yard, in their proper places, will give the whole diameter at those places.

After the same manner are the lines for masts, bowsprits, and mizen yards constructed, by an equilateral triangle, and from thence transferred

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to the rules; the proportions, or fractional parts, by which the lines are constructed, are as follows:

Quarters	1	2	3	Hounds.	Head.	Heel.	Capo.	Yard Arm.
Mafts	60	14	5	9	4	—	—	—
Bowsprits	50	13	4	11	7	—	—	—
Mizen Yard	37	10	3	—	—	2	1	—
	21	11	3	—	—	—	—	9
	12	12	8	—	—	—	—	17
lower } arm	14	11	2	—	—	—	—	1
	15	13	3	—	—	—	—	—
upper }								

Another method of proportioning the quarters to the slings, is taken from the divisions of the quarter of a circle, but the diameter at the yard arm must be first determined; which suppose $\frac{2}{3}$ of the slings, as before, and let G F (*Plate 3. Fig. 3.*) represent the diameter at the slings. Now to find the diameter at the quarters, take the following rule.

1st. With the radius G F, describe the quadrant G F E.

2^d. Upon the line G E, set off $\frac{2}{3}$ of the diameter at the slings, from G to *e*, and thro' *e* draw a line parallel to G F, to intersect the arch in *a*; from *a* draw a line parallel to G E, to intersect the line G F in *y*; then will *y a* be the diameter at the yard arm.

3^d. Divide the line G *y* into four equal parts, in the points 1, 2, 3, and draw the lines 1 qr. 2 qr. 3 qr. parallel to G E, which will be the diameters at those quarters.

There is also another line on each leg; these meet at the center of the joint, from whence they are numbered 1, 2, &c. to 12; the space betwixt the figures is nearly $\frac{1}{3}$ of an inch, each divided into 12 equal parts: So the spaces betwixt the figures may represent feet, and the intermediate divisions inches. But if it was required to make a scale of feet and inches, where one quarter of an inch, one eighth of an inch, or any other space shall represent one foot; it may be done by the same lines, as before directed. Thus, take with the compasses 12 times the space intended to represent one foot, and open the rule till that reaches from 12 upon one leg to 12 upon the other leg. Now, whereas the feet and inches of the scale on the rule, are taken from the center of the joint; those of the other scale, must be taken across at those points.

S E C T. II.

Of the Construction of the Ten, Twelve, and Eight Square Lines.

THE use of these lines, is in order to hew a piece of timber so that it shall be a prism contained under 10, 12, or 8 equal planes, which are called squares; their bases will be polygons of the same number of equal sides.

The first thing to be done, is to hew the piece four square; the four sides for a ten square will not be equal, for its base will be a rectangle; whereas the bases for a twelve and eight square will be squares; so they must first be hewed into parallelopipedons.

There are three lines necessary for making a ten square, *viz.* The first for determining the longest side of the rectangle; the second for determining the side of the polygon; and the third for determining how much wood must be taken off the corners to reduce the rectangle to a polygon. Their chief use is for making barrels of capstans.

To Construct the Ten Square Lines.

Upon the line AB (*Plate 3. Fig. 9.*) erect a perpendicular CF, which make any number of inches, suppose one and a half. Thro' F draw the line DG parallel to AB: From C draw the lines Cm and Cn, making each an angle of 18 degrees with the line CF, so shall mn be the side of a polygon of ten equal sides; for the angle at the center is 36 degrees, the tenth part of 360. With the radius Cm (equal to Cn,) describe the semi-circle AmnB, and erect the perpendiculars AD, BG; so shall the rectangle ADGB, be half the base of the piece when hewed into four squares. Now it is plain, that the piece will be thicker one way than the other; for CF is one half of one of the diameters, and CB half the other; and Fm half the side of one of the ten squares: To reduce this rectangle to a polygon, make ms, sA, nt, tB, equal to mn, and produce the lines ms and nt, to b and o, and cut off the triangles Dmb and Gno; cut off also the triangles b s A and f o B, so shall AsmntB, be half the barrel of a capstan of three inches diameter. If Fm be divided into three equal parts, we shall have the divisions of the line marked BS, (*Fig. 2.*) which may be continued to any length. The use of this, is to find the side of
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the ten square, which is done by setting off as many divisions of this line as the barrel is inches thick on each side of the middle line on the two big sides.

If the line *A b* (*Plate 3. Fig. 2.*) be divided into three equal parts, we shall have the divisions of the line marked *S S*, which serves to shew how much must be set off from the middle line on the two small sides, viz. one of these divisions, for every inch of the thickness of the barrel, which in this case will be to *b* and *o*; and when the wood is taken off from *m* to *b*, and from *n* to *o*, we may make *m s* and *n t*, each equal to *m n*, also equal to *S A* and *t B*.

If the line *A B* be divided into three equal parts, it will give the divisions of the line *L D*, (*Fig. 2.*) which serves to shew how much the barrel must be the biggest way.

In making the barrel, the first thing to be done, is to hew it to the designed thickness in inches; which is what is called siding of it, and afterwards to square it; that is to hew it the other way. In order to do this, we must take as many divisions of the line *L D*, as the barrel is to be inches thick, so when it is thus hewn, it will have two big, and two small sides: There must be a middle line struck on each side, and then proceed as directed, by setting off the proper distances from the middle lines; and when all the wood is taken off, there will remain nothing of the small sides but the middle line; but of the two big sides there will remain the side of the ten square.

In constructing these lines. it will be proper to produce the lines *C F* and *C m*, till *C F* is six inches or more; then will *F m* be half the side of the ten square of a barrel of 12 inches thick, which may be divided into 12 equal parts; the same must be done with the lines *S S* and *L D*.

To Construct the Lines for a 12 Square.

These are likewise chiefly for making barrels of capstans, (*Plate 3. Fig. 10.*) but here the piece must be hewed exactly 4 square; for when all the wood is taken off, the whole side of the 12 square will remain on all the four sides.

There are only two lines necessary for reducing the 4 square to a 12, and formed after the same manner as those for a 10 square; for since the angle at the center of a polygon of 12 equal sides is 30 degrees, if at *C* two angles of 15 degrees each be made, and the lines *C r* and *C s* be drawn, it is plain *r s* will be the side of the 12 square, of which *F r* is one half; which being divided into 3 equal parts, will give the divisions of the line 4 *S*: (*Fig. 2.*) There must be as many of these divisions set off from
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the middle line on all the 4 sides, as the barrel is inches thick, and therefore must be set off from A to *b*, and from B to *o*, as well as from F to *r* and *s*; but the whole wood from *r* to *b* must not be taken off. It must first be taken from *r* to *c*, the point where the side *r d* produced intersects the line A D, which is half the side of the square. A *c* divided into three equal parts, will give the divisions of the line 2 S, (*Fig. 2.*) which are to be set off from the middle line, on two of the opposite sides, as from A to *c*, and from B to *n*; and when the wood is taken off from these points to *r* and *s*, the points *d* and *t* may be found, by making *r d* and *s t* equal to *r s*; and when the wood is taken off from *d* to *b*, and from *t* to *o*, we have A, *b*, *d*, *r*, *s*, *t*, *o*, B, the half of a 12 square of 3 inches thick; but the lines C F and C *r*, may be produced as directed in forming the lines for the 10 square.

To Construct the Eight square Lines.

These are on most of the shipwright's rules. Their use is in making masts and yards, and so well known, that we need give no directions how to apply them; and shall only remark, that tho' masts and yards are round, they must be first hewed four square, each side being equal to the diameter of the mast or yard; which, suppose A B, (*Plate 3. Fig. 11.*) then will the rectangle A G D B, be half a four square, in which a semi-circle may be inscribed of the same diameter with the mast or yard. If from the center C be drawn the lines C *m* and C *n*, making each an angle of 22 $\frac{1}{2}$ degrees with the line C F, we shall have *m n* the side of a polygon of eight equal sides, of which F *n* is one half; and if this is divided into as many equal parts as the diameter has inches, it will give the divisions of the 8 square line; one of which for every inch diameter must be set off from the middle line on all 4 sides: And because the diameter is supposed 3, when that number is set off from the points A, B and F, we shall have the points *r*, *m*, *n*, *b*: But these points may be found by setting off their distance from D and G, the side of the square; for which purpose the line *n G* or *m D*, must be divided into 3 equal parts; so either of the lines may be used.

These lines are on foot rules; but there are two lines on the slider of the two foot rules; the one for finding the diameter at the quarters; the other for finding the length and diameter at the slings of masts and yards. We have already shewn how to find the quarters by the sliding rule, which requires an operation for every quarter; but this line shews them all at once, only by setting a point in the slider to the given diameter of the slings;

slings. It is the upper line upon the slider, and has a space for masts, one for yards, one for bowsprits, and one for each arm of a mizen yard.

To find the Quarters by this Line.

For masts, set P on the slider against the diameter at the partners, and against the figures 1, 2, 3, you will find the diameters of the 1st, 2d, and 3d quarters; against H S will be the diameter at the hounds, and against *m b* the diameter of the lower mast head, but if it is a top mast, the diameter will be against T *m*.

After the same manner the quarters of the yards may be found, by setting S against the diameter at the slings; and against the figures 1, 2, 3, will be found the diameters of 1st, 2d, and 3d, quarters; and against Y A, the diameter at the yard arm. The same may be said of the bowsprit and mizen yard: One example will be sufficient to illustrate the whole.

Required the bigness at the Quarters, and Yard Arm, of a Main Yard 30 Inches at the Slings.

Set S against 30, then against 1 will be 29, nearest, for the first quarter; against 2 will be 27, for the second; against 3 will be 21 for the third; and against Y A will be 12 for the yard arm.

Note. The slider must be applied to a double line of numbers.

Before this line can be constructed, the proportion that the quarters bear to the slings must be determined, which suppose to be as before.

	In decimals.
First quarter	$\frac{27}{28}$.964
Second quarter	$\frac{21}{24}$.875
Third quarter	$\frac{12}{16}$.750
Yard arm	$\frac{3}{4}$.750

At any convenient place upon the slider assign a point for S, and draw a score at that point for the slings; set that point or score against 28, and against 27 draw a score upon the slider for 1; then move the slider till S is against 10, and against 9 draw a score upon the slider, for 2 against 7 draw a score for 3; then move S to 5, and against 2 draw a score for the yard arm.

The reason of this will appear very plain, if we find the quarters by a pair of compasses; for then we take the distance betwixt 28 and 27, (which will always be equal to the distance betwixt the diameter at the slings, and that at the first quarter) but this is equal to the distance betwixt

twixt S and 1 upon the slider; therefore if S be set against the diameter at the slings, 1 will be against that at the first quarter; the like may be said of the rest.

If the fractions are all reduced to decimals, the strokes for the quarters, &c. may be remarked, without moving the slider after S is set to 1, as will be found upon examination; and the construction of the line will appear plainer by the decimals; for the points 1, 2, 3 and Y, are exactly the same distance from S, that the numbers .964, .9, .7, .4 are from 1 on the line of numbers. There is no difference therefore between the line on the slider, and the line of numbers, only when the proportion of the quarters, &c. is determined by numbers; as the distances properly set off from 1, to decimal parts, will reach from S to the points 1, 2, 3 and Y A; these points are used instead of the numbers .964, .9, .7, .4.

The lower line on the slider, is for finding the lengths of masts and yards, but before this can be done, there must be a given proportion; the following is generally allowed for masts, *viz.* As 100 is to 76, so is the extreme breadth in feet, to the length of the main mast in yards.

The lengths of the other masts are proportioned to the main mast, and the points or strokes upon the slider, are laid down from *B r b*, by those established proportions, in the same manner as the diameters at the quarters of the yards are proportioned to that of the slings.

The main mast is allowed to be in proportion to the fore mast, as 100 to 89; in like manner there must be proportions for the mizen mast, bowsprit, main, fore and mizen top masts, topgallant mast and sprit fail top mast; by these proportions, the points or strokes, are set off for the different masts at proper distances from *B r b*.

To find the Lengths of the Masts by the Line.

Set *B r b* against the extreme breadth in feet; then the lengths of all the masts will be against their respective divisions, which are all distinguished by letters peculiar to each; and at the same time, the lengths of their heads and hounds may be found, there being peculiar letters for that purpose, and those for distinction's sake are across the slider.

Thus $\begin{matrix} Mb \\ Mb \\ Mb \end{matrix}$ } For main mast head.
 $\begin{matrix} Mb \\ Mb \\ Mb \end{matrix}$ } For main mast hounds.

E X A M P L E.

Required the lengths of the masts of a ship whose extreme breadth is 50 feet.

Set *B r b* against 50; then the lengths of all the masts, mast heads and hounds, will be found against their respective divisions, distinguished by letters as follows, *viz.*

	Lengths, Yards.	Head, Feet.	Hounds, Feet.
M Main mast	38 $\frac{1}{2}$	15 .8	6 .6
F Fore mast	33 $\frac{3}{4}$	14 .1	5 .3
Z Mizzen mast	32 $\frac{1}{4}$	12 .2	4 .85
B Bowspit	24 $\frac{1}{2}$		
Mt Main top mast	22 $\frac{3}{4}$	6 .75	2 .7
Ft Fore top mast	20 $\frac{3}{4}$	6 .1	2 .4
Zt Mizzen top mast	16 .1	4 .75	1 .9
Mg Main top gallant mast	11 $\frac{1}{2}$	23 .25	1 .28
Fg Fore top gallant mast	9 .9	2 .9	1 .26
St Spritsail top mast	8 $\frac{1}{2}$		

The lengths of the yards are proportioned to the length of the gun deck, and estimated in the same measure that the gun deck is, and are found in the same manner as the masts are.

Set *G D* against the length of the gun deck, and the lengths of all the yards will be found against their proper letters, as in the following examples, *viz.*

Admit the gun deck to be 174 feet; required the lengths of all the yards.

Set *G D* against 174; then the several lengths will be found as follows, *viz.*

	Feet.
M The main yard	103
F The fore yard	90 $\frac{1}{2}$
Z The mizen yard	82
Mt The main top sail yard	71
Ft The fore top sail yard	62 $\frac{1}{2}$
Mg The main top gallant yard	49
Zt The mizen top sail yard	47
Fg The fore top gallant yard	43 $\frac{1}{2}$

To find the diameter of any mast or yard, first find the length in feet. Set *F L* against that length, and you will find the diameter in inches against

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gainst the letters, representing the mast or yard, whose diameter is required as in the following, *viz.*

	Fect.		Diameter.
Main mast	114	L <i>m</i>	38
Fore mast	102	ditto	34
Bowspit	97 $\frac{1}{2}$	B	48 $\frac{3}{4}$
Main top mast	69	T <i>m</i>	20 $\frac{1}{2}$
Fore top mast	62 $\frac{1}{2}$	ditto	18 $\frac{3}{4}$
Mizen top mast	48	ditto	14 $\frac{1}{4}$
Main top gallant mast	33	—	9 $\cdot$ 9
Fore top gallant mast	30	—	9 $\cdot$ 0

The letters L *m* signify all lower masts. B the bowspit. T *m* all top masts, and top gallant masts.

The line for finding the lengths and diameters of masts and yards, is put upon the inside of the slider on the foot rules.

The End of the First Part.

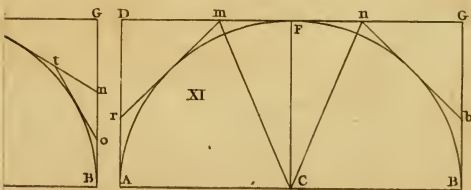
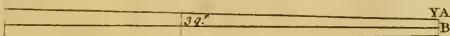
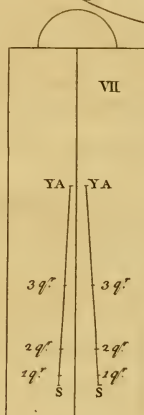
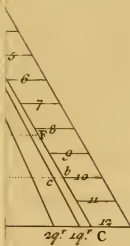
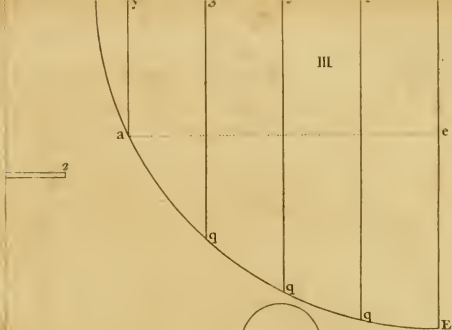


Fig I

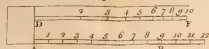
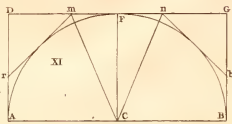
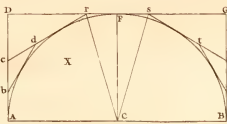
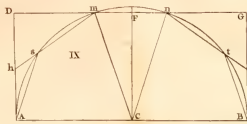
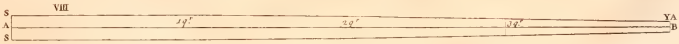
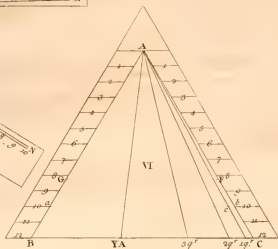
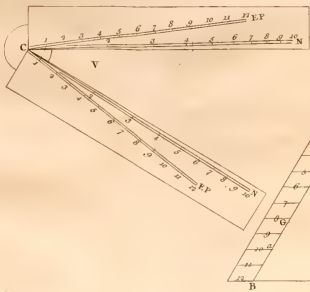
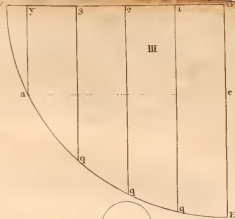
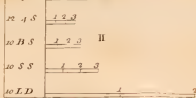
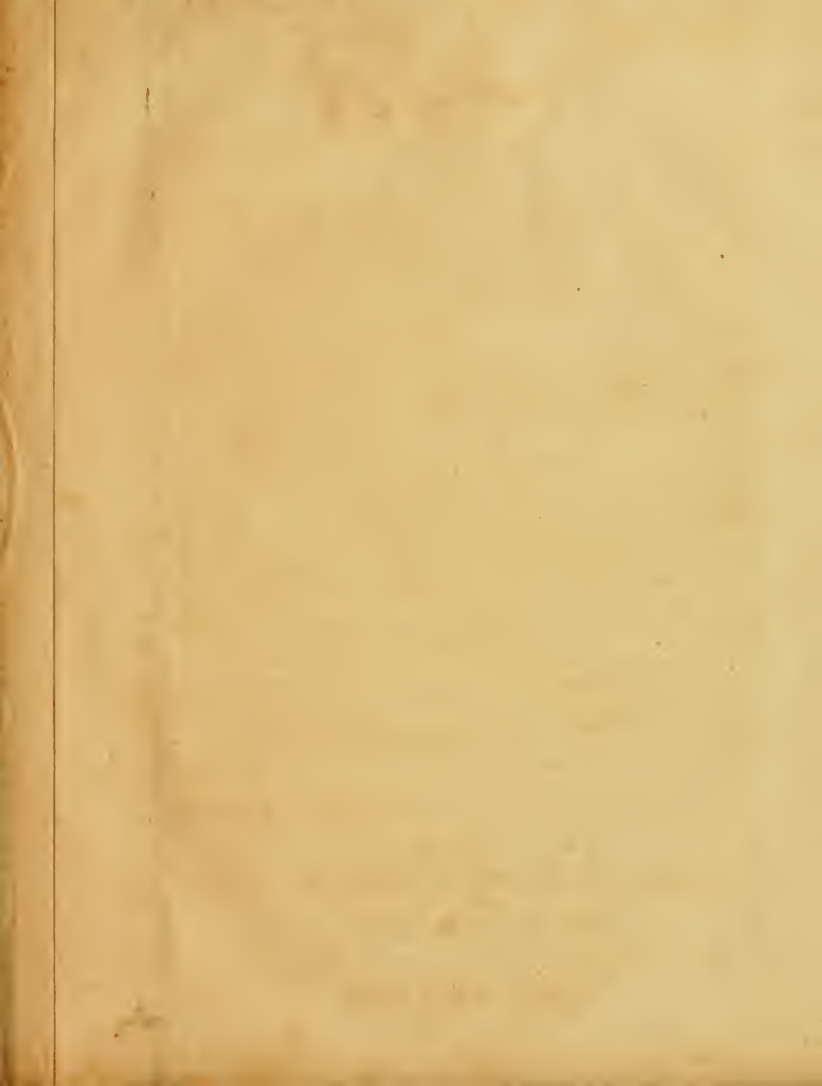
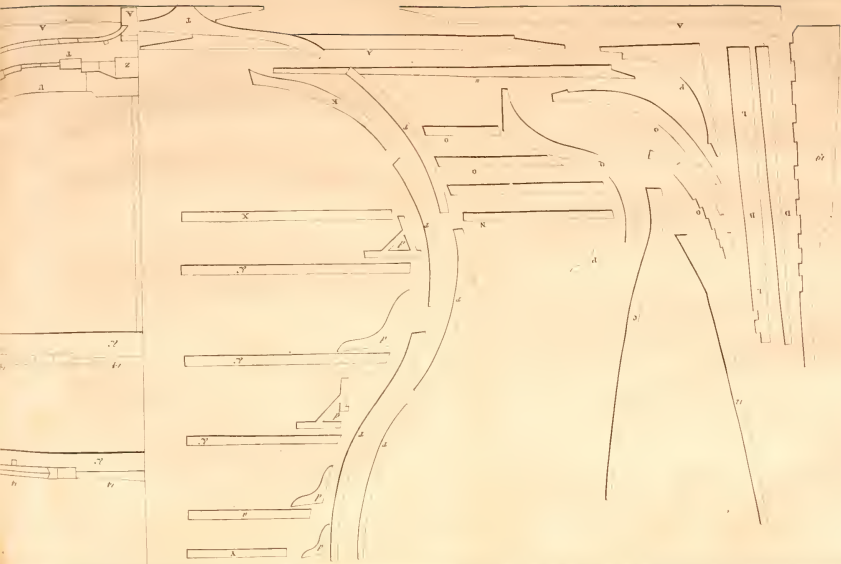


Fig II









THE THEORY OF SHIPBUILDING and NAVIGATION.

PART II.

C H A P. I.

Of the Orthographick projection of Solids on a Plane.

THE chief design of delineating a house, ship, or any other solid upon a plane, is to settle the just dimensions, and symmetry of its parts according to the scheme of the builder. When this is done by mathematical rules, we can find the exact length, breadth and heighth, not only of the whole, but also of any particular apartment on a sheet of paper. However, as a plane has but two dimensions, *viz.* length and breadth, and a solid three; they cannot all be represented by only one projection on the same plane.

A plane is an even surface, to which a right line may be every way applied, and upon which there are several ways of projecting solids. We shall only treat of the orthographic projection, as best suited to our purpose.

Before any solid can be represented by this way of projection upon a plane, it must be supposed to be cut by several planes: These are called plain sections, and will form even surfaces, which having but two dimensions, may be delineated upon a plane: And when the solid is cut so as to form an uneven surface, it is always supposed to be covered with an even one before it can be represented upon a plane; so that in effect, we only represent one plane upon another.

The thing to be represented is called the original, and the plane upon which it is to be represented, the plane of the projection.

When several lines parallel to one another, are drawn from all the parts of an original, to cut the plane of the projection; they will upon it describe a figure, which is called the projection of that original. The lines producing this figure, are called the projecting lines or rays; and this manner of representing any object, is called the orthographick projection of that object.

This parallelism of rays is the essential property which distinguishes the orthographick from all other kinds of projection: And tho' it is indifferent in what direction the projecting lines are drawn; it will be more convenient to make them perpendicular to the plane of the projection, and when this is parallel to the horizon, the length and breadth of any solid can be found by a plummet carried round it with a thread, so as to touch all the parts of it; but the height cannot be represented by this operation. This is what is called a plan of a building.

If another plane be erected perpendicular to the horizon, and the solid in the same position, supposed to be cut length ways by several planes parallel to one another, and perpendicular to the horizon; we can upon it, represent the true lengths and heights of all these sections; but instead of a plummet, we must make use of a square. This is what is called the plane of elevation, or side view of a building.

If another plane be erected perpendicular to the two former, we can upon it, represent the height and breadth of any section, cutting the solid right across, perpendicular to the horizontal and side planes. This, in a building, is called the profile, being an end view; in a ship, the head or stern view. By these three planes all the parts of a solid may be represented; and if two of the planes be known, the third may be found without having recourse to the solid.

By this description, it may seem that a house or ship, cannot be thus delineated till actually built. But it must be observed, that the extreme length, breadth and height, must be determined; by which the three planes aforesaid may be delineated. These may be called the outlines. The several parts contained within them may be delineated so as to answer the intended use; by which means we shall have a distinct view of the whole design, and may discover any inconveniencies that may attend such a disposition of the parts, which may be easily remedied upon paper; and the true dimensions of every particular may then be had upon the draught: Whereas, if we go to erect the structure without a draught, we run the hazard of pulling down several parts in order to make them uniform and convenient for the rest.

The

The delineating a ship upon a plane is called drawing, and the representation is called a draught.

Properties of the Orthographick projection.

1. All right lines upon an original plane, that are perpendicular to the plane of the projection, will be represented by points.

2. All lines on an original plane, that is inclined to the plane of the projection, will be represented by shorter ones, but if they be parallel on the original, they will be so when projected; and when the original is parallel to the plane of the projection, they will be exactly of the same length in both.

3. All planes, whether limited by right or curve lines, when perpendicular to the plane of the projection, will be represented by right lines.

4. All planes parallel to the plane of the projection, will be represented by equal and similar planes; but if they be inclined, their representations will be less than their originals.

5. The mutual intersection of two planes, is a right line common to both planes, and is called their common section; and if several planes intersect one another in the same line, it will be common to all the planes.

6. In the orthographick projection, the distance of the original from the plane of the projection, makes no alteration in its representation.

7. Before any plane can be represented upon another, its position with respect to the plane of the projection, must be determined.

8. The inclination of one plane to another, is the angle formed by two lines, one in each plane, both drawn perpendicular to the common section; where they meet at the same point.

In order to illustrate and demonstrate these properties, we shall in the following problems, shew the different representations of a plane according to its position, in respect of the plane of the projection; and to assist the imagination in conceiving why the same plane will have different representations; the figures are so contrived that they may be cut, and erected to any required angle with the plane of the projection.

P R O B. I.

Given, the plane $Ard a B C$; (*Fig. 5.*) upon which are drawn the lines $r1, d2, a3$, parallel to $A C$, and perpendicular to $B C$; required, its projection upon the plane $B M N T$, to which it is supposed perpendicular; so that $a C G$ shall be a right angle.

Draw

Draw the line Cg perpendicular to CG , and make it equal to CB ; (*Plate 4. Fig. 5.*) then will Cg be the projection of the plane $ArdaBC$. But if it were required to project it so that ACD shall be a right angle; draw the line CB perpendicular to CD , and BC will be the required projection of the plane $ArdaBC$: This will be manifest when the plane is erected, till the point A is perpendicular to the point C ; for then parallel lines drawn from the points r, d, a , will fall upon the line CB , and be represented by the points $1, 2, 3$; and the whole plane $ArdaBC$, will be represented by the right line BC , as by properties 1 and 3: For AC and CD being perpendiculars to CB , the common section of the two planes, will form a right angle at C , when the plane $ArdaBC$, is turned upon the axis BC till the point A is perpendicular to C ; and if then a thread be stretched through the points A and D , we shall have a right angled triangle, of which the thread is the hypotenuse, AC the perpendicular, CD the base, and ACD the right angle; tho' it was a right line before the plane was erected.

Note. The arch $BadrA$, and the radius AC , are to be cut through, and then the plane may be turned upon the axis BC .

P R O B. II.

Given the plane $ArdaBC$, to find its representation on the plane $BMNT$, to which it is supposed parallel.

This is only making a plane equal and similar to the given one. Therefore produce the lines $AC, r1, d2, a3$, to D, t, f, c ; make CD equal AC , $1t$ equal $r1$, $2f$ equal $d2$, and $3c$ equal $a3$; so shall $BCcftD$ be the plane required. For if the plane $ArdaBC$, be turned round upon the axis BC , till it is parallel to the plane; the point A will come to the point D , r to t , &c. and the parallels $r1, d2, a3$, will be projected into equal and parallel lines, as by properties 2 and 4. And the projection will be the same, if the original is lifted up to any distance above the plane of the projection, so it be parallel to it, tho' the projecting lines be oblique to the plane: As if it were required to project the plane $ArdaBC$ upon the plane $RPOS$, so that the point H shall represent the point B . Draw the line BH , and parallel to it, lines from the points $A, r, d, a, 3, 2, 1, C$. Make these lines each equal to the line BH ; so shall $H654kK$, be the plane required, equal and similar to the plane $ArdaBC$; we have omitted drawing the lines from $3, 2, 1$ and C to avoid confusion.

If the original is inclined to the plane of the projection, its representation will be less than the original, but its true dimensions may be found

found by the following problem; provided its inclination and representation be known, and the direction of the projecting lines.

P R O B. III.

Let the angle GCE be the inclination, and $BbesE$, the representation of a plane, and the projecting lines perpendicular to the plane $BbesE$; required $ArdaBC$, the original plane.

It is evident by the properties before described, that when the original is inclined according to the given angle, the projecting lines let fall from A, r, d, s , will fall in the points E, s, e, b , to which they are supposed perpendicular; and of consequence $EC, s1, e2, b3$, will the bases, and $r1, d2, a3$, hypotenuses of right angled triangles; of which the angles and bases being given, the hypotenuses may be found by constructing the triangles; or if the base and hypotenuse is given, the angle of inclination may be had in the triangle.

At the point E erect the perpendicular EG ; at C make the given angle, and draw the line CG to intersect the perpendicular in G ; then will the hypotenuse CG be equal to AC . This will easily be conceived by erecting the triangle GCE , till it is perpendicular to the plane $BbesE$; and when the plane $ArdaBC$ is inclined to it, according to the given angle, the lines AC and CG will coincide: In like manner the lines $r1, d2, a3$, may be found, by erecting perpendiculars at the points s, e, b , and drawing lines parallel to CG , from the points $1, 2, 3$, to intersect these perpendiculars, which will form so many right angled triangles: Their hypotenuses will be equal to the lines $r1, d2, a3$.

Note. The lines CG and GE , in the triangle are to be cut through.

P R O B. IV.

Given, the plane $ArdaBC$, and its inclination (the angle GCE) to the plane $BMNT$, to project it upon that plane.

This is only the reverse of the former, for here the hypotenuses and angles, are given to find the bases. Make therefore the given angle at C , and CG equal AC , from G let fall the perpendicular GE ; so shall the point E be the representation of the point A . Having thus constructed the triangle GCE , right angled at E ; the base CE will be the projection of the line AC . In like manner, the bases $1s, 2e, 3b$, may be found; by making $G1, G2, G3$, in the triangle, equal $r1, d2, a3$, in the plane $ArdaBC$: Then draw the lines $1s, 2e, 3b$, parallel to CE : These set off from the points $1, 2, 3$, in the line BC , upon perpendiculars drawn to those points on the plane $BMNT$, will give the lines

lines 1 s, 2 e, 3 b; so that B b e E C, will be the projection of the plane A r d a U C.

Our readers should be well acquainted with these problems: For we shall have frequent occasion to find the inclination of one plane to another. The next thing necessary to be understood is, when several planes are given, and their inclinations to one another, to find the dimensions of another plane which will intersect them in any assign'd position.

This will admit of several varieties; in order to explain which, we shall shew how to lay down a solid upon a plane. For this purpose we shall chuse one limited by six planes, like a chest, as being the simplest, and easiest to be represented of all solids; for if the ends be two equal right angled parallelograms, supposed to be parallel to one another, and perpendicular to the bottom; the sides will be equal, and parallel planes; the top and bottom will also be equal. But in order to make all the planes different, we shall suppose it broader at one end than the other, and the top sloping.

Let then C s u A, be the narrow, and D t b B, (*Plate 4. Fig. 1, 4.*) the broad end, right angled at u and A, also at b and B; their height at the back side B D and A C equal. The ends being supposed perpendicular to the bottom and back side, will occasion the sides likewise to be perpendicular to the bottom, tho' not parallel to one another; the angle T D t, or its equal s C S, will give the slope of the top; or its inclination to the back side. Now all the planes are different, and before their dimensions can be determined, the length of the chest must be given; which suppose the line A B. This will determine the dimensions of all the planes. And first to find the bottom. At the points A and B, draw the perpendiculars A E and B F; make B F equal B b, and A E equal A u, and draw the line E F; so the plane A B F E, will be the bottom. Secondly, to find the back side, produce the perpendiculars E A and F B, to C and D; making A C and B D, equal the given height of the ends, and draw the line C D; so shall the plane A C D B, be the back side. The bottom is the horizontal plane, and the back side, the plane of the elevation; and when this is erected perpendicular to that, the angles D B F and G A E, will be right. And if the plane D t b B be turned about upon D B as an axis; and the plane C s u A, turned round upon the axis C A, till they are perpendicular to the horizontal and elevation planes: Then perpendicular lines drawn from the points D and t, will fall in the points B and F, and from C and s in the points A and E; so that the plane A E F B, will be the projection of the top on the horizontal plane; which will be less than the original, because it is not parallel to the plane upon
which

which it is projected. But as its inclination is given, the true dimensions may be had by *Prob. 3.* The triangles being constructed, produce the perpendiculars BD and AC , to K and I , making DK equal to the hypotenuse Dt , and CI equal to the hypotenuse Cs ; and draw the line IK ; so shall the plane $CIKD$ be the top.

In like manner, the foreside may be projected upon the plane of elevation, which will be the plane $ASTB$: But this will be too short. Therefore from the points F and E , draw the perpendiculars EG and HF ; making FH equal to $t b$, and EG equal to su , and draw the line GH ; so shall the plane $EFHG$, be the foreside. The planes being thus found, they may be cut by the dotted lines, and erected to their proper position; and when the line IK , and the dotted perpendiculars are cut, the top may then be turned round upon the line CD , till it lies flat upon the plane $ACDB$, where it may be pressed down; by which means the line CD will remain immoveable, and acquire such a crease or fold, that the plane $IKDC$, may be turned round upon it as upon a hinge. In like manner, the dotted lines Cs, su, uA , (*Fig. 1.*) and the dotted lines Dt, tb, Bb , (*Fig. 4.*) may be cut; and the ends turned round upon the lines CA and DB , till they are laid flat upon the plane $ACDB$. And when the top is likewise laid flat upon these, the plane $ACDB$, may (together with the ends and top) be turned round upon the line AB , till it lies flat upon the plane $AGHB$; after which all the planes may be turned round, and laid in their first situation; and so they will all lie in one plane. We may now easily erect the chest; and first erect the ends till they are perpendicular to the back side; and the top may be turned round till it lays upon the lines Cs and Dt . And if the tongue x (*Fig. 1.*) be put in the slit x in the top, near I ; and the tongue a (*Fig. 4.*) in the slit a in the top near K ; then the backside, together with the ends and top, may be turned round upon the line AB , till it comes to be perpendicular to the bottom; and the point u (*Fig. 1.*) will be in the point E : And to keep it fast, the tongue y may be put in the slit y . In like manner, the point b (*Fig. 4.*) will be in the point F , and the tongue e may be put into the slit e ; so we have now the ends, top and backside fastened to the bottom: And if the dotted lines EG, GH, HF be cut, the plane $EGHF$ may be erected perpendicular to the bottom. The point G will be in the point s , and the point H in the point t ; and the tongue L , which may represent the hasp of a lock, may be put into the slit L in the foreside.

The chest being thus erected, it will be easy to make any partitions within it. But if the position of these partitions be known, their true di-

mentions, form and inclination may be found before the planes are erected; provided the dimensions and inclination of these planes to one another be given.

The horizontal and elevation planes are always perpendicular to one another. Now a plane may intersect these two in three different positions.

1st, When perpendicular to both. Its intersection in both planes will then be in a right line perpendicular to the line AB , the common section of the two planes, as *Fig. 1.* which intersects the elevation plane in the line CA , and the horizontal in the line AE . This is what the shipwrights call a square plane.

2^{dly}, When inclined to the plane of elevation, but perpendicular to the horizontal, as *Fig. 2.* which intersects the plane of elevation in the line GH perpendicular to AB , and the horizontal in the line Hb : So its inclination to the plane of elevation, will be the angle nHb . This the shipwrights call a canted plane. But if the plane $AEFB$, be the plane of elevation, and the plane $ACDB$, the horizontal; the plane intersecting them in the lines GH and Hb , would then be called a raking plane.

3^{dly}, A plane may be inclined to both, as *Fig. 3.* intersecting the plane of elevation in the line MN , and the horizontal in the line Nq . This plane is said to cant and rake. Their positions being thus determined, their dimensions are likewise determined.

P R O B. V.

To find the Dimensions of a square Plane, intersecting the common Section of the Horizontal and Elevation Planes in the Point A.

Through A draw the perpendicular IE , intersecting the common section of the top and backside in the point C , and the common section of the bottom and foreside in the point E . Make a right angle at A , because the required plane is supposed perpendicular to the horizontal; and make Au equal to AE , the breadth of the bottom at that place. Again, because the foreside is perpendicular to the bottom, make a right angle at u , and us equal to EG , which is the breadth of the foreside at that place: Lastly, draw the line sc ; so shall Cs be equal to CI , the breadth of the top at that place; and $Cs \perp A$ the required plane, the angle SCs , being the inclination of the top.

The profile, is the proper plane to project all square planes upon, as being parallel to it. Now the plane $DtbB$, is the profile: And if it were required to find the dimensions of a plane, intersecting the horizontal in the line AE , and the elevation in the line AC ; it is only setting
off

off $A E$ from B to p , and drawing $p o$ parallel to $B D$; then will $D o p B$, be the plane required, equal to $C s u A$; and $r o$ equal to $S s$. For $B D$ and $C A$, are equal by supposition.

P R O B. VI.

To find the Dimensions of a Cant Plane, intersecting the Horizontal Plane in the Line $H b$, and the Elevation in the Perpendicular $G H$.

From the point b let fall the perpendicular $b w$, and produce it to intersect the line $S T$ in W , and draw the line $G W$; then will $G W w H$, be the projection of it upon the plane of elevation. Which will be less than the original, because it is inclined to it; but its true dimensions may be found by *Prob. 3*. Thus, make $H n$ equal to $H b$, and raise a perpendicular at n : Make $n m$ equal to $w W$, and draw the line $G m$; so shall $G m n H$, be the required plane. The angle $n H b$, its inclination to the backside; and the angle $H b E$, the inclination to the fore-side. The inclination of the top will be the same as that of the bottom, which in this is a right angle.

P R O B. VII.

To find the Dimensions of a Plane that Rakes and Cants; intersecting the Horizontal in the Line $N q$, and the Elevation in the Line $M N$.

1st. From q let fall the perpendicular $q z$; thro' z draw the line $P f$, perpendicular to $M N$, produced to f . From the center N , with the radius $N q$, intersect the perpendicular $f p$ in P ; and draw the dotted line $N P$. Again, thro' M draw the line $M R$ parallel, and equal to $N P$, and draw the line $P R$; so $M N P R$, would be the required plane, if the bottom were as broad at the point v , as it is at the point z , and the top parallel to the bottom. But as this is not the case here, the required plane will be narrower at the top than at the bottom.

2^d. From M let fall the perpendicular $M v$, and draw the line $v V$, parallel to $N q$; make $M m$ equal to $v V$, and draw the line $P m$. So $M N P m$, would be the required plane, if the top was parallel to the bottom.

3^d. Make $N P$ in the line $N q$, equal to $v V$, and draw the perpendicular $P r$, and make $M n$ equal to $N r$. So a line drawn from r to n , would be parallel to $M N$, and would intersect the line $S T$ somewhere; and a line drawn from that point of intersection to M , would be the projection of a plane intersecting the plane of elevation in the line $M N$; and the hori-

zontal plane in the line NP . But there will be no occasion to draw this line, because Nq is the length of the line of intersection of the required plane; and z the projection of the point q . Therefore a line must be drawn from z to n , which will intersect the line ST in x ; so shall $MN \approx x$, be the projection of the required plane, upon the plane of elevation, which will be less than the original. To find which, thro' x draw a perpendicular to the line MN , to intersect the line Pm in the point d ; so shall $MdPN$, be the plane required. And to find its inclination to the plane of elevation, draw the line Zz parallel to fM , and produce it to t : Make ft equal to fp ; so shall zft , be a right angled triangle; and the angle zft , the inclination to the backside. For if the triangle be erected perpendicular to the plane, as mentioned in *Prob.* 3. the lines ft and fp , will coincide. To find the inclination to the fore-side, make the angle zNo , equal to the angle zft ; so shall NoV , be the required angle. For if the lines AB and EF were parallel; the inclination to the backside and fore-side, would be equal. Lastly, to find its inclination to the bottom; thro' the point v draw a perpendicular ab to Nq , produced to a . From N , with the radius NM , intersect the perpendicular in the point b . With the radius ab intersect the line vV , produced in the point c ; so shall avc , be a right angled triangle, and the angle cav , the inclination to the bottom. For if the plane $MNP R$, was projected upon the horizontal plane, the point M would be elevated till a perpendicular from it would fall upon the point v ; so av would be the base, and ab the hypotenuse of the right angled triangle avc .

DEMONSTRATION.

At N erect the perpendicular NX , make Ns equal to Nq ; then will $NsYX$, be the true dimensions of a plane intersecting the elevation in the line XN , and the horizontal in the line Nq , by the preceding problem; and the angle XNq , a right angle. For when the plane $ABCD$, is erected perpendicular to the plane $ABFE$; if the plane $XNsY$, be turned round upon the axis XN , the point s will describe the semicircle $sgcb$, as if a door be opened, and turned upon the hinges till it lies against a partition or wall, the bottom of it will describe a semicircle upon the floor. But if the upper hinge, suppose at X , be moved to M , it is plain the point q in the bottom of the door will not touch the floor.

And if the door be turned round upon the axis MN , till it lies flat upon the partition, or plane $XNsY$; the line Nq will lie upon the perpendicular Ne ; the angle MNq being always a right one; for the bot-
tom

tom of the door is square. But the required plane must touch the floor when it is in the direction of the line Nq ; therefore it cannot be square: And because the extremity of the bottom, when the plane is in its proper position, will be so far elevated above the plane $MNsR$, that a perpendicular from it will fall in the point z ; it is obvious, that in turning the door upon the axis MN , a perpendicular let fall from the extreme point of the bottom, will always meet the plane $MfPR$, somewhere in the perpendicular fz produced; and therefore when the door is laid flat upon the partition, or plane $MNPR$, that extreme point must lie upon the point P , because the dotted line NP , is made equal to Nq ; and when PR is drawn parallel to MN , and MR parallel, and equal to NP , we shall have the plane $MNPR$; so the angle MNP , will be that which the bottom makes with the side of the door. By which means the triangle NkP , will be added to the square bottom; and when the door is turned upon the axis MN , till it is in the direction of the line Nq ; the dotted line NP will lie close upon the floor on the line Nq : When the door is in this position, if the stock of a square be laid upon the plane $MNs y$, which may represent the partition of a room, and the tongue erected perpendicular to the plane, we may describe the line zn , by keeping the stock of the square perpendicular to the line MN ; and moving it along the plane in that direction, the tongue always touching the side of the door. All which will appear very plain when the planes are cut by the dotted lines, and erected to their proper positions.

Thus I have endeavoured to explain the principles of the orthographic projection, by laying down the simplest solid that can well be thought of, or conceived: And if I have not done it in such a manner as to make it intelligible to all capacities, it may be owing to their want of a sufficient knowledge of the principles of geometry and trigonometry.

If they cannot attain this by what has been said on those subjects in the first part, I would advise such to have recourse to a master for instruction. If they are at a loss how to lay down a solid limited by six planes, which requires no curves, it will be in vain for them to proceed any farther. For it will be impossible for them to comprehend the reason of the methods used in laying down irregular solids limited by various surfaces, some of which are plain, and others very irregular curves.

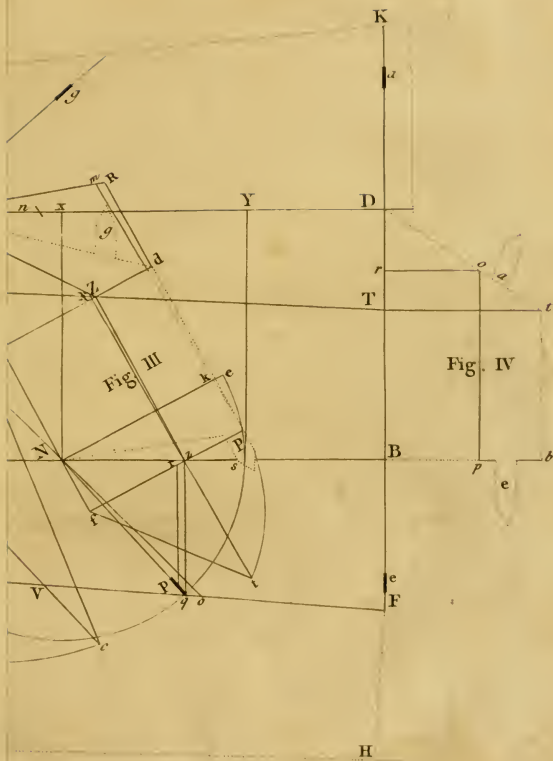
In laying down any irregular solid, we may suppose it to be inclosed within six planes, the opposites of which may be equal and parallel, and all the planes right angled; so we may then proceed upon the same principles as before, for we shall have the dimensions of the three necessary
planes,

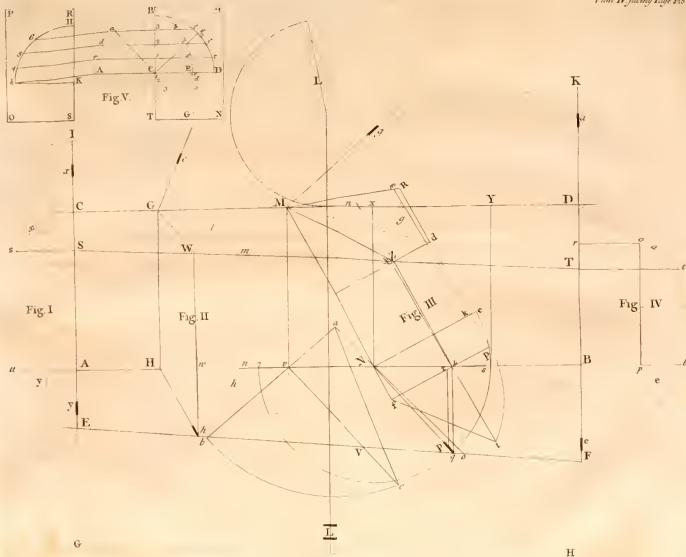
planes, viz. horizontal, elevation and profile. And if the true form and dimensions of two planes parallel to the profile, and likewise their distance from one another be given; we may find the true form and dimensions of any intermediate planes between them, in the same manner as we have done in that already laid down. But it must be observed, that the form of the surface, which limits that part of the solid intercepted betwixt the two given planes, must likewise be given; which shall be shewn plainly when we come to apply what has been now said to the actual laying down of ships on a plane.

C H A P. II. S E C T. I.

Explication of the Terms and Names of the Lines used in drawing Ships.

WHAT is chiefly intended in drawing of ships, is to find the form of all the timbers. Now if a ship's side were strait fore and aft, this could be done with as much certainty, as in the solid laid down in the preceding chapter. In that case we need only determine the form of the foremost and aftermost timbers, and then all the sections, either parallel or inclined to the horizontal, cutting the ship lengthwise, would be limited by strait lines; any two points of which being given, a ruler or strait edged batten would find the whole line. And tho' all sections parallel to the profile may be limited by curves, and these very irregular, yet if a sufficient number of points be found in each, a thin batten of pliable wood may be bent so as to touch all the points; and so describe what is called a fair curve. Now we may have as many points as we please, only by forming as many sections, either parallel or inclined to the horizontal, as shall be thought necessary to have points: For upon supposition that the side is strait fore and aft, these sections will all be limited by strait lines, their lengths will be the given distance betwixt the two parallel planes, and their breadths will be determined by the direction in which they cut the profile. This is so plain that it needs no example,





ample, and a case that cannot happen in laying down a ship; which being broader near the middle than at either end, cannot have a strait side. And therefore there will be a necessity of determining the form of the profile, by a section at the broadest place of the ship; and also the form of a section near each end parallel to the profile.

There can be no invariable rule given to determine the form of these three sections, because they must be conformable to the service the ship is designed for: However, determined they must be, before we can begin to find the form of the intermediates; if by no other means, by repeated trials, till they please the fancy of the artist, assisted by his judgment in discerning what form will best answer the proposed design. The next thing to be determined, is the form and dimensions of several sections, either parallel or inclined to the horizontal. They will all be limited by curves, some of which will be very irregular. However there will be three points in each section given, by the direction in which they cut the given planes.

Here we think it necessary to explain what is to be understood by a fair curve; a term frequently used in drawing: In order to which it must be observed, that the circumference of a circle seems to be the only curve that can with certainty be drawn; for this requires no art. But in describing an ellipsis we must find several points, thro' which the curve must pass according to the established properties of that curve; and then these points may be joined by a steady hand, which may be assisted by a mould or pattern, of which the artist should be provided with a variety of different sorts made of pear-tree, box, or some other wood proper for that purpose. Now if one that is not very well acquainted with drawing, should attempt to join these points without some such assistance, he would make rather a polygon, consisting of several very obtuse angles, than a curve; whereas a mould that will just touch three, four, or more of the points, will cut off all these angles and irregularities, by which means the curve will be clear of all breaches, or sudden turnings and deviations; and this is what is called a fair curve. From this description of it, we may plainly see that, if there be only three points of the curve given, there may be several curves drawn thro' them, and all very fair.

In forming an ellipsis, if the transverse and conjugate diameters be given, we may with certainty, find any number of points, thro' which the curve must pass. But the curves that are formed by the several sections of a ship, are very irregular; and as they have no properties peculiar to themselves, except that of being fair, there can be no invariable rule for describing them. We shall therefore attempt to do it by sector lines

lines taken from an approved body. This will undoubtedly form lines similar to the original; and the sector is so contrived, that it will form very fair lines quite different from the original. It is presumed this will be very useful to those that are not well acquainted with drawing, for whose use it is chiefly intended. However, before we describe it, we shall shew the methods generally used to regulate the form of all the curves that are necessary in drawing of ships; and in the first place shall explain their names in the following definitions. We shall make use of three planes, as described in the preceding chapter, but give them different names.

D E F I N I T I O N S.

1. The sheer plane is the same with that of elevation, and is a section of a ship supposed to be cut, by a plane passing thro' the middle line of the keel, stem and stern-post.

2. The floor plane is the same with the horizontal, and is that on which the whole frame is erected: The upper side of the keel is in this plane.

3. The body is the same with the profile. It is a section, supposed to cut the ship thro' the broadest place, and is perpendicular to the sheer and floor planes.

4. Water lines are supposed to be drawn on the surface of a ship, by the upper part of the water into which she swims, and are formed by the section of a plane cutting the whole body lengthways, perpendicular to the sheer plane, where they will always be represented by strait lines; and if these are parallel to the keel, they will be represented by strait lines on the body plane, called level lines. But these planes will be limited by curve lines on the floor plane, which in some cases will be inverted at the after end, and also at the fore end; but this last is avoided as much as possible. These curves limit the breadth of the ship at certain heights, expressed by lines drawn on the sheer plane for that purpose. But as the sheer plane cuts the ship exactly in two equal and similar parts, only one half of these sections are laid down; so that one side will always be represented by a strait line.

5. The heights of the breadth lines are described on the sheer plane; to determine the heights at which the half breadth of the planes of the timbers are to be set off; in which respect, the water lines on the sheer plane may be called the heights of the breadth lines. But because the extreme breadth of the plane of each timber rises gradually from the midships fore and aft; the lines representing their heights will be curves.

Such

Such are the main, and top timber heights of breadth lines, which are the principal ones that go by that name; but there may be as many more as shall be judged necessary to determine the form of all the timbers.

6. Half breadth lines are described on the floor plane, and are curves limiting the $\frac{1}{2}$ breadths of the planes of the timbers, at the heights expressed by the corresponding height of breadth on the sheer plane, in which respect the water lines may be called $\frac{1}{2}$ breadth lines; but that name is generally given only to such whose heights are expressed by curves on the sheer plane. They are formed by supposing the ship to be cut length ways, in a perpendicular direction to the sheer plane, thro' a curve height of breadth line. This will form an uneven surface; so the true length of it is not represented on the floor plane.

7. Ribband lines are either square or canted.

The square which is often called the horizontal ribband line, is in all respects the same with the above described $\frac{1}{2}$ breadth lines: The use of the ribband lines, is to fasten the timbers before the plank is brought on; for which purpose, they must be of a sufficient substance, and formed in such a manner, that they may fit the timbers without forcing or penning them. It would be very difficult to make a square one, because it rounds two ways; for when the ship is cut upon a level by this ribband, the surface produced, will be an uneven one. Upon this account, a plane must be so inclined to the sheer plane, that it shall intersect the timbers at the same points with the square ribband.

The cant or diagonal ribband, so called, because it cuts the body plane in a diagonal, is formed by a plane inclined to the sheer plane; and cutting the ship length ways in that direction, in such a manner, that it will intersect the timbers in the same points that the square ribband does. It will intersect the sheer plane in a strait line parallel to the keel, at the same height at the stem and post, with that of the square ribband. Its representation on the floor plane, will be the same as that of the square ribband. But because the plane of it is not parallel to the floor plane; this will not be the true breadth of it. This may be found by *Prob. 3.* of the preceding chapter, by which means we shall obtain the exact length and breadth of it; and being a plane, it will only round one way, and so a ribband may easily be made by it.

8. Sweeps, are arches of circles, described in the body plane to form the timbers, and are generally four.

1st. The floor sweep, which is limited by a line drawn in the body plane, perpendicular to the middle line, a little above the keel. The distance of this line above the keel at the midship timber, is called the dead rising; the upper part of this arch forms the head of the floor timber.

2d. The under breadth sweep; the center of which is in the line that represents the height of the extreme breadth of the timber. If there is a part of the timber strait, the center of the sweep will be in the lower line. From this center extend to the point that limits the $\frac{1}{2}$ breadth of the timber in the same line, and with that radius describe a circle downwards, till it comes near to the floor sweep.

3d. The reconciling sweep, which joins the two former in such a manner as to intersect neither; by which means we shall have a fair curve from the height of the breadth to the rising line: And if a strait line is drawn from the side of the keel at the upper edge, to touch the back of the floor sweep, we shall have the form of the midship timber below the breadth.

4th. The upper breadth sweep; the center of which is in the line that represents the extreme upper height of the breadth of the timber; from which a circle must be described to pass thro' the point that limits the $\frac{1}{2}$ breadth of the timber in the same line, and produced upwards discretionally to form the top timber. To these four some add a fifth, to form the hollow of the top timber; but this is generally done by a mould so placed as just to touch the above breadth sweep, and pass thro' the point that limits the $\frac{1}{2}$ breadth of the top timber: So that now the form of the midship timber is determined from the keel to the top of the side. The radius of the underneath sweep decreases, the farther the timber is from the midships; but the other sweeps have generally the same radius for all the timbers. There is no certain rule to determine the radii of these sweeps. Some do it by proportioning them to the extreme breadth of the ship, according to some given ratio. But there are various ways of forming this midship timber; sometimes by two arches, and instead of having a strait line from the edge of the keel to touch the back of the floor sweep in some ships it is made a hollow.

9. Half breadth of the floor is the distance of the center of the floor sweep from the middle line in the body plane, at the midship timber, which will always be less than the distance betwixt the point where the strait line drawn from the side of the keel to touch the back of the floor sweep is from the middle line. This last may be called the true $\frac{1}{2}$ breadth of the floor, and in sharp ships will be above the rising line.

10. Rising of the floor, is a curve drawn on the sheer plane, limited at the midships by the dead rising; and in flat ships it runs nearly parallel to the keel for some timbers before and abaft the midship, for which reason these timbers are called flats; but in sharp ships it rises gradually from the midship till it ends on the stem and post. To this line is some-

times;

times adapted an $\frac{1}{2}$ breadth of the floor line. The use of these two lines is to find the centers of the floor sweeps.

11. Cutting down line, is drawn on the sheer plane. It is limited in the midships by the thickness of the floor timber, and abaft by the breadth of the kelson; for it must be carried so high abaft as to leave room for the kelson, for which purpose the thickness of the timbers must be known. It must be carried up so high upon the stem as to leave sufficient substance for the breeches of the rising timbers. The lower edge of the kelson is in this line; so it limits the thickness of all the floor timbers, and likewise the height of the dead wood afore and abaft.

12. Timber and room, or room and space, is the distance betwixt the moulding edges of two timbers, which must always contain the breadth of two timbers, and sometimes two or three inches between them. It must be observed, that one mould serves for two timbers; the fore-side of the one being supposed to unite with the aft-side of the other, and so make only one line, which is actually the case in all the frames, which in some ships are every third, in others every fourth timber. The frames are first put up, and fastened to the ribbands, and afterwards the others are put up, which are called filling timbers. The midship timber is called dead-flat, and distinguished by this character $\oplus$; the timbers abaft the midship are distinguished by the figures 1, 2, 3, &c. and those before the midship by the letters of the alphabet A, B, C, &c.

S E C T. II.

OF WHOLE MOULDING.

THE length of the keel, extreme breadth, depth in the hold, height between decks, and in the waste; and sometimes the height and breadth of the wing transom are agreed on by contract in the merchant's service: From which dimensions the builder is to form a draught suitable to the trade the ship is designed for.

The first thing that is generally done, is to lay down the keel, stem and post, upon the sheer planes: Then to determine the proper station of the midship timber, where a perpendicular is erected: It is generally about $\frac{2}{3}$ of the keel before the post. On this line the given depth of the hold is set off from the upper side of the keel; to obtain which point, the thickness of the timber and plank must be added to that agreed on

by contract. This being fixed, will enable us to determine the upper height of the extreme breadth at that place, which sometimes is the very point itself. The lower height of the breadth must likewise be determined at this place. Then we may form the two main heights of the breadth lines which nearly unite abaft and afore. Abaft, these curves end at the wing transom, or above it; and afore, they are carried up sometimes as high as the hawse holes. The height of the breadth line of the top timber must likewise be formed. This is generally done by a bow, which makes nearly an arch of a circle. It is limited in midships by contract, afore and abaft only by the fancy and judgment of the artist, according to what sheer he designs: We must also form a line for the rising of the floor; for which purpose we must determine the dead rising, which is that of the midship timber. This limits it at that place, and in the whole moulding it is pretty near parallel to the lower height of the breadth line. These lines must absolutely be drawn on the sheer plane; and corresponding to the main and top timber height of breadth lines; there must be two half breadth lines formed on the floor plane.

The main half breadth at the midship timber is agreed on by contract, only observing that the thickness of the timber and plank must be deducted out of it, because it is the extreme breadth from outside to outside of the plank that is contracted for. Those in the draughts are called moulded half breadths: Then the breadth at the wing transom, if a square stern, is limited: It is generally about two thirds of the extreme breadth, but this is just as the artist shall think proper. He also fixes the breadth of the top timber, and then describes the two half breadth lines. In the due formation of these curves on the sheer and floor plane, the whole art of drawing chiefly consists; which must be acquired by practice, so that it will be scarce possible for one that is not very well acquainted with drawing, to form them, without having recourse to some other draughts. After these are formed, the stations of the timbers are fixed, if the room and space, and the breadth of the midship timber is agreed on by contract, this will determine the station of all the timbers; observing that the timbers abaft the midships must be set off from the fore side of the midship timber; and the timbers before the midship from the aft side of it. At every third or fourth timber there must be perpendiculars drawn on the sheer and floor planes, to the line that represents the lower edge of the keel, which is the common section of these two planes; tho' sometimes the half breadth lines are described on the sheer plane, when there is not space to produce the perpendiculars till they be of sufficient length to contain the height of the breadth and half breadth.

After

After the timbers are stationed, and the perpendiculars for the frames drawn on the sheer and floor planes; we proceed to the body plane, and draw a line equal in length to the whole breadth moulded. This line may be called the base of the body plane. A perpendicular is erected at each end of it, and one in the middle, which may be produced at pleasure. The next thing to be done, is to form the midship frame: The limits of it are had from the sheer and floor planes; the lower, upper and top timber heights of the breadth are taken from the sheer plane at the perpendicular, representing the midship frame, and set off on the middle line of the body plane from the base. Thro' these points, lines are drawn parallel to the base, and the respective half breadths corresponding to each, are set off on these lines, from the middle line in the body plane. The lower and upper main half breadths are limited by the perpendiculars already drawn at each end of the base. The half breadth of the top timber, is had from the floor plane on the perpendiculars representing the midship frame. The height of the dead rising is likewise taken from the sheer plane, and set up from the base upon the middle line in the body plane, thro' which point a line parallel to the base must be drawn; and upon this line the half breadth of the floor, is set off from the middle line, at which point a perpendicular is erected. The center of the floor sweep is in this line, from which a circle must be described that shall just touch the rising line. A proper radius for the under breadth sweep is next to be found: The center of it is in the lower breadth line, from which it is described to pass thro' the point which limits the half breadth. After which the radius, and center of a reconciling sweep to join the floor, and under breadth sweeps is found, and the circle described; and to compleat the frame below the breadth, the half breadth of the keel is set off from the middle line on the base; from which point, a strait line is drawn to touch the back of the floor sweep.

By this way of forming the frame, it is plain the centers and radii of the sweeps are arbitrary, but they must be determined before any of the other timbers can be formed; if by no other means, by repeated trials, till they are made to please the fancy and judgment of the artist. But there are various other ways of forming this frame; so that, tho' several ships may be of the same breadth, depth in the hold, and dead rising; they may all differ in the form of their timbers. After this midship timber is formed, a pattern or mould is made to fit exactly to the curve, and the dead rising line. By this, and a hollow mould, all the timbers are formed so far as the rising line, and lower height of the breadth line are parallel to one another in the sheer plane: This is what is called whole mould-

moulding, which we shall illustrate by laying down a long boat. And because in several mould lofts there is not sufficient length for the sheer plane, it is often laid down as if it were cut by the midship frame, and one part laid upon the other in such a manner, that the midship timber of the after part shall coincide with a perpendicular let fall from the fore part of the stem.

To lay down a Long-Boat 29 Feet 1 Inch long, and Breadth Moulded 9 Feet. (See Plate 5.) *Pl. 17.*

1st. Draw the strait line P $\oplus$, and erect the perpendicular P T. From the point P set off 29-1, the given length of the keel. But because the plate will not admit of the whole length, let the station of the midship timber be assigned; at which point erect the perpendicular $\oplus$ M. Let M be the upper, and N the lower height of breadth, at that place; T the height of breadth at the transom, and draw the curve T M to represent the sheer, or extreme height of the side. This in a ship would be called either the upper height of breadth line, or the upper edge of the wale. Draw also a curve thro' the point N, parallel to T M, to represent the breadth of the upper strake in a boat, or lower edge of the wale, if in a ship. The dotted line T N may also be drawn to represent the lower height of breadth.

2^d. Set off the rake of the post from P to *p*, and draw the line *p t* to represent the aft side of the post; so shall T *t* represent the round up of the transom. Set off the breadth of the post from *p* to *r*, and from T to *s*, and draw the line *r s* to represent the fore side of the post, which may either be a curve or a strait line at pleasure. Set up the height of the tuck from *p* to *k*. Let *k x* be the thickness of the transom, and draw the line *z x* to represent the fore side of the transom.

3^d. Set up the dead rising from $\oplus$ to *d*, and form the rising line *r i s*. We may then draw the line K L parallel to P $\oplus$, to represent the lower edge of the keel, and another to represent the thickness of the plank or the rabbet. The rabbet on the post may likewise be represented, and the stations of the timbers assigned; distinguished in the plate by their proper names, viz. $\oplus$, ①, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Thus have we completed the sheer plane, or side draught for the after body; and in like manner is that for the fore body to be done. First produce the line $\oplus$ M to *y* the height of the fore part of the stem, and form the stem either by sweeps or some other contrivance. The breadth of the stem must be known, and the aft side likewise formed. The stem being formed, we may set off from the fore part of it as much as the line

P $\oplus$ wanted of the whole length of the boat, which suppose $\oplus \oplus$. Erect the perpendicular $\oplus F$, and make it equal to $\oplus M$, the height of the sheer, and form the curve $F S$, which will represent the sheer or height of the side in the fore body. We may likewise draw a line to represent the lower part of the upper strake, and one for the lower height of the breadth. The rising line must also be formed, and the timbers stationed and distinguished by their proper names $\textcircled{A}$, $\oplus$, A, B, C, D, E, F, G, H. Now the whole sheer plane is completed; for if the line $S T$ was drawn afunder, till the point F came to the point M , we should have the whole length of the boat.

The next thing to be done is to form the half breadth line; for which purpose the perpendiculars $T P$, 9, 8, &c. must be produced. Then, from the point where the perpendicular $\oplus M$ intersects the line $K L$, set off 9 feet, the half breadth: Set off also the half breadth at the transom, from the line $K L$, and form the half breadth line $B b$. In like manner set off the half breadth, from the point where the perpendicular $\oplus F$ intersects the line $K L$, and form the half breadth line $R X$, according to the designed round of the harpin.

We may now proceed to form the timbers in the body plane: Where, let $A B$ be the breadth moulded at $\oplus$. Erect the perpendicular $C D$ in the middle of the line $A B$; and parallel to $C D$ draw the lines $n m$, the half thickness of the post, and $x y$ the half thickness of the stem. Then take off the several portions of the perpendiculars $\oplus$, 1, 2, &c. intercepted betwixt the upper edge of the keel, and the rising line in the sheer plane; and set them up from C upon the line $C D$. Thro' these points draw lines parallel to $A C$; take off also the several lower heights of breadth at $\oplus$, 1, 2, &c. from the sheer plane; set them also up from C upon the middle line in the body plane, and draw lines parallel to $A C$ thro' these points: Then take off the several half breadths corresponding to each, from the floor plane; and set them off on their proper half breadth lines, from the middle line in the body plane.

We must in the next place form the midship timber, either by two, or three sweeps, or some other contrivance which must be left entirely to the fancy and judgment of the artist: If he uses three sweeps, a proper center of each must be found. That of the under breadth must always be in the breadth line, as in this case at a . The center of the floor sweep, suppose at C , must be so, that when it is described, the back of the sweep may just touch the rising line. These two centers being found, and the arches described, we may with certainty find the center of the reconciling sweep, provided the radius be known, thus: Set off the given radi-

ns upon any strait line, as from A upon the base line to R; then take the radius of the lower breadth sweep, which set off from A to *a*; take also the radius of the floor sweep, and set off from A to *c*: Then with the radius *a* R, from the center of the under breadth sweep, describe an arch; and with the radius *c* R, from the center of the floor sweep, describe another arch to intersect the former in *r*, which will be the center of the reconciling sweep: For if with the radius A R, from the center *r*, we describe an arch, it will just touch the under breadth, and floor sweeps in the points *t* and *s*. A line drawn from *r* to *t* will pass through the center of the underbreadth sweep, and a line drawn from *r* to *s* will pass through the center of the floor sweep; so that it will be impossible for the reconciling to intersect either of the other two arches. The curve part of the timber being formed, a strait line must be drawn from the side of the keel to touch the back of the floor sweep; for which purpose the half breadth of the keel must be set off on each side of the point C, upon the base line. The form of the midship frame being determined, will in some measure determine the form of all the rest. For if a mould be made on any side of the middle line to fit the curve part of it; and the rising line, as that marked B E N D, and laid in such a manner that the lower part of it, which is strait, may be set upon the several rising lines, and the upper part just touch the point of the half breadth in the breadth line, corresponding to that rising upon which the mould is placed; a curve may then be drawn by the mould to the rising line. In this manner we may proceed so far as the rising line is parallel to the lower height of the breadth line. Then a hollow mould must be made, the upper end of which is left strait, as that marked H / *w*. This is applied in such a manner, that some part of the hollow may touch the side of the keel, and the strait part touch the back of the curve before described by the bend mould; and, beginning abaft, the strait part will always come lower on every timber till we come to the midship timber, where it comes to the side of the keel. Having thus formed the timbers, so far as the whole moulding will serve; the timbers abaft them are next formed. Their half breadths are determined by the sheer and floor planes, which is the only fixed point thro' which the curve of these timbers must pass. Some form these after timbers before the whole is moulded, and then make the hollow mould, which will be straiter than the hollow of either of these timbers. It is indifferent which are first formed, or what methods are used; for after the timbers are all formed, tho' every timber may appear very fair, when considered by itself, it is uncertain, what the form of the side will be. In order to find which, we must form several rib-

band

band and water lines; and if these do not make fair curves, they must be rectified, and the timbers formed from these ribband and water lines. In using the hollow mould, when it is applied to the curve of each timber, if the strait part is produced to the middle line, we shall have as many points of intersection as there are timbers: And if their heights above the base be transferred to the corresponding timbers in the sheer plane, a curve passing thro' these points is what is called a rising strait. This may be formed by fixing a point for the aftermost timber that is whole moulded, and transferring that height to the sheer plane. The curve must pass thro' this point, and fall in with the rising line, somewhere abaft $\oplus$: And if the several heights of this line be transferred from the sheer, to the middle line in the body plane; these points will regulate what is called the hawling down of the hollow mould. The timbers being thus formed, and proved by ribband and water lines; we may then form the transom. This may be done either by ribband or water lines. Here it is by water lines, of which there are three, formed in the following manner. (*Plate 5.*)

1st. Draw three lines in the body plane parallel to the base: These are called level lines, and may be equally spaced betwixt the tuck, and height of the sheer, taken upon a perpendicular to the keel. They may be likewise drawn on the sheer plane at the same heights, so they will be parallel to the keel. They are distinguished by 1st. 2^d. and 3^d. W^r.

2^{dly}. To form them on the floor plane. Take the distance betwixt the middle line in the body plane, and the several intersections of the level lines with the timbers: Transfer these to the corresponding timbers in the floor plane, which will give the points thro' which the curves will pass: So the portion of the first level line in the body plane, intercepted betwixt the middle line and timber 9, will be equal to the distance taken upon the perpendicular in the floor plane, drawn from the point 9, to intersect the curve of the first water line. The like may be said of all the rest.

The water lines being thus formed; the next thing to be determined, is the round aft of the transom, if any; if none, produce the line Tp to b , then will Kb be the half breadth of the transom at the height of the sheer: So the height PT in the sheer plane, must be transferred to the middle line in the body plane. Thro' this point a line Kb , must be drawn parallel to the base AC , upon which the half breadth Kb being set off, we shall have one point thro' which the curve must pass. In like manner there must be perpendiculars let fall from the several intersections of the water lines, and aft side of the post in the sheer plane, and produ-

ced to intersect their corresponding water lines in the floor plane; which will give their half breadths. These again being transferred, to their corresponding level lines in the body plane, we shall have the points through which the curve of the transom must pass; observing at the tuck to set off the half breadth of the post; or the depth of the rabbet may be deducted out of it. The transom being thus formed, it is plain it will be too short, by reason of the raking of the post: We must therefore take the height of the transom upon the rake, which will be the line pT in the sheer plane; and set up this on the middle line in the body plane. In like manner we must set up on the middle line in the body plane, the several distances of the water lines, from the point p in the sheer plane; and thro' these points draw the several dotted lines, upon which must be set off the half breadths as before: Then a curve passing thro' these points, will give the true form and dimensions of the transom, as is expressed by the dotted curve.

If the transom is to round aft, as the curve KnG on the floor plane, it may be formed after the same manner without regarding the round; and after it is properly trimmed the round may be worked out: But as this will require very thick plank; in such cases it will be proper to make use of a fashion piece, and wing transom: This fashion piece will be formed in the same manner as that for a ship which has a square tuck, so the same operation will serve for both.

Now the fashion pieces being always sided strait, their planes will intersect the sheer and floor planes in a strait line. In this case, it will be in the line Gg on the floor plane, which touches the transom in the point n ; Gn being supposed the thickness of the fashion piece. Having thus determined its direction on the floor plane, this will likewise determine its direction on the sheer plane. If the transom had no round, only what is called a slight or rising like a floor timber, the plane of the fashion piece would intersect the sheer plane in the rabbet of the stern post; and the floor plane in a strait line drawn from G to K . But here it is supposed to be in the line Gg , which will throw the head of the fashion piece aft to W on the sheer plane; the point where a perpendicular erected from g intersects the sheer line or breadth line produced. Now k being the height of the tuck, the line kW will be that in which the fashion piece intersects the sheer plane.

Having thus found the intersection of the plane of the fashion piece, both on the sheer and floor planes, it is evident it will rake aft and cant forward; so the true dimensions and form of it may be found by *Prob. 6. Chap. 1.* in the following manner.

1st. Pro-

1st. Produce all the water lines in the sheer plane, to the line kW in the points $a, s, b, ;$ and let fall the perpendiculars $a e, s o, b u.$

2^{dly}. From the points $e, o, u,$ draw lines parallel to $Gg,$ to intersect each corresponding water line on the floor plane in the points $3, 2, 1.$

3^{dly}. Transfer the several points $G, 3, 2, 1,$ on the floor plane, to the points $G, 3, 2, 1,$ on the sheer plane, in such a manner, that lines drawn from G to $G,$ from 3 to $3,$ &c. may be perpendicular to $gL;$ and let the point G be in the breadth line, the point 3 in the third water line, &c. on the sheer plane; so the plane $WG 3 2 1 k$ will be the projection of the plane of the fashion piece on the sheer plane. But it will be less than the plane of the fashion piece, because it is not parallel to the sheer plane. Therefore,

4^{thly}. Thro' the points $G, 3, 2, 1,$ in the sheer plane, draw the dotted perpendiculars $GF, 3 A, 2 S, 1 H,$ to the line $Wk.$

5^{thly}. Make the lines $WF, a A, s S, b H,$ in the sheer plane, equal to the lines $g G, e 3, o 2, u 1,$ in the floor plane; so that they may intersect the dotted perpendiculars in the points F, A, S and $H:$ So shall $WFAS H k$ be the true form and dimensions of the plane of the aft side of the fashion piece. When it is in its proper position, the line WF will be in the same plane with the sheer line; the line $a A$ in the same plane with the water line $a 3;$ the line $s S$ in the same plane with the water line $s 2;$ and the line $b H$ in the same plane with the water line $b 1.$

We have now formed all the timbers in the after body. Those for the fore body are formed in the same manner, by transferring the several heights of the rising and breadth lines from the sheer to the body plane; the half breadths corresponding to each heighth, must also be transferred from the floor to the body plane. The same hollow mould will serve both for the fore and after body; and the level lines by which the water lines, to prove the after body, were formed, may be produced into the fore body, and by them the water lines to prove the fore body may be described.

Another method of proving the body, is by ribband lines, which are formed by sections of planes inclined to the sheer plane, and intersecting the body plane diagonally, as before observed; of which there may be as many as we shall judge needful.

Here we think it sufficient to lay down one, represented in the body plane by the lines marked *dia*^l. These are drawn in such a manner, as to be perpendicular to as many timbers as conveniently may be. After they are drawn in the body plane, the several portions of the diagonal, intercepted betwixt the middle line and each timber, must be transferred

to the floor plane: Thus, fix one foot of the compasses in the point where the diagonal intersects the middle line in the body plane; extend the other foot to the point where the diagonal intersects the timber, suppose timber 9; set off the same extent upon the perpendicular representing the plane of timber 9, from the point where it intersects the line K L, on the floor plane; do the same by all the other timbers both in the fore and after body; and we shall have the points thro' which the curve must pass. And if this should not prove a fair curve, it must be altered, observing to conform to the points as nearly as the nature of the curve will admit: So it may be carried within one point, and without another, according as we find the timbers will allow: For after all the ribband lines are formed, the timbers must, if needful, be altered by the ribband lines, this is only the reverse of forming the ribband lines; for taking the portions of the several perpendiculars intercepted betwixt the line K L, and the curve of the ribband line in the floor plane, and setting them off upon the diagonal, from the point where it intersects the middle line; we shall have the points in the diagonal, thro' which the curves of the timbers must pass: So the distance betwixt the line K L, and the ribband at timber 3 on the floor plane, when transferred to the body plane, will extend on the diagonal, from the middle line, to the point where the curve of timber 3 intersects that diagonal. The like may be said of all the other timbers; and if several ribband lines be formed, they may be so contrived, that their diagonals in the body plane shall be at such distances, that a point for every timber being given in each diagonal, will be sufficient to determine the form of all the timbers.

In stationing the timbers upon the keel, for a boat, there must be room for two futtocks in the space before, or abaft $\oplus$; for which reason, the distance betwixt those two timbers will be as much more, than that betwixt the other, as the timber is broad. Here it is betwixt $\oplus$ and $\textcircled{A}$; which contains the distance betwixt $\oplus$ and $\textcircled{1}$, and the breadth of the timber besides.

This method of whole moulding will not answer for the long timbers afore and abaft: They are generally canted in the same manner as those for a ship, of which we shall treat in their proper place; and here shew in what manner the timbers are moulded after they are laid down in the mould loft, by a rising square, bend, and hollow mould.

It was shewn before how to form the timbers by the bend and hollow moulds, on the draught. The same method must be used in the loft, but the moulds must be made to their proper scantlings in real feet and inches. Now when they are set, as before directed, for moulding each
timber,

timber; let the middle line in the body plane be drawn across the bend mould, and draw a line across the hollow mould at the point where it touches the upper edge of the keel; and let them be marked with the proper name of the timber, as in the figure (*Plate 5.*): So the graduations of the bend mould will be exactly the same as the narrowing of the breadth; for the distance betwixt $\oplus$ and 7, on the bend mould, is equal to the difference betwixt the half breadth of timber 7, and that of $\oplus$. The height of the head of each timber is likewise marked on the bend mould, and also the floor and breadth firmarks. The floor firmark is in that point where a strait edged batten touches the back of the bend mould, the batten being so placed as to touch the lower edge of the keel at the same time. The several risings of the floor, and height of the cutting down line, are marked on the rising square, and the half breadth of the keel set off from the side of it, as in the figure.

Note. The cutting down is omitted in the plate to avoid confusion.

The moulds being thus prepared, let it be required by them to mould timber 7.

The timber being first properly sided to its breadth, lay the bend mould upon it, so as may best answer the round according to the grain of the wood: Then lay the rising square to the bottom of the bend mould, so that the line drawn across the bend mould at timber 7, may coincide with the line representing the middle of the keel upon the rising square; and draw a line upon the timber by the side of the square, or let the line be scored, or cut by a tool made for that purpose, called a raseing knife. The term raseing is used when any line is drawn by such an instrument instead of a pencil. This line so rased will be the side of the keel. Then the square must be moved till the side of it comes to 7 on the bend mould, and another line must be rased in by the side of it, to represent the middle of the keel. The other side of the keel must likewise be rased after the same manner, and the point 7 on the rising square be marked on each side of the keel, and a line rased across at these points, to represent the upper edge of the keel. From this line the height of the cutting down line at 7 must be set up, and then the rising square may be taken away, and the timber may be rased by the bend mould, both inside and outside, from the head to the floor firmark: Or it may be carried lower if needful. After the firmarks, and head of the timber are marked, the bend mould may likewise be taken away; and then the hollow mould applied to the back of the sweep in such a manner, that the point 7 upon it may intersect the upper side of the keel, before set off by the rising square: And when in this position, the timber may be rased by it, which will compleat the outside

side of the timbers. The inside of the timbers may likewise be formed by the hollow mould. The scantling at the keel is given by the cutting down before set off. The mould must be so placed, as to touch the sweep of the inside of the timber formed before by the bend mould, and pass thro' the cutting down point.

The use of the firmarks, is to find the true places of the futtocks; for as they are cut off 3 or 4 inches short of the keel; they must be so placed, that the futtock and floor firmarks may compare, or coincide: Notwithstanding which, if the timbers are not very carefully trimmed, the head of the futtock may be either within or without its proper half breadth; to prevent which a half breadth staff is made use of.

The half breadth staff may be one inch square, and of any convenient length. Upon one side of it are set off, from one end, the several half breadths of all the timbers in the after body; and those of the fore body upon the opposite side. On the other two sides are set off the several heighths of the sheer; the after body on one side, and the fore body on its opposite. Two sides of the staff are marked half breadths, and the other two sides, heighth of the sheer, as in the figure. (*Plate 5.*)

The staff being thus prepared, and the floor timbers fastened on the keel, and levelled across; the futtocks must next be fastened to the floor; but they must be set first to their proper half breadth and heighth: The half breadth staff, serves to set them to the half breadth; for which purpose a small line, called a ram line, is stretched from the middle line of the stem, to that of the transom or post; to which a plummet is hung by a line, so tied round the ram line, that it may slip easily along upon it, and may be moved to the plane of any timber, and as the plummet will occasion this line to be always perpendicular to the keel, which in a boat is generally parallel to the plane of the horizon: We may, by it, likewise set the timbers perpendicular to the keel, and then set them to their proper half breadth by the staff; and when the two firmarks coincide, the futtock will be at its proper heighth, and may be nailed to the floor timbers, and likewise to the breadth ribband; which may be set to the heighth of the sheer by a level laid across, taking the heighth of the sheer by the staff from the upper side of the keel; by which means we shall discover if the ribband is exactly the heighth of the sheer; and if not, the true heighth may be set off by a pair of compasses from the level, and marked on the timbers. The next thing to be explained, is the construction and use of the bevelling board; but we shall first shew how to form the timbers by sweeps, because the same method for bevelling serves for both.



S E C T. III.

Of forming the Body by Sweeps. (Plate 7.)

IN ships of war the general dimensions are established by the authority of those appointed by the government for that purpose, which I have collected into a table, together with the principal dimensions of ships for the merchant service, to which we refer our readers.

The sheer and floor planes are laid down in this, exactly in the same manner as in that of whole moulding. We may have a sufficient number of points from the tables, to determine the heights of the breadth, and half breadth lines. A rising of the floor line must likewise be formed on the sheer draught. We may then go to the body plane, and form the midship bend or frame timber; the limits of which, we have from the sheer and floor planes, and it must be formed in the same manner as before directed in whole moulding, either by two, three, or more sweeps, as the artist shall think most suitable to the service the ship is designed for. The lower, upper, and top timber heights of breadth, and risings of the floor, are set up on the middle line in the body plane, as in whole moulding, and lines drawn thro' these points parallel to the base upon which the half breadths are set off. A mould may then be made for the midship frame as before, and laid upon the several risings in the same manner as in whole moulding, with this difference; that here an under breadth sweep is described to pass thro' the point which limits the half breadth of the timber; the center of which will be in the breadth line of that timber. The proper centers for all the frames being found, and the arches described, the bend mould must be so placed on the rising line of the floor, that the back of it may touch the back of the under breadth sweep. But the general practice is to describe all the floor sweeps with compasses as well as the under breadth sweeps, and to reconcile these two by a mould which is an arch of a circle; its radius being the same with that of the reconciling sweep, by which the midship frame was formed. It is usual for all the floor sweeps to be of one radius; and in order to find their centers, a line is formed on the floor plane for the half breadth of the floor: This, as was before observed, is only an imaginary one; for it cannot be described on the surface of the ship: Instead of it some make use of a diagonal in the body plane, to limit the half breadth of the floor upon every rising line, and erect perpendiculars at the several intersections in the

the same manner as for the midship frame, as in the draught; where it is very plain the floor sweep constitutes no part of the after timbers abaft the square body.

After the sweeps are all described, we must have recourse to moulds, or some such contrivance, to form the hollow of the timbers, much in the same manner, as in whole moulding; and when we have thus formed all the timbers, they must be proved by ribband and water lines, as before directed; and altered, if needful, to make these lines fair. Hence it is obvious, that the form of the ribband lines must be determined, before we can with certainty have the true form of the timbers. But there will be a necessity of determining, at least, the form of three timbers, *viz.* the midship, foremost and aftermost, before we can form a ribband line. These will give three points, thro' which the curve of each ribband must pass. The points in the intermediate timbers may be found by forming timbers as before directed; and by repeated trials, altering them till they make fair ribbands; for it is by them that the whole structure is regulated, when every frame is erected into its proper place.

S E C T. IV.

Description and Use of the Sector in forming the Body.

THE sector has seven lines on each leg, meeting at the center of the joint, numbered I, II, III, IV, &c. so that every line upon one leg has a corresponding one upon the other leg, both divided and numbered alike. The after body is upon one side, and the fore body on the other.

(Plate 6.) *The Lines for the after Body are as follows.*

I. Has five divisions, *viz.* $\oplus$, 4, 1st dl, 8, 1st; and marked at the end H B T', denoting the height of the top timber breadth line at four timbers; 1st is the stern timber, and 1st dl, the first diagonal in the body plane.

II. Has eight divisions, *viz.* $\frac{L' C}{u, A}, \frac{U' C}{A, u}, S^m$, 8, 4, $\oplus$, and marked at the end $\frac{1}{2}$ B T', denoting the half breadth of the top timber at three timbers. L' C signifies the lower counter, and U' C, the upper counter; u denotes

denotes the heighth, and A the rake of the counters, both taken from the wing transom; S' is the rake of the stern timber, which is likewise taken from the wing transom at the heighth of the sheer rail.

III. Has eight divisions, *viz.* dⁿ, u' S. R 1 s, $\oplus$, 3, 5, 7, 8: It is marked at the end L H B, for the heighth of the lower breadth line for five timbers: dⁿ is for the distance betwixt the frames, and R 1 s, for the distance betwixt the lower breadth line, and the dead rising in the body plane: u' S. is the radius of the upper breadth sweep.

IV. Is in two parts. The innermost has four divisions, *viz.* 7, 5, 3, $\oplus$, expressing the points where these timbers intersect the second diagonal in the body plane: It is marked at the end 2 R for the second ribband.

The outermost part has six divisions, *viz.* $\oplus$, 3, 5, 7, 8, W T, and marked at the end U H B for the heighth of the upper breadth line at five timbers, and at the wing transom denoted by W T.

V. Is likewise in two parts. The innermost has four divisions, *viz.* 7, 5, 3, $\oplus$, expressing the points where these timbers intersect the first diagonal: It is marked 1ⁿ R for the first ribband.

The outermost part has eight divisions, *viz.* T, 8, 7, 5, 3, $\oplus$, k l, and then marked M¹ Bⁿ, for the main half breadth of five timbers; T for that at the wing transom, and k l for the half breadth of the keel in midships; without which there is another division marked 3ⁿ d l for the third diagonal.

VI. Has four divisions, *viz.* 7, 5, 3, $\oplus$; for the points where those timbers intersect the third diagonal, it is marked 3d R, denoting the third ribband; without which, there is another division marked 2d d l, for the second diagonal.

VII. Has five divisions, *viz.* 7, $\frac{1}{2}$ B F l, 5, 3, $\oplus$. $\frac{1}{2}$ B F l, denotes the half breadth of the floor: The other four are for the points where those timbers intersect the fourth diagonal: It is marked 4ⁿ R, denoting the fourth ribband; without which there is another division for the rake of the post marked Rⁿ P.

The heighth of the gun-deck is betwixt N^o IV and V. G D $\oplus$ for that in midships, and G D a for that at the post. There is likewise another division betwixt N^o I and II for the fourth diagonal: It is marked $\frac{1}{2}$ 4ⁿ d l, which must be doubled, because the length of the sector will not contain the whole.

I N D E X to the AFTER BODY.

		N ^o		N ^o
Height of breadth	{ lower upper top timber }	III	Ribbands	I
		IV		2
		I		3
rising		III		4
Half breadth	{ main top timber floor }	V	Diagonals	1
		II		2
		VII		3
Counter and stern timber, height		II		4
and rake			betwixt I and II	
Rake of the post		VII	Distance betwixt the frames	
Upper breadth sweep		III		

The Lines for the FORE BODY are.

I. Has three divisions, viz. H¹ stem, $\frac{G^4, H^4}{\text{rake } F^4 G}$ denoting the height of the stem; and its rake from timber G at the gun-deck and head.

II. Has four divisions, viz. *d, b*, and these marked H¹, T¹, for the height of the top timber line at these timbers; and again *d, b*, for the $\frac{1}{2}$ breadth of those timbers; and the line marked $\frac{1}{2}$ B T¹.

III. Has four divisions, viz. d¹⁶ F for distance betwixt the frames, and *c, e, g*; it is marked L H B, denoting the height of the lower breadth line at these timbers.

IV. Is in two parts. The innermost has four division, viz. *g, e, c, ⊕*; for the points of intersection of these timbers, with the second diagonal, is marked 2^d R, for the second ribband.

The outermost part has three division, viz. *c, e, g*: It is marked U H B, denoting the height of the upper breadth line at these timbers.

V. Is in two parts. The innermost has four divisions, viz. *g, e, c, ⊕*; for the points of intersection of these timbers, with the first diagonal: It is marked 1^d R for the first ribband.

The outermost part has four divisions, viz. *g, e, c, ⊕*: It is marked M $\frac{1}{2}$ B^h for the main half breadth at these timbers.

VI. Has four divisions, viz. *g, e, c, ⊕*, for the points of intersection of these timbers with the third diagonal: It is marked 3^d R for the third ribband.

VII. Has four divisions, viz. *g, e, c, ⊕*, for the points of intersection of these timbers, with the fourth diagonal: It is marked 4th R, for the fourth ribband. The sweep of the stem is betwixt N^o III and IV; and the height of the gun deck betwixt IV and V.

INDEX to the FORE BODY.

	N ^o		N ^o
Height of breadth { lower } III			V
{ upper } IV		Ribbands { 1 } IV	
{ top timber } II		{ 2 } VI	
Half breadth { main } V		{ 3 } VII	
{ top timber } II		Distance of frames III	
Stem height and rake I		Sweep of the stem betwixt I. I and IV	

Having thus described the lines, we shall now shew their use in laying down a ship. (*Plate 7.*)

The general dimensions being determined, and a scale adapted to the draught, take the half breadth with a pair of compasses, and placing one foot in the proper point for the half breadth of $\oplus$, which will be found in N^o V. open the sector till the other foot reaches to the same point in the corresponding line on the other leg.

The sector being thus set, it will be indifferent whether we begin with the body or sheer plane: Let it then be the sheer.

1st. Draw the line X Z to represent the upper edge of the keel, and length of the gun deck; but it may be produced to the aft side of the wing transom, and fore part of the stem.

2^d. Erect a perpendicular to the line X Z, upon which set up the height of the wing transom to W; taken from N^o IV. on the sector.

3^d. Take the rake of the post from N^o VII. on the sector, and set it forward from the perpendicular of the wing transom to the point 7, where a perpendicular must be erected, which will be the station of that timber.

4th. Take the distance of the frames from N^o III. on the sector, and set it off from 7 to 8; and erect a perpendicular at that point for timber 8. Draw also a line from 8 to the wing transom, to represent the fore part of the post.

5th. Take the height and rake of both counters, also the rake of the stern timber from N^o II. The height of the stern timber is on N^o I, and by these form the counters, and upright of the stern.

6th. Station the timbers, by taking the distance betwixt the perpendiculars at 7 and 8; which at eight times will reach to $\oplus$; and erect perpendiculars at 5, 3 and $\oplus$. Then for stationing the timbers in the fore body, we must turn the sector, and take the distance of the frames from N^o III. which, set eight times from $\oplus$, will reach to H. Erect perpendiculars at C, E, G and H; and from G set off the distance of the gun-

deck before G. It is in N^o I. on the sector; which will reach to Z; at which point erect a perpendicular, and set off the height of the gun-deck, taken from the sector; and from the gun-deck set up the height of the head of the stem, also its distance before G; both taken from N^o I. We may then form the stem. The center of the sweep is in the perpendicular of timber F, and the radius of the sweep is upon the sector betwixt N^o III and IV. which set up from the point F, will give the center: So the sweep will just touch the upper edge of the keel in the point F. And as the sweep will not reach to the gun-deck, we must make use of a mould to break in fair with the back of the sweep.

7th. Set up the heights of the lower, upper, and top timber breadths lines upon the perpendiculars erected for the stations of the several timbers. The points corresponding to each, are on their proper lines on the sector.

Having thus finished the sheer plane, we may then go to the floor plane; and producing all the perpendiculars for the timbers, we may upon them set off the main, and top timber half breadths. The points corresponding to each, are on their proper lines upon the sector; which must be set off from the line W K, representing the lower side of the keel, and may be produced both ways, as far as shall be needful. We must in the next place form all the ribband lines, which are the dotted ones in the draught; beginning with the fourth ribband. But it will be more expeditious, first to draw all the diagonals in the body plane.

Let A B be the whole breadth, on the middle of which erect the perpendicular K O; so shall A K, or K B, be the half breadth. Upon the line K O, set up the several heights of the breadth lines, taken from the sheer plane, and draw lines parallel to the base, as directed in the preceding sections; and likewise set off the half breadths, corresponding to each, taken from the floor plane. We may also set off the height and half breadth of the wing transom; all which may be done without the sector, but we must have recourse to it for the dead rising. This is in N^o III. in the after body, and must be set off upon the line K O, from the lower height of breadth to *i*. Thro' *i* draw the line *r i s*, parallel to the base, and set off the $\frac{1}{2}$ breadth of the floor from *i* to *r*, and from *i* to *s*: It is upon N^o VII. on the sector. Then taking *r i*, set it up from *i* upon the line K O, to which point draw the dotted diagonal marked 1^a R^d. This regulates all the other diagonals: For if one line be drawn from the point of its intersection, with the middle line, to the half breadth of the wing transom; and another from the point *r*, its intersection with the rising line, to the point $\oplus$ at the lower height of breadth;

breadth; each of these may be divided into four equal parts by the dotted diagonals $2^a R^a$ $3^a R^a$ $4^a R^a$.

Note. The lines from the ends of the first diagonal to the lower height of breadth, and to the wing transom, were drawn only with a black lead pencil, and wiped out after the diagonals were drawn. The diagonals being thus drawn, we may form the midship frame, for which purpose we must find a point in each diagonal, thro' which the curve of the timber must pass. These points we have from the sector; which must be set off from the intersections of the diagonals with the line KO. That in the first diagonal is in N^o I. The point in the second diagonal, is in N^o VI. The point in the third diagonal, is in N^o V. And the point in the fourth diagonal, is betwixt N^o I. and II. This last must be doubled, because the sector will not contain the whole length. The midship frame being formed, we must in the next place form the after and foremost timber; which the sector does, by giving the distance on every diagonal betwixt these timbers, and the midship frame now formed: So that we shall have a point in each diagonal, thro' which the curve of the timber must pass. To find the point in the first diagonal for the after timber; extend from $\oplus$ $1^a R$ in N^o V. to the corresponding point on the other leg. Set off this distance from $\oplus$ on the first diagonal: Do the same upon the second, third and fourth diagonals. The point on the second diagonal, is in N^o IV. That on the third, in N^o VI. And that on the fourth in N^o VII. The curve must pass thro' these points, and likewise thro' the point for the half breadth, which was before set off from the sheer and floor planes; by which means we have determined the form of the after timber; and the foremost timber is to be formed by the same method. These two timbers being formed, we may find points in the diagonals for all the intermediate timbers. Thus, to find the point for timber 3 in the first diagonal; extend from the point 3 in the inner part of the line N^o V. to its corresponding point on the other leg. Set off this in the first diagonal from the after timber, already formed; which will give the point thro' which timber 3 must pass; and to find the point in the second diagonal, we must extend from 3 in the inner part of the line N^o IV. and set off this distance in the second diagonal from the after timber. The same method must be used to find the points in the third and fourth diagonals. In like manner we may find a point in each diagonal for the timbers 5 and 7, which will be sufficient for the after body: And the same process must be used to find points in each diagonal for the timbers in the fore body.

Having now found the points, before we form the timbers, it may be proper, by them, to form the ribbands: For now we may take the distance
of

of each point in the diagonal, from its intersection with the middle line K O, and transfer it to the floor plane upon the perpendiculars that represent the planes of the timbers, as directed in *Señ. 2.* In order to limit the ends of the ribband lines in the floor plane; we must set off half the thickness of the post, on one side of the middle line K O, and half the thickness of the stem on the other side of it, in the body plane; first deducting the depth of the rabbet out of it. We must likewise determine the inner part of the rabbet on the stem, and upon the post in the sheer plane. In the stem, it is generally in the middle betwixt the lines that represent the outside of the rabbet. It may be also so on the post, from the wing to the lower transom; and from thence the line may be continued fair to intersect the line that represents the after side of the rabbet, at the upper edge of the keel; for there the rabbet is cut square into the post.

Now, it is obvious that when the plane of any diagonal ribband is in its proper place and position, the line W K will be in the sheer plane, parallel to the upper side of the keel; and its height will be the same with that of the point where the diagonal intersects the middle line in the body plane. But by reason of its inclination, and of the half thickness of the stem and post; the height of the plane of the ribband upon the post and stem, will be in the point where the diagonal intersects the line that represents the rabbet in the body plane. This then must be transferred from the body to the sheer plane; and set up from the upper edge of the keel upon a perpendicular that will intersect the line that represents the inside of the rabbet at that height. This perpendicular may be produced into the floor plane; and if that part of the diagonal intercepted betwixt the middle line, and the line that represents the inside of the rabbet, in the body plane, be set off upon the perpendicular, it will give the proper point for the end of the ribband line, as may be seen in the plate; where all the ribbands are dotted lines, and they are marked 1^a D R, 2 D R, &c.

Note. The scale in the plate is so small, that we have taken the outside of the rabbet to limit the end of the ribband.

The ribbands being thus formed, we may from them form all the timbers below the breadth.

The next thing to be done, is to form the top timbers. We have the height and half breadth of each from the sheer and floor planes; and the timbers below the breadth, are carried up by a sweep, which forms the lower part of the top timber. The center of this sweep is in the upper height of the breadth line of the timber, and may be taken from the sector: It is on N^o III. after body. The midship top timber has generally a hollow, which is left intirely to the artist; for some, especially small

small ships, have none. The general practice is to make a mould for this hollow, either by a sweep, or some other contrivance, and produce it considerably above the height of the top timber in a strait line, or very near one. The midship timber is formed by this mould, and so placed, that it breaks in fair with the back of the upper breadth sweep. All the other timbers are likewise formed by the same mould; observing to place it so that the strait part of it may be parallel to the strait part of the midship timber; and moved up or down in that direction till it just touches the back of the upper breadth sweep. Some begin at the after timber after the mould is made for the midship one, because they think it easier keeping the strait part of the mould parallel to this; than to the midship timber; and by this means the top side is kept from winding.

Others again, make a mark upon the mould where the breadth line of the midship timber crosses it; and with the same mould they form the after timber. This will occasion the mark that was made on the mould, when in midships, to fall below the breadth line of the after timber; and so another mark is made at the height of the breadth of the after timber. The next thing to be done, is to lay the strait part of the mould obliquely across the breadth lines of the top timbers, in such a manner that it may intersect the breadth line of the midship timber at one of these marks, and the breadth line of the after timber at the other mark. Then the several intersections of the breadth lines of the timbers, are marked upon the mould. The mould being thus marked, must be so placed in forming each timber, that the proper mark may be applied to its proper breadth; and the mould be turned about so as just to touch the upper breadth sweep. Any of these methods may make a fair side; but it may be easily proved by forming another half breadth line.

C H A P. III. S E C T. I.

Of the CANT TIMBERS.

Hitherto we have considered the timbers, as having their planes perpendicular both to the sheer and floor planes. These are called square timbers; and when they are all formed, we may from them form as many ribband and water lines as shall be necessary to form the cant timbers. Their planes are inclined to the sheer, but perpendicular to the floor planes. The reason of canting these timbers, is that they may nearly be equally spaced at the breadth ribband: For if the post has a considerable rake, and the timbers all square, there will be a great space at the breadth ribband, betwixt timber 8 and the wing transom: Besides the timber may be so canted, that it may be square to some of the ribbands; whereas, if they were perpendicular to the sheer plane, they would intersect the ribband lines so as to form very oblique angles; which would occasion a very great bevelling. Another advantage that attends canting the timbers, is that they will not require such compass timber.

It is usual to begin the cant timbers from the aftermost floor timber; and space them near equally on the breadth line, to the wing transom: And in order to space them upon the keel, the cant of the fashion piece must be determined. Now if we suppose the plane of the fashion piece to intersect the sheer and floor planes in the point F, it must intersect the floor plane in the line F P, because the point P is supposed to be the end of the wing transom. So the angle s F P will be its inclination to the sheer plane. It will intersect the sheer plane in a perpendicular erected from the point F; and if the space betwixt the point F, and the foremost cant timber upon the keel, be divided into the same number of equal parts, that the space betwixt the same timber, and the wing transom upon the breadth line, is divided into; this will determine the cant of all the timbers, only by drawing lines from all points in the line W K, to the corresponding points in the breadth line, in the same manner as the line F P, determines the cant of the fashion piece.

It would be needless to draw all these lines in the plate; the only intent of drawing them being to shew how to form the timbers by them: And as one method serves for all the cant timbers, which are supposed perpendicular

dicular to the floor plane; it will be sufficient to shew the formation of the fashion piece.

Before any of the cant timbers can be formed, there must be a sufficient number of water lines, or diagonal and horizontal ribband lines formed from the square timbers; and when these are absolutely determined, we may, with certainty, form all the cant timbers, either by water, or ribband lines,

If we make use of the diagonal ribbands, which are distinguished by the dotted curves in the floor plane, we must form an horizontal ribband corresponding to each. We have only laid down one of these horizontals in the plate, *viz.* that corresponding to the third diagonal: It is marked 3^d H R. To form this ribband, fix one foot of the compasses in the point where the third diagonal intersects the midship frame in the body plane; and extend the other foot to touch the middle line K O; so that if a line were drawn from one foot of the compasses to the other, it would be perpendicular to the line K O: This distance set off from the line W K, upon the perpendicular that represents $\oplus$ in the floor plane, will give the point thro' which the curve must pass at that place. The same method must be used for finding the points on all the other timbers.

Now, tho' the diagonal and horizontal ribbands seem to be quite different curves in the plate, they will make but one line upon the timbers; for the one intersects them in a direction perpendicular to the sheer plane, and the other is so inclined as to intersect the timbers in the very same points. The horizontal one is too short upon the plate, but the true length of it might easily be had by transferring to the sheer plane, the several heights at which the diagonal intersects the timbers in the body plane. By these we might form a height of breadth line to correspond to this horizontal ribband, which is only a half breadth line; and the length of this height of breadth line may be taken by a penning batten, and all the timbers marked upon it. Now when the batten is applied to a strait line, and all the timbers transferred to this line from the batten, we may erect perpendiculars at each, and set off the same half breadths as before; by which means we may have the true length of the horizontal ribband: But as this will be of no manner of service, we shall omit forming it. We only mention it, because several imagine these two curves to be as different on the surface of the ship as they are upon the draught. The horizontal ribband corresponding to the first diagonal one, is formed to timber 7, and marked 1st H R; but the horizontals for the second and fourth diagonals were formed by a black-lead pencil, and only the point in which they intersect the line F P, is in the plate; which is sufficient for our purpose.

There are likewise five water lines formed, four of which are represented by level lines in the body plane; and by lines parallel to the keel in the sheer plane: Three of them represent the planes of the transoms in the sheer plane, *viz.* D^k , 1^a , 2^a : But the plane of the third transom is perpendicular to the post. The lower water line is drawn parallel to the keel from the stem to the post, and produced into the body plane, as in the plate, where it is marked MN: The plane of the third transom intersects the timbers at different heights, which are transferred from the sheer to the body plane, where it forms a curve.

The water lines being now drawn in the sheer and body plane; our next business is to form them in the floor plane, where they will be curves. The points thro' which the curve of the lower water line is to pass, are had by transferring the several portions of the level line, intercepted betwixt the line KO, and the curve of each timber, from the body plane to the corresponding perpendiculars in the floor plane, where it is marked *Wa't'L*. It must be observed that the line KO, in the body plane, represents the several perpendiculars that are drawn in the sheer plane to represent the planes of the timbers: For the spaces in the body plane, contained betwixt the line KO, and the curves of each timber, are so many different planes; and when in their proper places, they will be parallel to one another, if perpendicular to the sheer and floor planes. Thus the plane contained betwixt the line KO, and the curve of timber $\oplus$, is (when in its proper place) supposed to be erected perpendicular to the sheer plane, in the line which represents the plane of $\oplus$; and the like may be said of all the rest. The planes of the cant timbers will not be parallel to one another, because they are differently inclined to the sheer plane; but as they are perpendicular to the floor plane, they will intersect the sheer plane in a line perpendicular to the keel: So the plane of the fashion piece intersects the sheer plane in the dotted perpendicular erected from the point F, which is the same with the line KO, in the body plane. We thought it necessary to take notice of this, because some who are learning to draw, mistake the line KO; for they imagine it only represents the post or stem.

Another error which they frequently fall into, is about forming the water lines when their planes are not parallel to the keel. They imagine that the half breadths must be set off from the line WK, which represents the lower edge of the keel; whereas it is indifferent what strait line they are set off from, so the timbers be exactly spaced, and perpendiculars drawn to represent their planes. Now when the water lines are supposed parallel to the keel, all the timbers are properly spaced, and the perpendicu-

pendiculars ready drawn to the line $W K$; which is the reason it is used in such cases: Tho' when the plane is in its true place, the line $W K$, will be in the line $M N$. But the case will be quite different when the water lines are not parallel to the keel; for then their planes will intersect the sheer plane in a strait line, forming oblique angles with the planes of the timbers; this is the case in the plane of the third transom. The distances betwixt the timbers will be more in this line than in the line $W K$; so the half breadths cannot be set off from the line $W K$, upon the perpendiculars that represent the planes of the timbers unless they be properly spaced at the same distance they are upon the line that represents the plane of the third transom in the sheer plane; upon which account we have made use of that line to set off the half breadths from, and drawn the dotted perpendiculars at the points where it intersects the planes of the timbers 8, 7, and at the point where it intersects the lower height of breadth line. The heights of the points of intersection are transferred from the sheer to the body plane; and the half breadths at these heights, transferred from the body plane to the dotted perpendiculars before drawn: The half breadth to be set off upon the perpendicular where it intersects the lower height of breadth line, is had from the floor plane, and the dotted perpendicular aa , will shew the place where the half breadth must be taken: This perpendicular, if produced, will intersect the plane of the third transom in the lower height of breadth.

Having now formed four diagonal ribbands with their corresponding horizontals, and also two water lines; we may by these, form the fashion piece, either upon the body plane or sheer plane: But as the plane of the fashion piece is parallel to neither of these, it will require two operations.

Now the line $F P$, will intersect all the ribband and water lines; but because the diagonal ribbands are not in their proper position, the line $F P$ will not intersect them in the point where the plane of the fashion piece intersects them. The first thing then to be done, is to find the true place of the fashion piece on each diagonal ribband: And first, to find its place upon the fourth diagonal ribband, from the point t , where the fourth horizontal ribband intersects the line $F P$, let fall a perpendicular to the point s , and produce it to intersect the diagonal ribband in r ; so shall r be the true place of the fashion piece upon that ribband; that part of the perpendicular betwixt t and s is not drawn in the plate to avoid the confusion of too many lines. The reason of this will be very evident, if we suppose the whole plane of the ribband to be turned round upon the axis $W K$; for then the point r will always be right over some point of the

perpendicular $r t s$; and when the ribband is in its proper inclination, a perpendicular from r will fall into the point t , and the plane of the fashion piece will intersect the floor plane in the line $t F$, and the plane of the diagonal ribband in a strait line drawn from r to F : For it must be observed that when the ribband is in its proper place, the line $W K$ will be in the sheer plane, in a line parallel to the keel; the height of which may be had from the body plane. In this case it will be the distance betwixt K and r , but it will be needless to draw this line in the plate.

Having now found the place of the fashion piece on the fourth diagonal ribband, we must by the same method find its place on the other diagonals, as in the plate, where lines perpendicular to $W K$, are drawn to the points o, o, o , in the diagonal ribbands, from the points where the line $F P$ intersects the corresponding horizontal ribbands.

These points being now found, we may take the nearest distance of each point to the line $W K$, and set off those distances on the proper diagonals in the body plane. Thus, for the fourth ribband, place one foot of the compasses in the point r , and the other in the point s in the floor plane; and set off that distance from r to S , on the fourth diagonal in the body plane: Do the same by all the rest of the diagonals; and a curve intersecting the diagonals in these points would be the projection of the fashion piece in the body plane, but we have not drawn this in the plate; for as the plane of the fashion piece is not parallel to that of the body plane, its projection will be less than the original: However this may be found by *Prob. 6. Chap. 1. Part 2.* by the following method.

1st. Draw a perpendicular to the line $K O$, in the body plane, to pass thro' the point S to F .

2^d. Take the distance from r to F , in the floor plane, and set it off from r to F , in the body plane. In like manner draw perpendiculars to the line $K O$, in the body plane, thro' the points before found on the diagonals, as in the plate, where only that part of the perpendicular is drawn which lies without the diagonal; and take the several distances betwixt the points o and F , in the floor plane, and set them off from the intersections of their corresponding diagonals, with the line $K O$, to the points o, o , in the body plane: So we have the points o, o, F , thro' which the curve must pass.

3^d. To find the point P in the body plane, thro' which the curve must pass. Transfer the point P in the floor plane, to the point P , in the sheer plane, by a perpendicular to the line $W K$, to intersect the height of breadth line in the point P ; and set off this height upon the line $K O$, in the body plane, which will be a little above W , the height of the

the wing transom: Draw a perpendicular at this point, to the line K O; take the line F P, in the floor plane, and set it off upon this perpendicular, from the line K O, to the point P: So shall the curve P' P o o, be the form of the fashion piece.

These points may all be found without the diagonal ribbands, by half breadth lines and water lines, formed on the floor plane, as for instance: To find the point F; place one foot of the compasses in *t*, the point where the horizontal ribband intersects the plane of the fashion piece, and the other in *s*; *ts* being perpendicular to W K: With that extent, move the compasses with one point in the line K O, and the other point perpendicular to it, till it intersects the fourth diagonal, in the point S; thro' which draw the perpendicular *t* F. Then take the distance from *t* to F, in the the floor plane, which set off from *t* to F, in the body plane; so shall F be the point required, as before: In like manner the points *o*, *o*, may be found: But this, as well as the other method, requires two operations; whereas, if several water lines were formed, with their planes parallel to the keel, we might find the points by one operation. Thus, suppose it was required to find a point in the level line, that represents the plane of the water line formed in the floor plane, which is marked *W a t*, L. Fix one foot of the compasses in the point *f*, where the line F P intersects the water line in the floor plane, and the other foot in the point F. Set off this upon the level line in the body plane, from the line K O to *f*, which will be the point required. All the other cant timbers, both in the fore and after body, are formed after the same manner as the fashion piece. We have formed but one more in the plate, which is abaft the fashion piece, to assist us in forming the transoms.

S E C T. II.

Of the TRANSOMS.

THE transoms are fastened to the stern post, in the same manner that the floor timbers are to the keel; and as the floor timbers have a rising, so likewise have the transoms, which is called the flight; and besides this flight, the wing transom has a round aft, and a round up, both which are arbitrary: The deck transom has a round up, the same with that of the beams: But in forming the transoms, there is no regard had

had to the round up; for that may be done by the beam mould, after the transom is properly hewed the moulding way.

In forming the transoms, the first thing to be done, is to assign each its proper place upon the post, and then to determine the position of their planes with respect to the floor plane; for their planes are always perpendicular to the sheer plane. In the plate there are five transoms: Their upper sides upon the post are in the points W , D^* , 1^a , 2^a , and 3^a : The planes of the wing, deck, first and second transoms, are supposed parallel to the floor plane, and represented in the sheer plane by lines drawn parallel to the keel from the post, till they intersect the lower height of breadth line; and the plane of the third is represented by a line perpendicular to the post, as in the plate, So it will not be parallel to the floor plane.

The height and position of the transoms being determined, we have no more to do, but to form water lines for each. That for the third we have already formed: The rest being supposed parallel to the floor plane, may be formed in the same manner as the water line there laid down: The only difficulty will be to find a sufficient number of points to determine their forms; because in the deck and first transoms, their planes intersect the breadth; so that we could only have a point in timber 8, if the fashion piece and a timber abaft it, had not been formed by the ribbands; but now they are formed, we may have likewise a point in each of their planes, thro' which the curves of the water lines shall pass.

We shall begin with the wing transom. First determine the round aft which suppose the line WT , in the floor plane: Take its height from the sheer plane, and set it up in the body plane from K to W , and draw the line WT : Then take this line WT , and set it off on the floor plane, on the line FP , which will reach to the point n . A curve drawn thro' the point n , to break in fair with the breadth line, as in the plate, will intersect the line WT in T ; so shall WTn , be the aft side of the wing transom. Next for the deck transom, draw a level line in the body plane at the point D^* to timber 8. Set off this distance upon timber 8, in the floor plane, from the line WK ; which will give us a point thro' which the curve must pass: Then take the distance in the level line, betwixt the line KO , and the curve of the fashion piece; which set off from the point F , upon the line FP , in the floor plane; and this will give another point thro' which the curve must pass: Again, take the distance in the same level line, betwixt the line KO , and the curve of the timber abaft the fashion piece; which set off from the point G upon the line Gg , in the floor plane; and we shall have a third point thro' which the curve must

must pass. Lastly, let fall a perpendicular to the line $W K$, from the point D^* upon the post, and produce it into the floor plane, upon which set off half the thickness of the post, allowing for the rabbet, which will limit the end of the water line that forms the deck transom. After the same manner are the first and second transoms formed, by drawing level lines in the body plane, at their heights upon the line $K O$.

Now some are apt to mistake these level lines for the lengths of the transoms: The reason, as was before observed, is because they imagine the line $K O$ to be the stern post; whereas it is the perpendicular in which the plane of the fashion piece intersects the sheer plane; and so these lines are drawn upon the plane of the fashion piece.

All that now remains, is to determine the length of each transom; and this is done by the line $F P$, in the floor plane, which intersects the wing, deck, first and second transoms, to their proper lengths. But before we can find the length of the third, the plane of the fashion piece must be projected upon the sheer plane: Thus, take the nearest distance betwixt any perpendicular in the floor plane, and the point where the line $F P$ intersects the water line; and set that off from the same perpendicular upon the line that represents the same water line in the sheer plane. Now the curve $P F$ will be found to be the projection of the aft side of the fashion piece upon the sheer plane: For the distance betwixt the perpendicular of timber 8, and the point f , where the line $F P$ intersects the lower water line in the floor plane, is equal to the distance betwixt the same perpendicular and the curve $P F$, taken in the line $M N$. The distance betwixt the perpendicular of timber 8, and the point where the line $F P$ intersects the second water line in the floor plane, is equal to the distance betwixt the same perpendicular and the curve $P F$, taken in the line that represents the second transom in the sheer plane. And by the same method, we find points in the lines that represent the deck and first transoms. The point P is transferred from the half breadth line in the floor plane, to the height of breadth line in the sheer plane. The curve being thus drawn, will intersect the line that represents the plane of the third transom, in the point z : From which point draw the perpendicular $z F$, to the curve of the dotted water line; so shall $z' z, F$, be the true form of the third transom; and a line drawn from F to P , will be the plane of the fashion piece. It must be observed that the ends of the transoms are let into the fashion piece; for which there must be a proper allowance left without the lengths found by the line $F P$. We have in the plate only laid down the half of each transom. Those who incline

incline to lay down the whole transoms, may easily transfer the halves already described to the other side of the line W F.

Having now formed all the timbers, both square and cant, in the after body; we shall proceed to the fore body. The cant timbers are laid down in the same manner as those in the after body, by the diagonal and horizontal ribbands; where the dotted line K T represents the plane of the knuckle timber, canted upon the floor plane; from whence it is transferred to the body plane, and represented by the dotted curve betwixt timber H and G.

The hawse pieces are seldom laid down in the loft; it being the general practice to make moulds for them after the other timbers are put up, and the harpins are brought about; but they may be formed in the following manner.

Let P H represent the plane of the hawse piece on the floor plane, which may be produced to K T, the plane of the knuckle timber: In the plate let H be supposed the heel of the hawse piece; from which point erect the dotted perpendicular $b l$ into the sheer plane, and draw the dotted level lines $a l$, $c l$, in the body plane; by which, form the water lines $a l$, $c l$, in the floor plane, and draw the dotted lines $a l$, $c l$, perpendicular to the line $b l$, in the sheer plane; which will represent the planes of the water lines. Draw also the dotted perpendicular $b l$, to the point where the line $b l$ intersects the upper height of breadth line. Upon the line $c l$ in the sheer plane, set off the distance H P, taken from the floor plane, P being the point where the plane of the hawse piece intersects the water line $c l$. Then take the distance from H, to the point where the line H P intersects the $M \pm B^h$ line, and set it off upon the line $b l$ to t ; or, rather find the point upon the lower height of breadth line, where the hawse piece comes to; from which draw a perpendicular to the line $b l$, and upon this set off the distance, as before: Then take the distance from H to the point where the line H P intersects the water line $a l$ in the floor plane, and set it off upon the line $a l$, in the sheer plane to p . Lastly, to find the height of the heel, because we have not formed a timber at the point H, produce the line H P, to intersect the plane of the knuckle timber K T, in the floor plane, at the point r : Take $r K$, with a pair of compasses, and placing one foot in the curve of the knuckle timber in the body plane; so as that the other foot touch the line K O, or rather set off $r K$ from the line K O to k upon the base line; at which point erect a perpendicular to intersect the curve of the knuckle timber in the point k ; so shall $k k$ be the height of the heel, if the plane of the hawse be produced to intersect the plane of
knuckle

knuckle timber: But in the plate the heel of the plane of the hawse piece is supposed to be at the point H: Therefore a perpendicular must be erected from the point r , into the sheer plane, upon which setting up the height $k k$, we shall have the point k . We may by the same method find a point in the plane of timber H, in the sheer plane; through which the curve of the hawse piece must pass; and if produced to k , it will intersect the perpendicular $b l$, in the point b ; which is the height of the heel.

Tho' the hawse pieces are seldom laid down, yet by forming them on the sheer plane, we shall thereby discover if there be any faults in the half breadth lines or water lines: For if the timbers that are formed by these lines are not fair, some of those lines from which they are formed must certainly be the occasion of it; which therefore must be rectified before we can find the true form of the harpins; which is the next thing to be done.

S E C T. III.

To form the Harpins and Rails of the Head.

AS the harpins are level'd across, they will be formed by the section of a plane perpendicular to the sheer plane: But there is no necessity for these sections to be parallel to the keel. In the plate we have drawn only a strait line to represent the plane of the harpin above the wale. It is drawn from the stem to timber E, and marked harpin. Now in order to form the curve of this harpin, it would be proper to form timber F, in the body plane: Also to draw perpendiculars to the several points where the plane of the harpin intersects the planes of the timbers E, F, G and H, in the sheer plane; and upon these to set off the half breadths corresponding to each, taken from the body plane. This would give us the points thro' which the curve must pass, which would be a water line. But as this is performed exactly in the same manner as the water line that represents the third transom, we judge it unnecessary to form it in the plate.

The rails of the head are projected on the sheer plane, according to their true hangings; and in order to find their true lengths, draw the dotted line S T, parallel to the keel at the height of the rails, upon the head. We must then determine the station of the cat-head upon the

Y.

floor

floor plane; and likewise the thickness of the head at the rail; and let fall a perpendicular from the point T , where the line ST intersects the cat-head in the sheer plane, to the point T in the floor plane; and likewise a perpendicular from S in the sheer, to S in the floor plane; and draw the line TS : sS in the floor plane being half the thickness of the head of the figure at the rail; so shall TS , in the floor plane, be the true length of the rail. Let the line TS , in the sheer plane, be divided into any number of equal parts: Suppose into the points x, y, z ; from which points draw perpendiculars to the line TS , to be limited by the rail. Divide the line TS , in the floor plane, into the same number of equal parts, in the points x, y, z . Draw perpendiculars to these points, and make them equal to the corresponding ones in the sheer plane; so we shall have the points thro' which the curve of the rail must pass.

We have now shewn different ways of forming all the timbers; where it must be observed that we have always supposed every timber to be one intire piece of wood from the keel to the top of the side; whereas in reality, they are in several different pieces; the head of the lower piece being cut square to join to the heel of the next above it: And in order to support these joinings, another sett of pieces are cut, and joined together in such a manner, that if both the setts were fastened together, the joinings in one sett, would be nearly against the middle of the pieces in the other sett. In this manner are all the frames fastened and erected, as if each was one piece of wood. The pieces laid across the keel, to which they are fastened, are called floor timbers: The other pieces are called futtocks, except that which goes to the top of the side, which is called a top timber. Hence it is plain that the mould which serves for the floor timber, will serve for the lower part of the corresponding futtock. The mould for the upper part of the first futtock, will be the same with that for the lower part of the second; and the mould for the lower part of the top timber will be the same with that of the upper part of the corresponding futtock. It is of great importance in building, to give proper scarp to the timbers; for which we refer our readers to the table of scantlings at the end of this part.

C H A P. IV. S E C T. I.

Of Bevelling the TIMBERS.

IN the preceeding chapters we have considered the timbers as plain surfaces, without any regard to their thickness or breadth; whereas every timber consists of two planes, and the space contained betwixt them is the breadth of the timber. We have already shewn how to find the form of one of these planes, which is called the moulding side of the timber. The form of the other side will be different from the moulding side, except in midships. Now if the timber be properly hewed from the moulded side, we shall have the form of the other side: This is what is called bevelling the timbers; a term so well known that it needs no explication. We shall only remark that the bevelling is the angle made by the meeting of two planes limiting a solid; and as this angle cannot be measured by scale and compasses, without cutting the solid by another plane perpendicular to both; it is done by an instrument called a bevel. When the angle is a right one, the timber is said to be square, and is measured by an instrument of that name.

In order to hew any piece of timber to its proper bevel, it will be very proper first to make one side fair, and out of winding; a term used to signify that the side of the timber should be a plane. Now if this side be uppermost, and placed horizontally, or upon a level; it is plain if the timber is to be hewed square, it may be done by a plummet and line; but if the timber is not hewed square, the line will not touch both the upper and lower edge of the piece; or if a square be applied to it, there will be wood wanting either at the upper or lower side. This is called within or without a square. When the wood is deficient at the under side, it is called under bevelling; and when it is deficient in the upper side, it is called standing bevelling; and this deficiency will be more or less, according to the depth of the piece; so that before the proper bevellings of the timbers are found, it will be sometimes very convenient to assign the breadth of the timber; nay in most cases it will be absolutely necessary, especially afore and abaft; tho' the breadth of two timbers, or the timber and room, which, as was before observed, includes the two timbers, and the space betwixt them, may be taken without any sensible error; as far as the square body goes. For as one line represents the

moulding side of two timbers, the fore side of the one being supposed to unite with the aft side of the other; the two may be considered as one intire piece of timber.

Notwithstanding it is usual in draughts to lay down only every third or fourth timber; yet in the loft it will be necessary to lay down all the timbers: But as our plate will not admit of this, let us suppose the line $a a e$, betwixt the timbers 5 and 7, in the floor plane, to represent the moulding side of two timbers; and the lines $m n$ and $r s$, the moulding sides of other two timbers. Draw the lines $b c$ and $k l$, the one in the middle betwixt $e a$ and $m n$, and the other in the middle betwixt $e a$ and $r s$; so shall the distance betwixt the lines $b c$ and $k l$, be the breadth of two timbers, together with the space betwixt them: The portion $b k$ of the ribband may be taken for a strait line, and then the angle that is made by the line $b c$ and $b k$, or the angle made by the line $k l$ and $k b$, will be the bevelling according to the side on which the timber is moulded; the one being as much standing as the other is under bevelling. In order to find how much this is from a square, draw the lines 1, 2, 3, 4, perpendicular to the lines $b c$ and $k l$; and the portions of the line $e a$ intercepted betwixt the ribbands, and these perpendiculars will be what the timber is either within or without a square; so $4 e$ will be that at the fourth ribband: And because the line $e a$ represents the moulding side of both timbers, the timber before it will be standing, and the timber abaft it, under bevelling.

It is very necessary to observe that the planes of the ribbands should be perpendicular to the planes of the timbers, which is the case in all the square timbers; But the planes of the cant timbers are inclined to the planes of the ribbands; therefore their bevelling cannot be had by the ribband lines in the same manner as these of the square, because when the stock of the bevel is laid upon the moulding side of the timber, the tongue of the bevel will be out of the plane of the ribband.

Another thing to be carefully observed, is in what direction the stock of the bevel is to be laid upon the moulding side of the timber. This is found in the body plane: If we bevel by ribband lines, the diagonals will give the line of direction; but if the bevellings are taken by water lines, the level lines in the body plane will give the direction in which the stock of the bevel is to be laid upon the timber: When these lines in the body plane are very oblique to the curves of the timbers, if the bevel is not kept exactly in the same direction, it will occasion a very great error; and only the very sharp edge of the tongue will touch the timber. For this reason, the best way to take the bevellings will be, so
that

that both stock and tongue may be square to the timber; but this will alter the bevelling, and bring it likewise nearer to a square, which is another advantage we shall gain by altering the direction of the stock; and the true bevelling may be found by the following method.

Let the distance betwixt timber 7 and timber 8, in the floor plane, be supposed the breadth of a timber; then the perpendicular at 8 will represent the plane of the aft side; and the perpendicular at 7, the plane of the fore side of the timber in the floor plane: The curve of timber 8 in the body plane, will be the form of the aft side; and the curve of timber 7, the form of the fore side of the timber: So that the nearest distance betwixt these two curves, will certainly be what the bevelling differs from a square; for if the timber were square, the same curve would represent both sides of it.

Now if it were required to find the bevelling of this timber by the water line, (*W a t' L* in the floor plane) it is evident it will be the angle $8 i u$, if the moulded side be aft; and $x u$ will be what it is without a square. This will be in the direction of the level line in the body plane, where it is $x u$; but $x v$ being the nearest distance betwixt the curves taken from the point x , that must be set off from x to v , on the floor plane; so shall $8 i v$, be the true bevelling, when the bevel is set square to the timber at the point v , where the firmark must be placed. But if the moulded side be forward, the angle $x u i$, will be the bevelling, and $x u$ what it is within a square; the same as that which was without a square when the moulding side was aft. Here $u z$ is the nearest distance betwixt the curves, taken from the point u ; and when this is set off from x to z , in the floor plane, the angle $x z i$, will be the true bevelling at the point z , in the body plane, where the firmark must be placed. This method will be very useful for the cant timbers, when they are bevelled by water lines, and may be done by the workman, if the bevelling is given in the direction of the plane of the water line, by observing the following directions, which are in effect the same with these now prescribed.

1st. Apply a square to the bevelling board, at the point where the line that determines the bevelling of the timber intersects the side of the board; and the distance of the other end of the line from the square upon the opposite side of the board, will be what it is within or without a square.

2^d. If the timber has an under bevelling, take the quantity of it, found by the square, with a pair of compasses, and set it off upon the line of direction, on the timber, from the point where the line intersects the moulded side, which is rased in upon the timber: One foot of the compasses being fixed in this point, let the other foot rest in a point in the

line

line of direction: From this last point take the nearest distance to the outside of the timber, and mark the firmark at that place.

3d. Take the nearest distance found on the timber, and set it off from the square upon the same side of the bevelling board, from which the distance set off upon the line of direction, was taken; and mark that place upon the board. A line drawn from that point, to the point where the square was applied on the opposite side of the board, will give the true bevelling, to be taken square to the timber, observing to set it to the proper firmark.

If the timber has a standing bevelling, we must apply a strait edged batten to the line of direction upon the timber, upon which we must set off what the bevelling is, without a square; and proceed in the same manner as before.

Note. If the bevelling board is not exactly the breadth of the timber, the bevelling must be transferred from the board to two parallel lines, the breadth of the timber being the distance betwixt them.

But if it be required to bevel the cant timbers by the diagonal ribbands, the angle Fob , will be that which the fashion piece will then make with the third ribband: For o being the point where the plane of the fashion piece intersects the third ribband; a line drawn from o to F , will be that in which the plane of the timber intersects the plane of the ribband: But then as these planes are not perpendicular to one another, the angle Fob will not be the true bevelling, unless the bevel be so applied that the tongue may be in the direction of the ribband, and then the stock cannot lay flat upon the side of the timber: For which reason this method will not do for practice; for the surest way to take any bevelling, is when both the stock and the tongue of the bevel are square to the timber.

In order then to find the true bevelling upon a square, the direction in which the ribband intersects the timber must be given, as well as the angle Fob ; and likewise the breadth of the timber: Now if these three be given, the angle upon the square may with certainty be found by the following method.

Let the distance betwixt the parallel lines AB and EF , be the breadth of the timber; Ba the direction of the ribband; and ab what the bevelling is without a square. (*See the Fig. under the Scale, Plate 7.*)

Now, that we may the easier conceive how this bevelling may be found, let us suppose the timber to be quite strait, and first trimmed square. Then, because ab is what it is without a square, it is plain there must be so much lined off the aft side of the timber, and when this is hewed

hewed off; the line aB will be the breadth of the outside of the timber; and if BD be made equal to Ba , and Dd equal to ab , and the angle at D a right one; then it is plain the angle ABd , will be the bevelling, if the tongue of the bevel can be kept in the direction of the line Ba : But when the stock of the bevel is laid flat on the side of the timber, the tongue will naturally be perpendicular to the plane of the timber, which will be in the line BF ; and if Ff be made equal and parallel to Dd ; then will the angle ABf , be the true bevelling upon a square: But if the outside of the timber is a curve, the stock must be placed at the point t , and tB made equal to Fa ; and the tongue will come to the point a .

To apply this to find the bevelling of the fashion piece at the third ribband, the angle Fob , is given in the floor plane; and to find the direction in which the plane of the ribband intersects the plane of the timber; we must find the angle, or the inclination of these two planes to one another: For tho' the plane of the timber is perpendicular to the plane of the water lines; it will not be so to the planes of the ribbands: And what was asserted in *Prob. 7.* in regard to the angle at the top and bottom of the chest, *viz.* that they would be equal, must be understood so, as that both the stock and tongue of the bevel be kept parallel to the back side of the chest; which might be easily done, when the partition is properly bevelled to the backside of the chest: But here the case is different, therefore we must find it by the following method.

1st. From the point o draw the line eo perpendicular to Fo ; the line in which the plane of the timber intersects the plane of the ribband on the floor plane.

2^d. Thro' the point o in the body plane, draw the line eo perpendicular to KO , o being the point where the ribband intersects the fashion piece.

3^d. Take the line oe from the floor plane, and set it off from the point 3 , where the third diagonal intersects the line KO in the body plane to the point e in the line oe .

And lastly draw the line $3e$; so shall $O3e$, be the angle the plane of the ribband makes with the plane of the fashion piece.

Now let the distance betwixt the line KO and pf , be supposed the breadth of the timber; then will $3q$ be the breadth of it upon the plane of the ribband; which set off upon the floor plane from o to m , and draw the line bmB , parallel to Fo ; so bm will be what the bevelling is without a square, when taken in the direction of the ribband; and the angle Fob , the bevelling. In order to find the bevelling upon a square, set

set off the breadth of the timber from o to l , and make $l\ 2$ equal to $m\ b$; so shall $F\ o\ 2$, be the true bevelling on a square. The line $2\ o$ is omitted in the plate, because it would be too near to the ribband line $b\ o$.

We have now shewn how to bevel the cant timbers either by water or ribband lines; and much after the same method, may the fashion piece of a square tuck be bevelled. It both rakes and cants, and of consequence will be inclined to the planes of the water lines, if they are parallel to the keel; as that of the long-boat. (See Plate 5.)

Now, in a ship the planes of the water lines may be represented by lines perpendicular to the plane of the fashion piece, in the sheer plane; and formed in the same manner as that of the third transom in the ship (Plate 7.): And then we shall have the true bevelling in the same manner as that of the cant timbers. It may likewise be done by water lines parallel to the keel, as in the long-boat; where we cannot form all the necessary water lines perpendicular to the plane of the fashion piece, because there are no top timbers. We may therefore find the angle the planes of the water lines make with the plane of the fashion piece; and from thence find the true bevellings, by the method directed for finding the bevellings by the ribband lines, when the plane of the fashion piece is perpendicular to the floor plane: So that all that seems now necessary, is to shew how to find the angle the plane of the fashion piece makes with the floor plane, to which the planes of the water lines are supposed parallel; in order to which;

1st. Produce the line $W\ k$ to the point m , in the line $g\ L$, the common section of the sheer and floor planes.

2^d. Thro' the point m draw the line $d\ n$ parallel to $g\ G$.

3^d. Let fall the perpendicular $k\ y$, and thro' the point y draw the line $l\ n$ perpendicular to $d\ n$. Draw also the line $y\ v$ perpendicular to $l\ n$.

Note. The point k may be assumed at pleasure.

4th. From the center m , with the radius $m\ k$, intersect the line $l\ n$ in the point l . We may also draw the line $m\ l$; so $k\ m\ l$, will be the angle formed upon the plane of the fashion piece by its intersection with the sheer and floor planes.

Lastly. With the radius $n\ l$, from the center n , intersect the line $y\ v$ in v ; so shall $l\ n\ v$, be the angle which the plane of the fashion piece makes with the plane of the water lines; as in Prob. 7. Chap. 1. Part 2.

Having thus found the angle, let $d\ m$ be the breadth of the fashion piece: Thro' d draw a line parallel to $n\ l$, to intersect the line $n\ v$ in the point c ; so shall $c\ n$ be the breadth of the fashion piece upon the plane of the ribband. If then lines be drawn on the floor plane, parallel to $g\ G$,

gG , $e3$, $e2$, $u1$; as xf parallel to gG , and if xG be equal to nc , and perpendicular to gG ; then the angle gGf , will be the bevelling upon the plane of the ribband, and xf what it is without a square. Again, if from the point G we set off dn , the breadth of the timber, and draw the dotted line ii , equal and parallel to xf ; we shall have the angle gGi , the true bevelling upon a square.

After the bevellings of the timbers are found, they are put on a board provided for that purpose, called a bevelling board. This board should be made the exact breadth of the timber, which suppose x, z, s, t , in the figure bevel (*Plate 5.*). If upon one edge of the board we set off as many points, as we intend it shall contain timbers, and place them at any convenient distance from one another, whether equal or unequal is indifferent, and distinguish them all by their proper names; we may then lay the graduated edge of the board to the line that represents the moulding side of the timber, so that the proper point be at the intersection of the plane of the ribband, and the plane of the timber; and when in this position, if we mark the other edge of the board where it crosses the ribband; a line drawn across the board from these two points, will be the true bevelling of the timber at that place. So when the board is applied to timber 7 in the floor plane, we shall find the bevelling to be as much from a square as is expressed by the dotted square line drawn across the board at that place. It must be observed that the perpendicular at timber 7. represents the fore-side of the floor timber, and aft-side of its corresponding futtock: So that the floors will be under, and the futtocks standing bevellings, and where the space of the ribband, containing these two timbers, is strait, the one will be as much standing as the other is under bevelling; as in the plate, where the perpendiculars drawn to timber 7, from the points in which the aft-side of the floor, and fore-side of the futtock intersect the ribband, are parallel to that drawn thro' the point, where the line that represents the moulding side of both timbers, intersects the ribband: This method will answer for all the timbers, whether we bevel by half breadth lines, water lines, or ribbands, only observing the directions given before, to transfer oblique to square bevellings. We shall likewise hereby find, that in some cases, where the ribbands are very round, one bevelling will not do for two timbers, and even when it is only taken for one timber an allowance ought to be made for the round of the timber; for which purpose it will be necessary in some cases to make a mould to fit the round or hollow, and fasten this to a strait edged batten in the proper direction.

Another method practised to find the bevellings for the square tim-
Z
bers.

bers, is by the diagonals in the body plane; without regarding the curves of the ribbands in the floor plane: But this cannot be used, unless we first form all the timbers in the body plane.

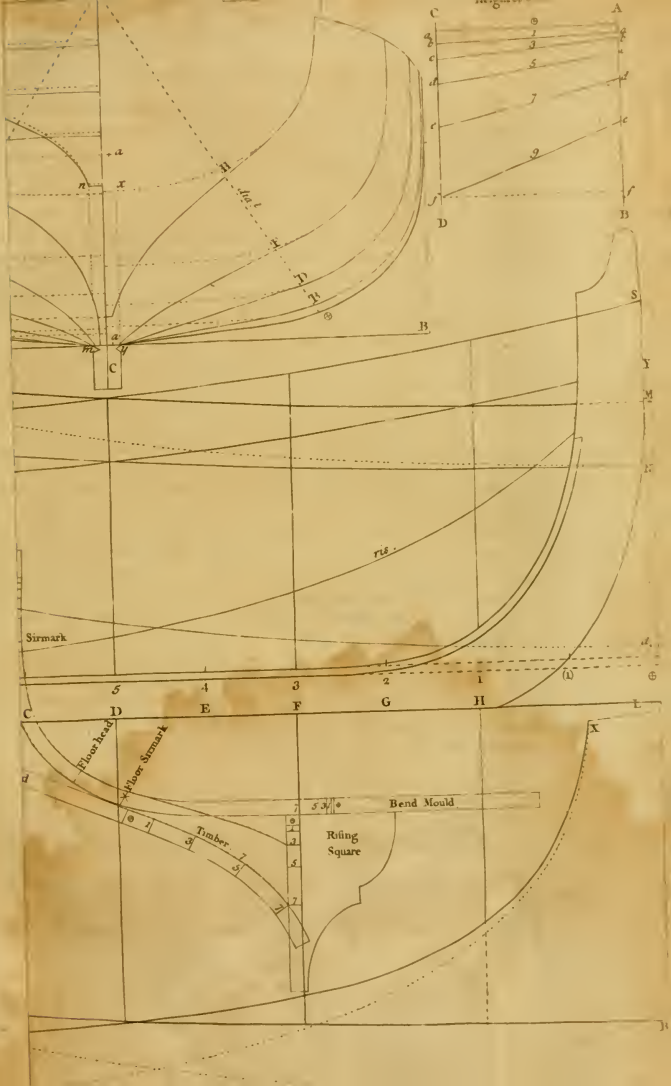
Let then the distance betwixt the parallel lines AB and CD , be the timber and room, that is, as before observed, the breadth of two timbers, and the space betwixt them, and if this be equal to the distance betwixt the perpendiculars representing the planes of the timbers, we have all the timbers ready formed in the body plane in *Plate 5.* and may find the bevellings in the following manner.

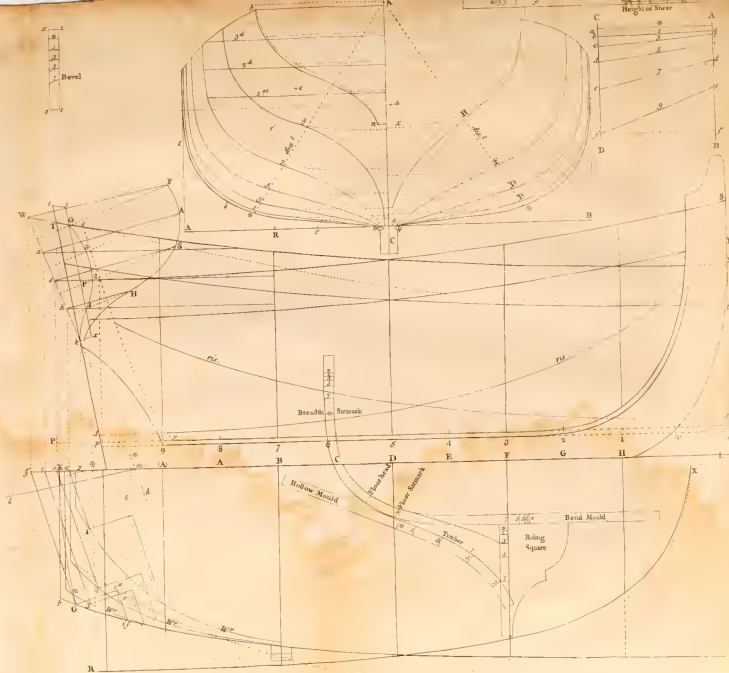
Draw the line AC , perpendicular to AB and CD , for the bevelling of $\oplus$: Then take the several distances in the diagonal, from $\oplus$ to the points where it intersects the timbers 1, 3, 5, 7 and 9, and set them off from the point a , to the points b , c , d , e and f , in the lines AB and CD ; so that lines drawn from f to b , from e to c , &c. would be parallel to AC . Then draw the lines ab , bc , cd , de , ef ; which will give the bevellings of the timbers 1, 3, 5, 7 and 9: That is to say, of two timbers, viz. a floor timber and a futtock. So fe will be what the bevelling of timber 9 is within a square, as may be seen in the floor plane, by producing the line that represents the plane of timber 9, till it is equal in length to that which represents the plane of timber 7: But it must be observed that the perpendicular at 9, represents the foreside of the floor timber; therefore the futtock corresponding to it, is before the perpendicular at 9; the futtock corresponding to timber 7, will likewise be before its perpendicular: So that tho' this method may give us nearly the bevellings of two timbers, yet these are not the two that are to be fastened together. Therefore this method ought to be rejected, unless we set off the breadth of the timbers on each side of the line that represents the moulding edge, and draw perpendiculars betwixt each on the floor plane. We might then indeed find points in the diagonal, betwixt the timbers formed, which would give the bevellings of each floor timber with its corresponding futtock.

S E C T. II.

To find the Bevellings of the Transoms. (Plate 7.)

There are two ways of doing this. One is by forming curves on the sheer plane, by sections of planes cutting the ship fore and aft, parallel to the sheer plane. These planes will be represented by strait lines in the





the floor plane, parallel to W K; and in the body plane, by strait lines parallel to K O. In the plate we have only formed the two dotted curves in the sheer plane, to timber 7; the intersections of these with the planes of the transoms, which in the sheer plane are represented by strait lines, will give the bevellings; so that all that is now necessary, is to shew how these curves are formed, and in what direction the stock of the bevel is to be placed upon the transom. In order to this, first draw the two dotted lines in the body plane, parallel to K O, to intersect the timbers. Transfer the heights of these intersections to their corresponding timbers on the sheer plane; which will give the points thro' which these curves must pass. Secondly draw two dotted lines parallel to W K in the floor plane, to intersect all the transoms: These transferred to the planes of the transoms in the sheer plane, will give the points where the curves intersect these planes. These dotted lines in the floor plane, must be the same distance from the line W K, that the corresponding ones are from the line K O, in the body plane. They will intersect the transoms in the direction in which the stock of the bevel is to be laid upon the transoms; and if this should be judged too oblique, it may be transferred to a square one, as before directed. As the third transom is not formed in the floor plane, a line must be drawn parallel to the plane of it, where it is formed, to find the proper place and direction of the bevel.

The other method is by forming more timbers abaft the fashion piece. Their planes will be represented by strait lines in the floor plane, where they will intersect all the transoms already formed. In the plate we have only drawn one G g, by which we have formed another cant timber in the body plane, in the same manner as the fashion piece was formed. The angle formed by the curve of this timber, and the level lines that are drawn at the height of each transom in the body plane, will be the bevelling; and the strait line which represents the plane of the timber, will give the direction in which the bevel is to be placed upon the transoms. This may likewise be transferred from an oblique to a square bevelling; and if needful, a mould made for the hollow of the transom.

The only thing that remains in regard to bevellings, is to shew how the ribbands are bevelled. Now as their planes are represented by diagonals in the body plane, the angles that are formed by the diagonals, and the timbers in the body plane, will be the bevellings; and the perpendiculars representing the planes of the timbers in the floor plane, will give the direction for the bevel. The harpins are bevelled by level lines in the body plane; but as they are not parallel to the keel, when the stock is laid flat upon the upper side of the harpin, the tongue will not be in

the direction of the timber; yet as the harpins are not above four or five inches broad, this need not be regarded. Those who incline to greater exactness may use the same method as in finding the bevellings of the fashion piece for a square tuck, or form the harpins by diagonals in the body plane, which may be so contrived as to intersect the timbers nearly in the same points with the sheer.

C H A P. V.

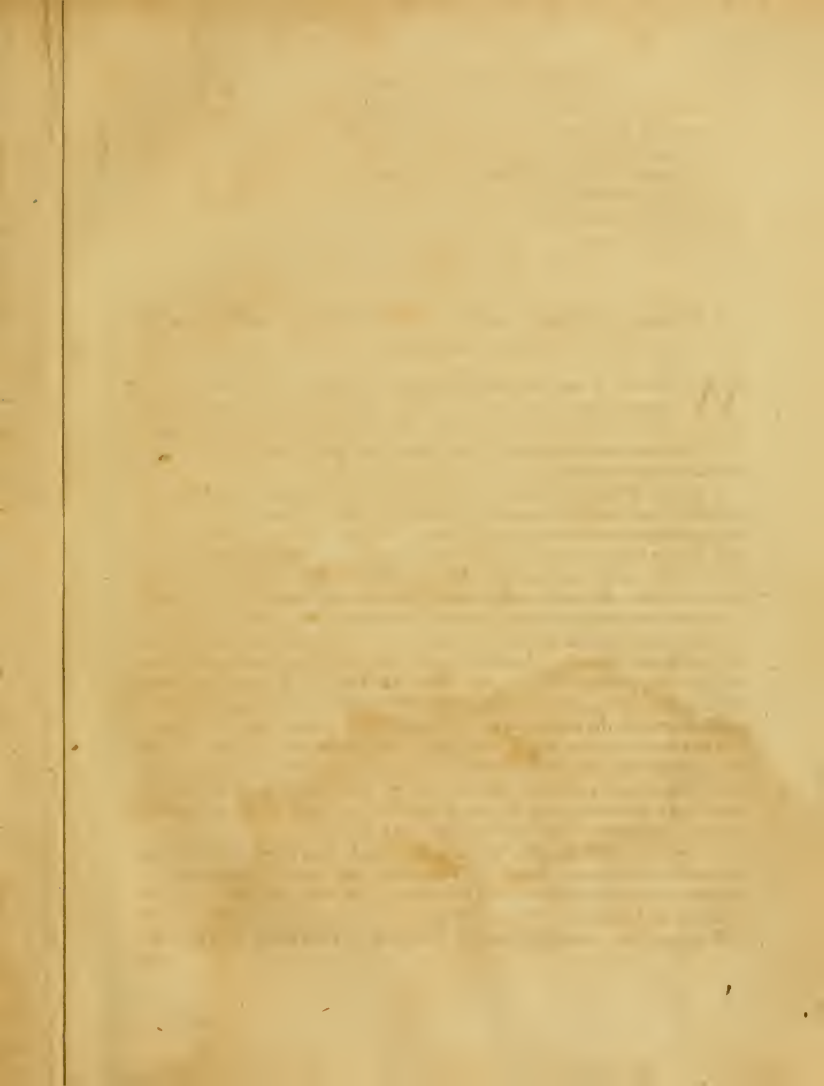
Of forming Bodies not similar to that by which the Lines on the Sector were constructed.

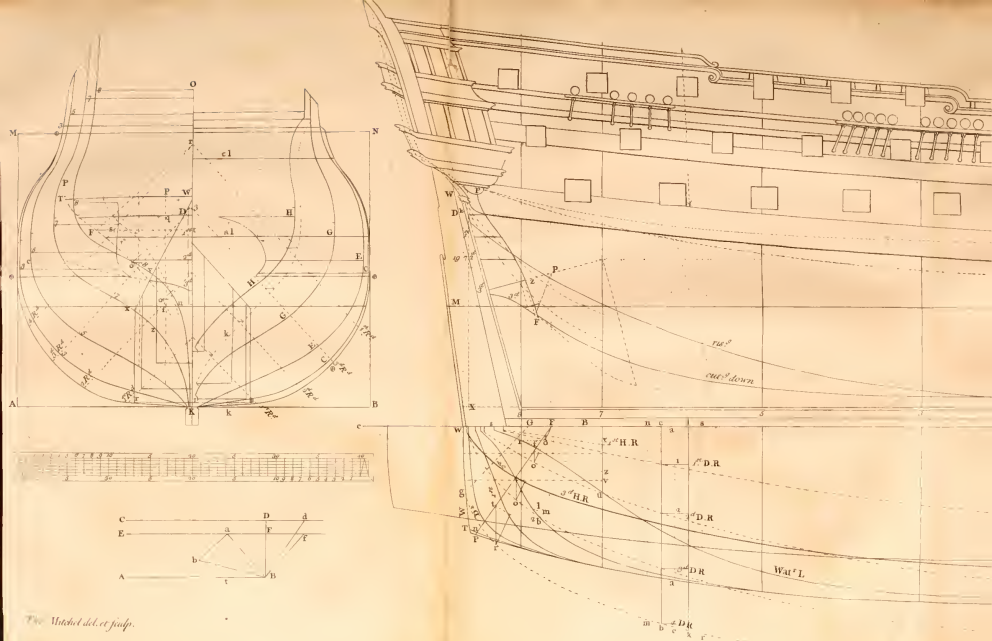
WE have, as was proposed in this second part, shewn the general methods used in drawing of ships, and how to lay down a ship by the sector, if similar to that from which the lines were constructed. We shall now shew the use of the sector in forming bodies that are not similar to one another.

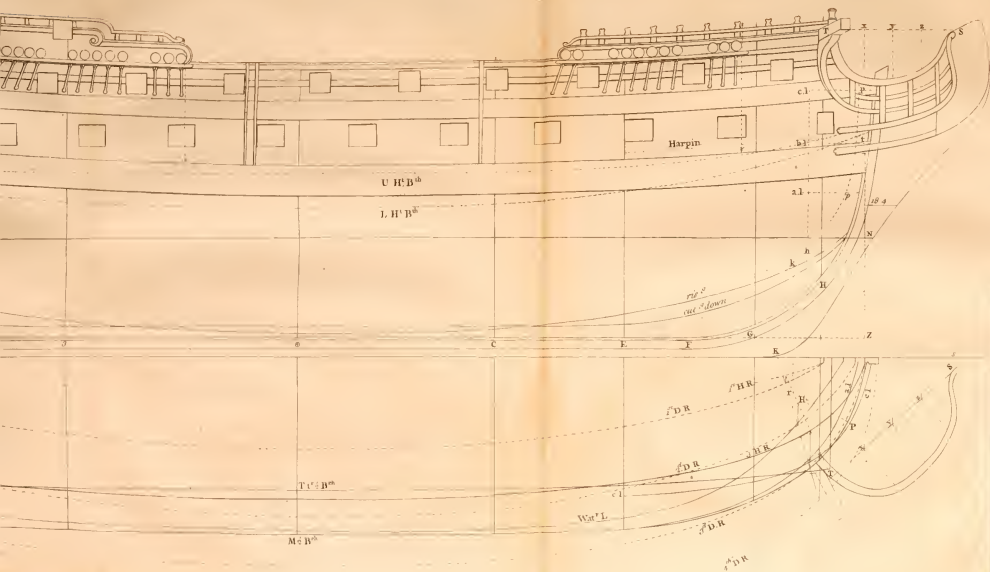
The first thing to be done, is to set the sector by the proposed half breadth, and draw the diagonals as before directed. Then we must form the midship frame by the points, as in *Plate 7*. By this we shall discover, that it will be either too full or too sharp; and therefore must be altered by the artist, according to the service the ship is designed for. The foremost and aftermost timber must likewise be formed by the artist; and then the portions of the diagonals intercepted betwixt these timbers and the midship, will not be the same as that given by the sector, when set to the half breadth. In order then to find the points in the diagonals for the intermediate timbers, the sector must be set to each seperately. Thus, take with a pair of compasses, the portions of the diagonals intercepted betwixt the midship and aftermost timber, now made conformable to the service for which the ship is designed; and by these set the sector seperately for each, till the distance taken by the compasses reach from the proper points in one leg, to the corresponding points in the other leg; and being thus set, we may find the proper points in that diagonal; and then set the sector for the next diagonal.

In order to illustrate this, we have in *Plate 8*, laid down the midship, foremost and aftermost timbers of two ships; the one an *East India* ship, the other a *French* privateer. Their bodies are very different from one another, and likewise from that by which the sector was formed; and if the intermediate timbers formed by the sector in both ships, will produce

fair







fair ribbands, it may be presumed that it may be very useful in any other cases.

Now because we cannot open the sector in the plate, we have taken the several divisions of the four ribband lines upon the after body of the sector, and set them off from the point C, upon the four lines C 1^a R, C 2^a R, &c. intersecting one another in the point C: So that the point C is the same distance from the points 7, 5, 3 and $\oplus$, upon these lines, that the same points are from the center of the joint upon the sector. We have likewise drawn four other lines to intersect the former in the point C. But the angle formed by the two lines marked 1^a R, is not equal to the angle formed by the two lines marked 2^a R; nor to that formed by the two lines marked 3^a R, or by those marked 4^a R. The angle formed by each two lines of the same name, is determined in the following manner. From the center C, an arch of a circle is described to intersect the line 1^a R in the point $\oplus$. From this point, the distance betwixt the midship and aftermost timber now formed, taken upon the first diagonal, is set off upon the arch; and the other line C 1^a R is drawn thro' this point. In like manner the other three lines are drawn, by describing arches of circles from the center C, to meet each line in the point $\oplus$, and setting off upon each arch the distance betwixt the midship, and aftermost timber, taken upon the diagonal corresponding to each line. So the distance betwixt the points $\oplus$ and $\oplus$, in the two lines marked 1^a R, is equal to the portion of the first diagonal intercepted betwixt the midship and aftermost timber; the distance betwixt these points in the lines marked 2^a R, is equal to the portion of the second diagonal intercepted betwixt the aforesaid two timbers, &c. The lines being thus drawn, and each divided in the same proportion as its corresponding one of the same name is, the points on each diagonal for the timbers 3, 5 and 7, will be found upon examination to be at the same distance from the after timber, that these points are from one another, in the two lines corresponding to each diagonal.

The like process may be used for forming the intermediate timbers in the fore-body: But it must be observed, that in forming the ribbands from these timbers, their stations in the sheer plane must be determined by the sector, as in *Plate 7.* and after the ribbands are all formed, the proper stations of the timbers may be assigned.

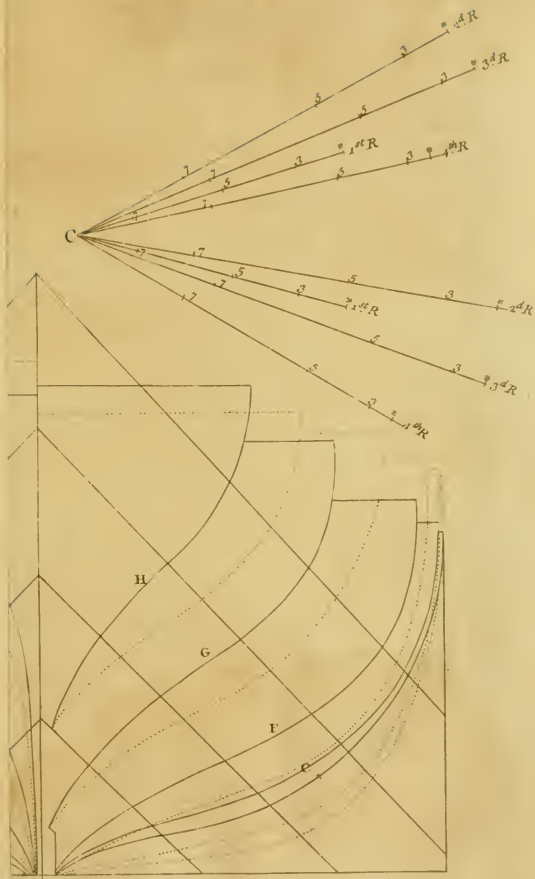
Now tho', both the ribbands and timbers thus formed may prove fair, yet neither this method by the sector, nor any other method, which has been published, can be established as a certain invariable rule; because the curves by which they are formed have no properties peculiar to themselves to distinguish them from all other curves, as was before observed.

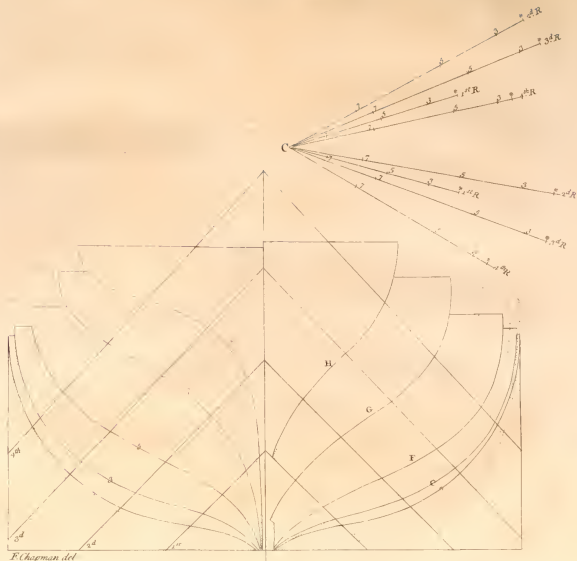
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The only way to make any considerable improvements in this art, we presume, will be, by carefully examining the different bodies of several ships that have been actually built, and whose good or bad qualities have been discovered by experience. We have therefore, for the sake of such of our readers as are not furnished with a sufficient number of draughts for that purpose, collected into the following table all the dimensions that are necessary to determine the form of fourteen different ships; and we may venture to affirm, that the youth will receive more benefit by delineating these from the dimensions, and thereby sooner acquire the art of drawing, than by all the rules and directions that have been hitherto published on that subject.

The dimensions in the following tables are taken from the diagonal scale, *Plate 2.* and the number of equal parts contained in twelve feet, is specified at each ship.

PRIN-





PRINCIPAL DIMENSIONS OF FOURTEEN SHIPS.

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	RATES OF SHIPS OF WAR.							MERCHANT SHIPS.							Tons.
	No 1. 1ft.	No 2. 2d.	No 3. 3d.	No 4. 4th.	No 5. 5th.	No 6. 6th.	No 7. Sloop.	No 8. 630.	No 9. 398.	No 10. 340.	No 11. 332.	No 12. 162.	No 13. 114.	No 14. 50.	
Length from the fore-side of $\oplus$ to the keel	19976	19680	20619	17669	14888	12802	7556	15300	12994	11952	13104	9658	6240	10993	
fore-part of the post on the keel	21656	21272	21870	19155	16102	13752	8260	16686	14744	13414	14360	10426	7840	12614	
Ditto to the wing transom	11195	12105	11006	9800	8616	9129	4034	7756	6820	6820	5004	4580	2514	5420	
Ditto to the touch of the stem	17440	17413	10458	14270	12844	11945	6474	11708	10150	9720	8940	7378	5148		
Ditto to affside of stem on the l. deck	17556	17995	16458	14280	12844	12097	6634	11700	10266	9762	8168	7380	5012	8494	
Ditto to ditto at the head	16200	16485	15084	13200	12076	11256	10044		9200	8468					
Ditto to the knuckle															
From the after timb. to the l. counter	3504	3120	2700	2816	2350	2120	1546	2046	2250	2500	2006	1478	—	3518	
Ditto to the second counter	3950	3410	3060	3164	2638	2390	1800	3010	—	2870	2270	1656	—	—	
Ditto to upright of the stem at hr. rail	5370	4440	3610	3806	2952	2674	1800	3360	2370	3350	1477	1960	—	3900	
Height of the wing transom	6000	6280	6060	5428	4742	3690	2194	4485	5440	3770	3571	3048	2240	3034	
Height of the stem	7750	9060	8000	6568	6130	5242	3472	5900	5526	5031	4776	4130	5950	4893	
Height of the lower deck; $\left\{ \begin{array}{l} \oplus \\ \text{item} \end{array} \right\}$ on the post	5408	5800	5458	4756	4318	3260	—	4936	3080	3548	3155	3048	—	—	
Height of the lower deck; $\left\{ \begin{array}{l} \oplus \\ \text{item} \end{array} \right\}$ on alter tim.	4940	5130	4939	4278	3930	2930	2130	3500	2860	3172	2690	2541	2810	—	
Height of the lo. edge $\left\{ \begin{array}{l} \oplus \\ \text{item} \end{array} \right\}$ of the lower wale	5150	5641	5304	4342	4200	3200	2600	3766	3080	3120	2834	2770	3000	—	
Height of the lower counter	3510	4196	4080	3450	3250	2630	1675	3133	2400	2464	2123	2310	1730	2236	
Ditto the upper	4400	5068	4782	4100	3740	3170	2250	3400	3080	2860	2720	2004	2320	3130	
Heig. of heig. of hr. line on the stem	7300	7400	7150	6355	5710	4020	2500	5510	5680	4720	4506	3596	4321	—	
Upper breadth fore-cp	7990	8070	7812	6880	6510	5320	2680	6100	—	5424	4970	3871	—	—	
Sweep of the stem $\left\{ \begin{array}{l} \text{upper} \\ \text{lower} \end{array} \right\}$	4770	3866	2756	2532	2370	2450	1750	2710	2354	2280	2082	1440	1500	3570	
Height of the upper water line $\left\{ \begin{array}{l} \text{a-liaft} \\ \text{a-lare} \end{array} \right\}$	6380	5190	5370	4420	4200	2780	1416	3310	3826	3920	2790	3100	1854	3950	
From fore-part $\left\{ \begin{array}{l} \text{fore-mast} \\ \text{ma-mast} \\ \text{miz-mast} \end{array} \right\}$ to cen. of deck	3786	4006	4250	3226	3108	2780	1560	3850	2700	2700	2700	2200	1960	—	
	3596	4400	4014	3770	2950	9145	1330	3380	2430	2750	2450	2150	1000	—	
	3878	3800	4384	3950	2800	2422	4570	8936	7240	8700	5840	5100	—	—	
	4582	14464	15145	13213	10954	9364	1968	3850	2240	2560	3500	1600	—	1950	
							6000	1596	9830	7250	5940	3370	—	—	

Di.

DIMENSIONS for forming the BODIES.

N^o 1.

A first RATE 100 Guns. 2262=12 Feet.

Timbers names.	Diagonals.					Height of bread.		Half brea.		Distance of frames.	
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top ti.	main	top.	fr. ⊕
⊕	1620	3114	5104	6432	7380	4178	4790	9790	5476	4030	—
8	1460	2790	4678	6094	7200	4316	4844	10080	5388	3886	7232
16	1186	2212	3930	5552	6822	4646	4998	10438	5214	3661	11862
24	0766	1532	2980	4470	5940	5290	5490	10934	4742	3374	16492
30	0311	0577	1780	2940	4636	6070	6137	11423	4110	3135	19976
D	1586	3016	4998	6322	7316	4212	4776	9822	5457	4028	4626
M	1346	2538	4398	5848	7000	4430	4777	9965	5262	3940	9270
W	0518	1386	2878	4362	5736	5148	5240	10240	4418	3728	13870
X	—	0828	2251	3668	5046	5526	5578	10325	3924	3652	15006
Height of the diagonals on the middle line	1178	2281	4142	6003	7904	—	—	—	—	—	—
Distance from ditto on the side produced	1184	2300	4128	5944	7780	—	—	—	—	—	—

N^o 2.

A second RATE 90 Guns. 2731=12 Feet.

Timbers names.	Diagonals.					Height of bread.		Half brea.		Distance of frames.	
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top ti.	main	top.	fr. ⊕
⊕	1589	3010	4974	6220	7164	4224	5070	9750	5410	3916	0000
9	1410	2728	4700	6008	7051	4416	5130	10041	5366	3830	6820
18	0973	1140	3812	5363	6650	4837	5310	10436	5228	3652	11944
27	0405	0813	2163	3894	5517	5714	5844	11000	4704	3312	17014
32	0105	0206	0880	2414	4223	6396	6425	11388	4010	3026	19842
I	1465	2804	4750	6072	7126	4355	5060	9774	5412	3848	6556
S	1022	2012	4184	5364	6624	4960	5295	10006	5216	3740	11645
W	0743	1580	3186	4786	6083	5360	5569	10136	4867	3686	13345
Z	0265	0930	2336	3872	5132	5927	6018	10272	4116	3612	15045
Height of the diagonals on the middle line	1164	2214	4060	5912	7774	—	—	—	—	—	—
Distance from ditto on the side produced	1164	2214	4084	5944	7823	—	—	—	—	—	—

N^o 3.

A third RATE 74 Guns. 2730=12 Feet.

Timbers names.	Diagonals.					Height of bread.		Half brea.		Distance of frames.	
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top ti.	main	top.	fr. ⊕
⊕	1533	2877	4658	5923	6864	4069	4632	7920	5166	4064	—
9	1462	2729	4533	5806	6743	4199	4717	8142	5103	3966	6840
18	1191	2199	3922	5241	6226	4580	5026	8482	4790	3762	11962
27	0586	1194	2500	3956	5182	5299	5572	8990	4228	3350	17083
32	0140	0340	1156	2512	3992	6086	6192	9430	3564	2952	20499
I	1464	2745	4547	5834	6778	4114	4662	7952	5140	4012	4812
M	1310	2426	4206	5504	6455	4270	4741	8063	4926	3867	8214
S	0962	1796	3267	4660	5722	4647	4996	8256	4353	3580	21628
Y	—	0694	1932	3210	4199	5295	5400	8468	3147	3197	14468
Height of the diagonals on the middle line	1152	2155	3928	5706	7462	—	—	—	—	—	—
Distance from ditto on the side produced	1144	2122	3942	5714	7484	—	—	—	—	—	—

N^o 4.

DIMENSIONS for forming the BODIES.

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N^o 4. A fourth RATE 50 Guns, 1034 Tons, 2722=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of traces.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	
⊕	1406	2600	4154	5216	6000	3444	3950	7010	4496	3400	0000
9	1260	2316	3884	4978	5803	3620	3968	7146	4398	3355	5646
18	0918	1676	2923	4120	5118	4050	4219	7500	4068	3094	10993
24	0532	1042	2062	3206	4386	4466	4598	7857	3654	2798	14559
29	0146	0349	1004	2151	3454	5000	5056	8240	3040	2456	17505
I	1264	2322	3900	4970	5754	3666	3962	7078	4390	3318	5342
P	0967	1742	3000	4191	5149	4108	4314	7240	4056	3020	8898
S	0640	1265	2358	3466	4520	4482	4582	7360	3626	2798	10688
W	0190	0640	1507	2452	3414	5024	5024	7512	2690	2540	12460
Height of diagonal on } the middle line											
Distance from ditto on } the bala produced											
	1058	1980	3480	4974	6471						
	1071	1984	3482	4976	6480						

N^o 5. A fifth RATE 40 Guns, 706 Tons, 2736=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of traces.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	
⊕	1208	2000	3333	4400	5090	3230	3766	6404	4006	3027	0000
10	1098	1818	3136	4230	5030	3254	3778	6570	4006	3027	5954
19	0680	1218	2346	3500	4416	3628	3960	6850	3712	2828	10424
25	0339	0600	1448	2558	3560	4220	4398	7114	3198	2511	13400
28	0128	0216	0761	1796	2864	4708	4760	7262	2950	2376	14888
G	1175	1958	3300	4360	5040	3290	3778	6436	3990	3027	4210
N	0911	1534	2764	3880	4682	3514	3862	6544	3762	2998	7178
T	0356	0772	1768	2880	3710	4015	4194	6680	3066	2800	10156
Y	0000	0282	1133	2200	2910	4414	4510	6776	2298	2574	11676
Height of diagonal on } the middle line											
Distance from ditto on } the bala produced											
	1020	1744	3066	4342	5564						
	1016	1716	3123	4634	6230						

N^o 6. A sixth RATE 20 Guns, 508 Tons, 2744=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of traces.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	
⊕	1064	1870	2988	3818	4316	2570	3126	5408	3590	3014	0000
9	0939	1640	2724	3556	4153	2680	3136	5511	3544	2950	4424
18	0610	1100	1944	2808	3600	2984	3298	5742	3222	2650	8834
24	0290	0532	1144	1964	2796	3385	3588	6034	2683	2286	11792
26	0119	0224	0680	1460	2350	3580	3729	6148	2383	2128	12802
I	0922	1620	2684	3520	4100	2722	3136	5511	3504	2864	5710
P	0642	1154	2052	2893	3526	3126	3330	5654	3094	2595	8665
S	0432	0828	1536	2254	2844	3494	3606	5784	2510	2372	10147
U	0000	0370	0894	1498	2016	3836	3900	5886	1764	2180	11136
Height of diagonal on } the middle line											
Distance from ditto on } the bala produced											
	0848	1508	2528	3450	4314						
	0860	1514	2720	4106	5600						

A a

N^o 7.

N^o 7.

A SLOOP of 150 Tons, 2696=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	tro. ⊕
⊕	0754	1262	1830	2290	2575	1448	1622	2510	2078	1976	—
4	0678	1145	1712	2168	2512	1500	1665	2550	2062	1906	2000
8	0518	0899	1405	1898	2315	1632	1758	2654	1939	1830	4150
12	0254	0476	0910	1403	1898	1895	1939	2819	1704	1600	6268
14	0115	0220	0538	1057	1590	2074	2090	2920	1446	1362	7334
D	0714	1194	1766	2217	2538	1500	1656	2538	2078	1961	1999
F	0616	1072	1651	2134	2481	1597	1717	2608	2048	1918	3070
H	0449	0836	1376	1872	2257	1793	1848	2702	1930	1848	4144
K	0110	0412	0870	1351	1773	2106	2106	2856	1586	1668	5212
Height of the diagonals } on the middle line } Distance from ditto on } the base produced }	0614	1066	1616	2136	2644	—	—	—	—	—	—
	0614	1046	1700	2378	3101	—	—	—	—	—	—

N^o 8. The LONDON EAST INDIA Ship 630 Tons, 2730=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	tro. ⊕
⊕	1430	2588	3886	4720	5210	3230	3654	6014	3674	2980	—
4	1374	2476	3790	4636	5133	—	3754	6126	3623	2966	4152
12	1140	2040	3306	4252	4816	—	4012	6360	3461	2826	8296
20	0560	1148	2182	3270	4110	—	4396	6698	3170	2580	12444
24	0228	0546	1372	2430	3528	—	4646	6920	2974	2420	14526
D	1376	2485	3784	4632	5140	—	3694	6012	3624	2958	3894
H	1264	2276	3586	4481	5020	—	3766	6036	3566	2914	5978
M	1012	1886	3120	4116	4712	—	3900	6090	3416	2812	8060
O	0578	1340	2577	3652	4346	—	4036	6152	3186	2712	9404
Height of the diagonals } on the middle line } Distance from ditto on } the base produced }	1110	2020	3330	4640	5950	—	—	—	—	—	—
	1106	2020	3226	4407	5571	—	—	—	—	—	—

N^o 9. The BONETTA 398 Tons, 2723=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	tro. ⊕
⊕	0915	1707	2935	3934	4616	2704	3026	5232	3260	2521	—
9	0842	1548	2797	3775	4462	2790	3084	5308	3140	2474	4599
18	0516	0956	1900	2898	3690	3410	3660	5508	2733	2238	9198
24	0206	0354	0890	1666	2422	4150	4336	5740	1836	1696	12264
26	0120	0168	0498	1082	1710	4410	4584	5832	1244	1242	13254
I	0810	1524	2766	3756	4460	2830	3136	5320	3142	2514	4364
P	0539	1044	2047	3060	3786	3292	3580	5550	2708	2330	7430
R	0220	0640	1550	2503	3220	3522	3800	5678	2290	2040	8470
S	—	0362	1174	2090	2756	3677	3956	5764	1942	1748	9000
Height of the diagonals } on the middle line } Distance from ditto on } the base produced }	0710	1320	2490	3740	5000	—	—	—	—	—	—
	0706	1315	2361	3400	4393	—	—	—	—	—	—

N^o 10.

DIMENSIONS for forming the BODIES.

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N^o 10. The THAMES 340 Tons, 2745=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fr. ⊕
⊕	1024	1731	2909	3962	4966	2500	2930	5211	3196	2464	—
3	0845	1441	2608	3694	4722	2722	3058	5333	3067	2378	4791
5	0510	0913	1920	3107	4249	3165	3341	5554	2846	2251	7966
7	0193	0334	0995	2113	3436	3661	3728	5872	2533	2023	11130
8	0106	0155	0653	1735	3096	3816	3854	5985	2410	1928	11952
B	0970	1647	2826	3870	4850	2624	2979	5220	3118	2400	3190
D	0722	1240	2315	3395	4382	3115	3233	5290	2876	2235	6157
E	0532	0951	1909	2986	3942	3396	3458	5354	2600	2130	7272
F	0345	0699	1561	2600	3536	3594	3622	5398	2310	2055	7950
Height of the diagonals on the middle line Distance from ditto on the base produced }											
	0824	1434	2682	4068	5581	—	—	—	—	—	—
	0858	1439	2419	3317	4110	—	—	—	—	—	—

N^o 11. A FRENCH Privateer of 372 Tons, 2730=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea		Distance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	top.	main	top.	fr. ⊕	
⊕	0807	1753	2746	3770	3686	2762	4514	3242	2734	—	
8	0700	1536	2480	3174	3578	2830	4536	3216	2711	3948	
16	0450	1010	1790	2530	3134	3082	4708	3030	2584	7876	
24	0164	0370	1754	1300	2040	3608	5062	2600	2280	11812	
27	0073	0116	0279	0598	1330	3907	5260	2364	2110	13294	
H	0750	1544	2449	3114	3464	2874	4600	3096	2660	3700	
M	0625	1238	1990	2628	3056	3122	4700	2810	2548	5674	
O	0428	0956	1616	2248	2710	3354	4752	2526	2486	6670	
Q	0106	0518	1088	1672	2146	3711	4834	2084	2398	7648	
Height of the diagonals on the middle line Distance from ditto on the base produced }											
	0800	1611	2472	3232	3906						
	0806	1620	2880	4363	6176						

N^o 12. A SHIP of 162 Tons. 2702=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	fr. ⊕
⊕	0914	1652	2500	3018	3218	2308	2590	3838	2494	2136	—
4	0814	1486	2340	2898	3138	—	2664	3926	2430	2086	3520
12	0535	0962	1590	2220	2692	—	2926	4170	2226	1910	7034
16	0260	0504	0956	1544	2090	—	3179	4340	2026	1776	8776
18	0090	0217	0510	1034	1670	—	3317	4444	1888	1682	9658
D	0800	1458	2334	2916	3175	—	2668	3898	2465	2110	3356
H	0630	1140	1892	2546	2900	—	2842	3974	2318	2010	5135
K	0298	0776	1486	2110	2506	—	3010	4026	2066	1848	6014
M	—	0100	0716	1264	1622	—	3232	4090	1361	1287	6912
Height of the diagonals on the middle line Distance from ditto on the base produced }											
	0716	1300	2116	2920	3652	—	—	—	—	—	—
	0712	1300	2187	3132	4160	—	—	—	—	—	—

A a 2

N^o 13

N^o 13. A Fishing S M A C K, 114 Tons, 2723=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea		Distance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	iro. ⊕
⊕	0681	1316	2100	2672	3143	1920	—	3310	2400	2051	—
6	0554	1101	1916	2500	2988	2042	—	3186	2361	2000	2142
12	0321	0660	1366	2090	2698	2211	—	3200	2158	1800	4298
15	0186	0424	1026	1728	2418	2336	—	3278	1930	1608	5374
18	0098	0216	0653	1300	1960	2504	—	3386	1552	1348	6440
C	0628	1234	2031	2600	3064	1930	—	3400	2361	2024	1078
F	0566	1102	1872	2450	2918	2020	—	3508	2248	1930	2164
I	0440	0861	1522	2108	2580	2176	—	3620	1968	1684	3240
U	—	0124	0726	1345	1814	2600	—	3800	1330	1000	4312
Height of the diagonals on middle line	0500	1000	1820	2644	3456	—	—	—	—	—	—
Distance from ditto the base produced	0500	1000	1820	2644	3456	—	—	—	—	—	—

N^o 14. A S L O O P of 50 Tons, 5462=12 Feet.

Timbers names.	Diagonals.					Height of brea.			Half brea.		Distance of frames.
	1ft.	2d.	3d.	4th.	5th.	low.	upp.	top.	main	top.	iro. ⊕
⊕	1422	2547	3684	4724	5698	1956	2770	4190	3570	3303	—
4	1400	2518	3657	4710	5676	2028	2782	4164	3570	3303	2968
8	1234	2235	3354	4452	5454	2295	2900	4257	3494	3252	5984
12	0854	1648	2665	3808	4882	2770	3216	4465	3216	3022	8898
14	0536	1174	2112	3244	4424	3052	3408	4606	2985	2812	10346
B	1372	2458	3598	4670	5634	2050	2800	4296	3548	3252	2662
D	1278	2280	3400	4484	5454	2193	2915	4412	3436	3154	4154
F	1116	1993	3001	4045	4981	2444	3113	4578	3112	2882	5624
H	0558	1338	2264	3214	3980	2873	3432	4790	2388	2200	7120
Height of the diagonals on middle line	1129	2024	3110	4352	5778	—	—	—	—	—	—
Distance from ditto the base produced	1120	2020	2838	3583	4250	—	—	—	—	—	—

These tables of dimensions are so particular that one example will be sufficient to illustrate their use in laying down any of the ships.

Let it then be required to lay down the *Bonetta* pink, which in the tables is N^o 9. 398 Tons.

1st. Draw the line A B, upon which erect the perpendicular C D; and because the main half breadth in the column is 3268, take that from the scale, and lay it off from C to A, and from C to B, and erect perpendiculars at A and B.

2^d. Look for the lower height of breadth, in the column; which will be found to be 2704. Set up this from A to L, and from B to L; and draw the line L L, which will be parallel to A B. Set up also the upper height 3026, from A to V, and from B to V; and lay off 2354, which
by

by the tables is the radius of the upper breadth sweep, from the points V and V; which will give us the center of the sweep.

3d. Set up 5232, the height of the top timber line, from C upon the line CD; thro' which point draw a line parallel to AB, and lay off 2521. the half breadth of the top timber, both ways upon it, from the line CD; and form the top timber by a mould, to break in fair with the back of the upper breadth sweep. This compleats the upper part of the midship frame.

4th. To form the lower part of the midship frame, draw the five diagonals; their heights from the point C, upon the line CD, and their distance from C, upon the line AB produced, is given in the proper columns. The height of the fifth diagonal is 5000, which set up from C to 5: The distance from the middle line upon the base is 4393, which lay off from C, upon the base produced; and draw a line to this point from 5; which will be the upper or fifth diagonal. In the same manner are the other diagonals drawn, by taking the numbers from the proper columns.

5th. Lay off 4616 from 5 upon the fifth diagonal, 3934 from 4 upon the fourth, 2935 from 3 upon the third, 1707 from 2 upon the second, and 915 from 1 upon the first diagonal: A curve passing thro' these points, and the main breadth at the lower height will form the midship frame to the floor head. A strait line to touch the curve in this point, drawn to the upper edge of the keel, compleats the whole midship frame. The points in the diagonals are in the column corresponding to $\oplus$.

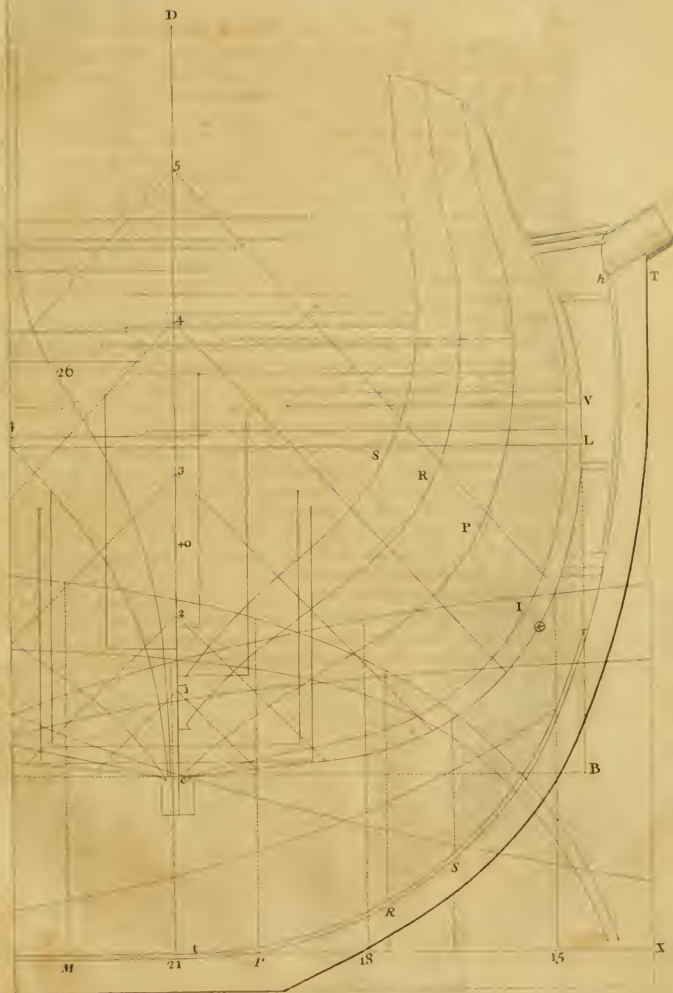
In like manner we may find points in the diagonals, for all the other timbers, and also for their lower, upper, and top timber heights of breadths and half breadths, from the dimensions in the proper columns corresponding to each; and by these form the timbers. We may likewise station the timbers in the sheer and floor planes, the distances of each from $\oplus$ being in the proper columns, and also form all the curves that are necessary upon these planes. But as our plate will not contain this, we shall only lay down the stem, post, counter and stern.

In order to do this, draw the line ZX, to represent the upper side of the keel; and at any convenient place erect a perpendicular for the after timber 26: The distance of this timber from $\oplus$, is 13254; and the wing transom from $\oplus$, is 14744; which therefore is 1490, abaft timber 26. The height of the wing transom is 5440. Set up this from the line ZX, upon a perpendicular erected to the point W 1490, abaft timber 26. The distance of $\oplus$ from the fore part of the post is 12994, which subtracted from 13254, remains 260. Lay this off from 26 to P, and
draw

draw the line P W for the fore side of the post. The counter is 2250 abaft 26, which lay off on the line Z X, and erect a perpendicular, upon which set up 5680 to c , which is the heighth of the counter. The upright of the stern at the sheer rail, is 2370 abaft timber 26, which set off from timber 26 upon the line Z X, at that point erect a perpendicular to s , and draw the line cs , which may be produced to the heighth of the stern, as in the plate. We may then form the counter; where it must be observed that a pink has no transom. We have only assumed the point W to determine the rake of the post. Timber 26 is 13254 from $\oplus$, and timber 24 is 12264 from $\oplus$. Therefore the distance betwixt them is 990, which set off from 26 to 24, gives the station of that timber; and by the same manner, the stations of the other timbers may be found.

Having thus laid down the stern, we shall in the next place lay down the stem. Erect the perpendicular X T, to limit the fore part of the stem; upon which set up 5526, the heighth of the aft side of the stem from X to T, and let b be the aft side of the head. The head of the stem is 10266, and the touch of the stem is 6826 before $\oplus$, therefore the distance betwixt them is 3440, which set off upon the line Z X, from a perpendicular let fall from b ; this will give the point t , the touch of the stem, where erect a perpendicular, and set up 3296 to o ; which will give the center of the lower sweep of the stem. The radius of the upper sweep is 7080, and n the center; and these two sweeps will form the stem. We may now station the timbers F, I, M, P, R and S, as in the plate; for as the touch of the stem is 6826, and timber P 7430 before $\oplus$; P will be 604 before the touch of the stem. We have in the plate laid down the main and top timber half breadth lines, also the rising and narrowing of the floor and floor sweeps. After the same manner any of the other ships may be laid down from the tables.

Having now given the principal dimensions, we shall in the next place give the scantlings.



Scantlings of the principal Pieces of Timber. In MERCHANT SHIPS. In SHIPS of WAR.

[illegible]

Scantlings of the principal Pieces of Timber. In MERCHANT SHIPS.

In SHIPS of WAR.

NAMES OF THE PIECES.		Tons.		70		100		200		300		400		600		800		1000		1200		1400		1600		1750		2000		
f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	f.	in.	
STEM.	Fore and aft below	0	9	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	
	At the head	0	10	1	1	1	3	1	4	1	5	1	6	1	7	1	8	1	9	1	10	1	11	1	12	1	13	1	14	
	Falle	0	10	1	1	1	3	1	4	1	5	1	6	1	7	1	8	1	9	1	10	1	11	1	12	1	13	1	14	
STEP.	Main	Sided to the diameter of their refect-	0	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
	Fore Mizzen	live masts in flups of war	0	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
STERN POST.	Fore and aft	At the keel	0	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
	At the head	0	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
WING	Thwarflups	At the keel	0	6	10	6	10	7	0	7	0	7	0	8	0	9	0	10	0	11	0	12	0	13	0	14	0	15	0	16
	At the head	0	9	0	10	10	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	
TRANSOMS.	Second	Lower futlocks	0	8	0	10	10	10	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	
	Third	Middle ditto	0	8	0	10	10	10	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	
	Upper ditto	0	8	0	10	10	10	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11	0	11
TIMBERS.	Sided	Top timber at the heel	0	6	10	7	0	8	0	9	0	10	0	11	0	12	0	13	0	14	0	15	0	16	0	17	0	18	0	19
	Ditto at the head	0	5	10	5	10	6	0	6	0	7	0	7	0	8	0	9	0	10	0	11	0	12	0	13	0	14	0	15	
	At the floor heads	0	6	10	7	0	8	0	9	0	10	0	11	0	12	0	13	0	14	0	15	0	16	0	17	0	18	0	19	
Main broad	In and out	At the breadth	0	5	0	5	0	3	0	3	0	4	0	4	0	4	0	4	0	4	0	4	0	5	0	5	0	5	0	5
	At the top of the side	0	3	0	3	0	2	0	2	0	2	0	3	0	3	0	3	0	3	0	3	0	4	0	4	0	4	0	4	
	Ditto thick	0	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
WALES.	Channel broad	0	4	0	5	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0	6	0
	Ditto thick	0	3	0	3	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0	4	0

N. B. The stern post is the same thwartships with the keel below, and the head square, if to be had.

Of the ROTHER.

THE general method is to make the breadth of the roller, at the lower end in large ships $\frac{1}{2}$ or $\frac{2}{3}$, and in small ships $\frac{1}{10}$ of the extreme breadth of the ship; but this cannot be established as an invariable rule, for a full built will require more roller than a sharp-built ship, and it will be necessary sometimes

to have a piece, fayed to the aftide of the rother, called a back.

As to the fcautlings, the head of the rother, if it can be had, may be 3 or 4 inches bigger than the head of the stern post thwartships, and two inches bigger afore and aft than thwartships.

For the sake of such of our readers as are not acquainted with the terms relating to shipbuilding, we shall conclude this second part with the following glossary.

A
G L O S S A R Y,
O R
E X P L I C A T I O N,
O f T E R M S relating to
S H I P B U I L D I N G.

BEAMS, are the large pieces of timber, which are laid across the ship; their ends are lodged on the clamps, and being bound by knees to the side, keep the ship to her breadth.

Bow, is the round part of the ship, forward. That on the right hand, with one's face forward, is called the Starboard, and that on the left the Larboard-bow; they both unite at the stem.

BREAST-HOOKS, are large knees fayed across the stem to both bows, into which they are bolted.

CARLINGS, are square pieces of timber, lying fore and aft from one beam to another, into which they are scored.

CATHEAD, is a large square piece of timber; one end of it is fastened upon the fore-castle, the other end projects without the bow so far as to keep the anchor clear of the ship when it is heaving up by a tackle, the block of which is called the Cat-block: The rope which passes thro' the several shivers of the block, and extremity of the cathead, is called the Cat-fall.

B b

CLAMPS,

CLAMPS, are thick planks, which support the ends of the beams.

COUNTER. The hollow part of the stern above the wing-transom is called the lower, and that part betwixt it and the lower part of the cabin-lights is called the upper or second Counter.

DEAD WOOD, consists of large pieces of timber laid one upon another, upon the keel, afore and abaft, where the ship is so thin as not to admit of sufficient substance for the two half timbers, which are therefore scored into this dead wood. The half timbers are used when, by reason of the sharpness of the floor, one piece of timber cannot be had which will make a floor-timber.

DECKS, are the same in a ship that floors are in a house, and are denominated, according to their height, lower, middle, and upper: Besides which, there is a deck which covers the cabin, and reaches from the stern near to the main-mast; this is called the Quarter-deck. In some ships there is an apartment above the great cabin, called the Round-house; the deck which covers it is called the Poop. Another deck covers the forecattle, which is an apartment in the fore-part of the ship, in which is the cook-room.

FAY, is to fit two pieces of wood so as to join close together. The plank is said to fay to the timbers when it bears, or lies close to all the timbers.

HARPINS, are the fore-part of the wales which go round the bow and are fastened to the stem.

HAWSE-PIECES, are broad timbers in the bow of the ship, thro' which there are holes cut for the cables to pass.

HEAD, is some figure, often that of a lion, carved as an ornament for the fore-part of the ship. There is a large piece of timber fayed to the stem upon which the figure rests; this is called the Knee of the head, and by reason of the great breadth at the the upper part, it is composed of several pieces: It is let into the head, and fastened to the bow on each side by knees, called the Cheeks of the head. The head is supported by rails, which extend from the crown of the figure to the cathead.

HEEL. The lower part of a mast, or any timber, is called the heel, and the upper part the head.

KEEL

KEEL, is the principal piece of timber first laid upon the blocks, which supports the whole structure. When this cannot be had of a sufficient depth in one piece, there is a plank fastened to the bottom, called the False Keel, which serves likewise to save the bottom of the main keel.

KEELSON, is fayed over the floor-timbers, and bolted thro' them into the keel.

KNEES, are crooked pieces of timber. One leg or arm is bolted to the beams, and the other to the ship's side. They are either lodging or hanging. The hanging knees are fayed up and down, and the others fore and aft the side, and rest upon the clamps.

LIMBER-BOARDS, are short pieces of plank, fayed next to the keelson, which may be taken out to clear the limber-holes, that are left either below or above the floor-timbers, for a passage for the water to the pump.

RABBET. When a plank is to be fastened to any piece of timber, such as the stem or post, there is so much wood cut out of the piece as the plank is thick, which is called the Rabbet; and when the plank is let into this rabbet, it will be even with the outside of the piece, as at the after-end of the keel, and lower end of the stern-post.

RAILS, are narrow planks, generally of fir, upon which there is a moulding stuck. They are for ornament, and nailed across the stern above the wing-transom and counters, &c. They are likewise nailed upon several planks along the sides; one in particular is called the Sheer-rail, which limits the height of the side from the forecastle to the quarter-deck, and runs aft to the stern and forward to the cathead. The wales are nearly parallel to this.

ROTHER, is a piece of timber, or several pieces fastened together, and fitted to the stern-post, to which it is hung by irons, whereon it moves, and thereby the ship is steered.

SCANTLING, is the breadth or thickness of a piece of timber.

SCARPHS. When two pieces of timber are joined together, so that the end of the one goes over the end of the other, being tapered so that the one may be let into the other and become even, they are said to be scarphed; such are the keel-pieces. But when the ends of the two

pieces are cut square and put together, they are said to butt to one another; and when another piece is laid upon, and fastened to both, as is the case in all the frame-timbers, this is called scarphing the timbers; and half the piece which fastens the two timbers together is reckoned the length of the scarph.

STEM, is that circular piece of timber where both the sides of the ship unite forward. The lower end of it is scarphed into the keel, and the bowsprit rests upon the upper end of it.

STEPS, are large pieces of timber fayed across the keelson, into which the heels of the masts are fitted.

STERN, is the after-part of the ship, in which are all the cabin-lights. It likewise includes the stern-frame, which consists of the stern-post, transoms, and fashion-pieces, all fastened together.

STERN-POST, is that strait piece of timber at the after-end of the ship, which unites both the sides. The heel of it is tenanted into the keel, and the wing-transom fastened at the head of it.

TIMBERS, in a ship, are as the ribs in the body, and serve to support the sides, the planks being all fastened to them; the two aftermost are called Fashion-pieces; they support the ends of transoms. The two timbers, forward, at the cathead, are called Knuckle-timbers. [For the names of the other timbers, see Chap. III. Sect. 3. Part II.

TUCK-square, is, when the heels of the fashion-pieces are let in upon the post, at which place the height of the tuck is fixed.

WALES, are planks, thicker than the rest, brought about the outside of the ship, in the wake of the decks.

THE THEORY OF
SHIPBUILDING and NAVIGATION.

PART III.

Of NAVIGATION.

CHAP. I. SECT. I.

Of Trigonometry by tabular Calculation from a Table of natural Sines, Tangents and Secants.

WE have in the first part explained the doctrine of trigonometry, so far as to give a solution to all the varieties geometrically by scale and compasses. We come now to shew how to perform the same arithmetically by a table of natural sines, tangents and secants; in order to which, it will be absolutely necessary to shew how this table may be constructed.

To Construct a Table of natural Sines, Tangents and Secants, to a Radius of 10000 equal Parts.

Describe a quarter of a circle, and let the radius be 10000 equal parts. Divide the arch into 90 degrees, and draw sines, tangents and secants to every degree, as directed in making the plain scale. Measure each off these seperately, by the same line of equal parts that the radius was taken from, and set down the numbers contained in each, in proper columns corresponding to every degree in the quadrant, as in the following table, where it is done only to every fifth degree, being sufficient to shew the nature of the table, which has been calculated to great exactness, by numbers
for

for every minute to a radius of 100,000; but in practice the logarithms of those sines, tangents and secants are used.

A Table of natural SINES, TANGENTS and SECANTS.

Deg.	Sines.		Tang.		Seca.		
5	871	9962	875	11430	10038	11473	85
10	1736	9848	1763	56713	10154	57588	80
15	2588	9659	2679	37320	10353	38637	75
20	3420	9397	3639	27474	10642	29238	70
25	4226	9063	4663	21445	11034	23662	65
30	5000	8660	5773	17320	11547	20000	60
35	5735	8191	7002	14281	12208	17434	55
40	6427	7660	8390	11917	13054	15557	50
45	7071	7071	10000	10000	14142	14142	45
		Sines.		Tang.		Seca.	Deg.

This table gives by inspection, the sine, tangent, or secant of any arch, or number of degrees therein expressed, if the radius of the circle be 10000; and because, as the radius of a circle is to the sine, tangent, or secant of any arch of the same circle, so is the radius of any other circle; to the sine, tangent, or secant of a similar arch of this other circle, as proved in *Prop. 8. Chap. 3. Sect. 2. Part 1.*

Therefore we may find the sine, tangent, or secant of any arch, in any circle, provided the radius be known, by the following proportion.

As the radius in the table.

Is to the sine, tangent, or secant of any arch in the table.

So is the radius of any other circle.

To the sine, tangent, or secant of a similar arch of this other circle.

E X A M P L E.

Required how many feet in length is the sine of 35 degrees, supposing the radius to be 130 feet.

The tabular radius is 10000, the sine of 35 degrees in the table is 5735; therefore by the rule of three.

$$10000 : 5735 :: 130 : 74 \frac{555}{10000} \text{ or } 74.555$$

$$\begin{array}{r} 130 \\ \times 5735 \\ \hline 910 \\ 9100 \\ 91000 \\ \hline 11000745550 \end{array}$$

If

If the length of the radius of any circle, and the length of a sine, tangent, or secant of the same circle be given, and it be required to find the arch; the proportion will be,

As the given radius of any circle

Is to the given sine, tangent, or secant of the same circle :

So is the radius in the table

To a sine, tangent, or secant in the table ;

which must be found in the table ; and corresponding thereto in the column of degrees, is the quantity of the arch required.

E X A M P L E.

Let the radius be 500, and the given sine 383 ; then

$$500 : 383 :: 10000 : 7600$$

$$500 \quad \begin{array}{r} 383 \\ \hline)38300100 \end{array} (7660, \text{ and this found in the column of}$$

sines in the table, corresponding thereto, is 50 degrees the quantity of the arch required.

It will be needless to give any more examples, as in practice we shall use the table of logarithms.

S E C T. II.

Of artificial SINES, TANGENTS and SECANTS.

WE observed in the first part, that the sides of a right angled triangle were distinguished by different names, *viz.* hypotenuse, perpendicular and base; and that by the angle, is understood that opposite to the base, which is supposed to be drawn across the paper; and the perpendicular up and down, making a right angle with the base, to which angle the hypotenuse is always opposite.

The sides may be considered likewise as sines, tangents, or secants, by which means, besides the aforesaid proper names, they will acquire another, which we shall call their surnames; and these will vary according to the side made radius.

If the hypotenuse be radius, the base will be the sine; and the perpendicular, the sine complement of the angle.

If

If the base be made the radius, the hypotenuse will be the secant complement; and the perpendicular, the tangent complement of the same angle as above.

If the perpendicular be made the radius, the base will be the tangent, and the hypotenuse the secant of the angle; all which has been demonstrated in the first part.

Hence, the whole business of trigonometry, may be said to consist, either in finding a sine, tangent, or secant, to a given arch, the radius being known; or if the radius, and either the sine, tangent, or secant be known, to find the arch; both which may be performed by a due attention to the two foregoing proportions: And as any side may be made radius, there may be different operations for each case.

All the various cases of right angled triangles, are express'd in the following table.

The

The PROPORTIONS for the several Solutions of the six Cases of Plane right-angled Triangles.

Given	Requir.	PROPORTIONS.	Radius.	Cases.
Hypo.	Base	R : line of the angle : : H : B R : sine comp. of the angle : : H : P	Hypo.	1 st .
and	and	Sec. comp. of the angle : R : : H : B Sec. co. of the angle : tan. co. of the ang. : : H : P	Base.	
Angle	Perpen.	Sec. of the angle : tang. of the angle : : H : B Sec. of the angle : R : : H : P	Perpen.	
Base	Hypo.	Sine of the angle : R : : B : H Sine of the ang. : sine comp. of the ang. : : B : P	Hypo.	2 ^d .
and	and	R : sec. comp. of the angle : : B : H R : tang. comp. of the angle : : B : P	Base.	
Angle	Perpen.	Tang. of the angle : sec. of the angle : : B : H Tang. of the angle : R : : B : P	Perpen.	
Perpen.	Hypo.	Sine comp. of the angle : R : : P : H Sine comp. of the angle : sine of the angle : : P : B	Hypo.	3 ^d .
and	and	Tan. co. of the ang. : sec. co. of the ang. : : P : H Tang. comp. of the angle : R : P : B	Base.	
Angle	Base	R : sec. of the angle : : P : H R : tang. of the angle : : P : B	Perpen.	
Hypo.	Angle	H : B : : R : line of the angle required	Hypo.	4 th .
and	and	B : H : : R : sec. comp. of the angle required	Base.	
Base.	Perpen.	After finding the angles the perpendicular is found by case 1 st . or 2 ^d .		
Base	Angle	B : P : : R : tang. comp. of the angle required	Base.	5 th .
and	and	P : B : : R : tang. of the angle required	Perpen.	
Perpen.	Hypo.	After finding the angles the hypotenuse is found by case 2 ^d . or 3 ^d .		
Perpen.	Angle	P : H : : R : sec. of the angle required	Perpen.	6 th .
and	and	H : P : : R : sine comp. of the angle required	Hypo.	
Hypo.	Base.	After finding the angles the base is found by case 1 st . or 3 ^d .		

It is evident from what has been said, that the first thing to be done, in order to give a solution to any of the cases, is, to make one of the sides radius; now if the thing required be a side, any of the three may be made radius; for there is no necessity, for making the given side radius, but after the radius is fixed, then both the given and required sides, will have particular surnames; the proportion for finding a side will always be,

C c

As

As the surname of the given side,
Is to the surname of the required side :
So is the given side
To the required side.

If the thing required be an angle, one of the given sides must be made radius, which will determine the surname of the other side; and the proportion will be

As one of the given sides, viz. that made radius
Is to the other given side :
So is the radius

To the surname of the second side.

And when this is found in the table, the quantity of the requir'd angle will be found in degrees and minutes in their proper columns.

Note. If the thing requir'd be a side, the two first terms will be either sines, tangents, or secants, to be taken out of the logarithmick table of sines, tangents and secants; the third term will be a natural number, as miles, yards, or any other measure, the logarithm of which, must be taken out of the table of logarithms; and, when the logarithms of the second and third terms are added together, if from this sum be subtracted the logarithm of the first term, look for the remainder in the table of logarithms; and corresponding thereto in the proper column, will be the natural number, expressing the length of the required side, taken by the same measure with the given side.

When an angle is required, we must not work for the angle itself, but for the sine, tangent, or secant of it; the two first terms of the proportion will be natural numbers, and their logarithms must be taken out of the table of logarithms; the third term will be a sine, tangent, or secant, and its logarithm must be taken out of the table of artificial sines, tangents and secants; and then the second and third terms must be added, and the first subtracted from their sum as before; the remainder must be found in the table of artificial sines, tangents and secants; and the quantity of the angle required will be found in degrees and minutes corresponding thereto.

We shall illustrate the whole by an example in each case, by the tables, and also by *Gunter's* scale.

The general rule by the pen, is the same as in any other question in the rule of three; and if we use the natural sines, it will be performed, by multiplying the second term by the third, and dividing the product by the first term, the quotient will be the fourth term required; observing to make the first term according to the aforesaid proportions, but it will be
indif-

indifferent which of the other two is made the second term, as they are to be multiplied into each other.

But as the natural sines, &c. are calculated to seven places, this would make the operations very tedious, upon which account their logarithms are used, and are had by the table of artificial sines, tangents and secants; if the table of logarithms, was made for natural numbers to seven places, we could find the logarithms of natural sines, &c. as easily as of any other number; but even in that case, we must have a table of natural sines, &c. and afterwards have recourse to the table of logarithms, whereas by the table of artificial sines, &c. we have the logarithm at once.

This table contains every degree, and minute, of the quadrant; if the number of degrees be less than 45, look for it at the head of the table; and for the minutes under Min. increasing downwards on the left hand of the page; but if the degrees exceed 45, look for them at the bottom of the page, and the minutes in the right hand column above M, increasing upwards; and when the degrees and minutes are thus found, the logarithmick sine, tangent, or secant, will be found in its proper column; observing if the degrees be found at the top, the word sine, tangent, or secant, must be found at the top, underneath which, right against the minutes, which must be found under Min. is the thing requir'd; but if the degrees are at bottom, these words must be found at the bottom, above which, and right against the minutes, which must be also found above M; is the thing required.

E X A M P L E.

Let it be required, to find the sine, tangent and secant of an arch, or angle of $33^{\circ} 45'$. Here the degrees are less than 45; therefore look for them at the top, and the minutes under M; right against which, and under the word sine, is 9.744739; under tangent, is 9.824893; under secant, is 10.080154; but if the sine, tangent and secant of $56^{\circ} 15'$ were required, look for 56 degrees at the bottom, and 15 minutes over M; right against which, and above the word sine, is 9.919846; above tangent, is 10.175107; above secant 10.255261.

The degrees at the top begin at 0, and increase to 44; the degrees at the bottom, in the first page, are 89, and decrease to 45; the one including the minutes, is always the complement of the other; so that if it was required to find the sine complement of any arch, look for the sine, and in the same line, you'll find the sine of the complement.

Thus to find the sine complement of 33° ; $45'$; look for the degrees 33° at the top, and $45'$ in the left hand column under Min. the sine will be under the word sine as before, 9.744739, and the sine complement 9.919846 in the same line above the word Sine; the like may be said of the tangent complement and the secant complement.

Having the logarithmick sine, tangent, or secant, to find the degrees and minutes corresponding thereto.

This is only the reverse of the former, for you must look over the table, till it is found, and if in a column that has the word sine, tangent, or secant, at the top; the degrees are at the top; and the minutes in the left hand column under Min. but if it is in a column, which has these words at the bottom, the degrees are at the bottom, and the minutes above M in the right hand column.

E X A M P L E.

Let it be required to find the degrees and minutes, answering to the tangent, 10.346337; this will be found over the word tangent, therefore the degrees must be at the bottom, viz. 65, and right against it above M, is 45; so 65° , $45'$; is the arch required: But if the complement was required, the degrees would be at the top, and the minutes under Min. viz. 24° , $15'$. Sometimes the exact number cannot be found in the table, in which case, all that can be done, is, to take the nearest to it; so we can never err a whole minute.

To work these by scale and compasses, there is a line of logarithmick sines, and a line of logarithmick tangents, upon *Gunter's* scales, constructed in the same manner as the line of numbers; for against 30 degrees, in the line of sines, is 5000 in the line of numbers, the natural sine corresponding thereto; and against 30 degrees, in the tangents, is 5773, the natural tangent; the line of sines is continued to 90 degrees, but the tangents to 45 degrees; the tangents above 45 degrees, are the same with their complements, for the radius, which is equal to the tangent of 45 degrees, is a mean proportional betwixt the tangent of any arch, and the tangent of its complement to 90 degrees, which is the reason, that the line of tangents is numbered 1 and 89, 5 and 85, 10 and 80, &c.

The tables being thus explained, we shall in the next place shew their use in the resolution of the six cases, of right angled triangles.

CASE I.

C A S E I.

Given the hypotenuse 60 miles, the angle $56^{\circ} 15'$; required the base, and perpendicular.

For the BASE.

As the radius, or sine of $90^{\circ} 0$	-	-	-	-	10.000000
Is to the sine of $56^{\circ} 15'$	-	-	-	-	9.919846
So is the hypotenuse 60 miles	-	-	-	-	1.778151
To the base 49.9 miles	-	-	-	-	+ .697997

For the PERPENDICULAR.

As the radius, or sine of 90°	-	-	-	-	10.000000
Is to the sine complement of the angle, or sine of $33^{\circ} 45'$	-	-	-	-	9.744739
So is the hypotenuse 60 miles	-	-	-	-	1.778151
To the perpendicular 33.3	-	-	-	-	+ 1.522890

By GUNTER's Scale.

Extend the compasses from 90 on the line of sines, to $56^{\circ} 15'$; the same extent will reach in the line of numbers, from 60 to 50 nearly, for the base; and the extent from 90 to $33^{\circ} 45'$ on the line of sines, will reach in the line of numbers, from 60 to $33\frac{1}{2}$ nearly for the perpendicular.

C A S E II.

Given the perpendicular 33.3 miles, and the angle $56^{\circ} 15'$; required the hypotenuse, and base.

To avoid working by the secants, let the hypotenuse be radius, and it will be,

For the HYPOTHENUSE.

As the sine complement of the angle, or sine of $33^{\circ} 45'$ com. arith.	0.255261
Is to the radius, or sine of $90^{\circ} 0'$	10.000000
So is the perpendicular 33.3 miles	1.522890
To the hypotenuse 60 miles	+ 1.778151

In this, and such like cases, where the first term is not radius, instead of the logarithm of the first term, use the complement arithmetical of it, which is, what any logarithm wants of the logarithm of the radius; now this being always 10.000000, the complement arithmetical, will be found, by subtracting each figure in the logarithm, from 9 excepting, the first

first towards the right hand, which must be subtracted from 10; as in this example, the logarithm of the sine of $33^{\circ} 45'$, by the table, is 9.744739, and by subtracting, as above directed, the complement arithmetical will be 255261; this is so plain and easy, that by a little practice, the complement arithmetical, will be as readily taken out of the table, as the logarithm itself, and then it must be added to the other two logarithms; and when the logarithm of the radius is subtracted from this sum, the remainder will be the logarithm of the fourth term required; and this will be the same thing, as if the operation was performed by the common method, *viz.* by subtracting the logarithm of the first term, from the sum of the logarithms of the second and third terms; in this example the sum of the two is

Sine $33^{\circ} 45'$	9.744739	Logar. of the first is	9.744739
Radius	10.000000		
Perpen. 33.3 miles	1.522890	Remainder	11.522890
	11.522890		
	9.744739		
	1.778151		

Now in subtraction as a lesser number is taken from a greater; it is plain if any number be added to both, and then the lesser subtracted from the greater, it will make no alteration in the remainder; and this is the very case here, for 9.744739, is to be subtracted from 11.522890; if to each of these be added the complement arithmetical, *viz.* 0.255261; the sum of the greatest will be 11.778151; the sum of the least will be 10.000000; and as all the figures in the lesser number are cyphers, except the first to the left hand; the subtraction is performed, only by cancelling the first figure to the left hand in the greatest number.

For the B A S E.

As the sine complement of the angle or sine of $33^{\circ} 45'$ com. arith.	0.255261
Is to the sine of the angle $56^{\circ} 15'$	9.919846
So is the perpendicular 33.3 miles	1.522890
To the base 49.9 miles	+ 1.697997

By the G U N T E R's Scale.

The extent from $33^{\circ} 45'$ to 90° in the line of sines, will reach in the line of numbers from 33.3 to 60, for the hypotenuse; and the extent from $56^{\circ} 15'$ to $33^{\circ} 45'$ in the line of sines, will reach in the line of numbers, from 33.3 to 49.9 for the base.

CASE III.

C A S E III.

Given the base 49.9 miles, and the angle $56^{\circ} 15'$ required the hypotenuse, and perpendicular making the hypotenuse radius.

For the HYPOTHENUSE.

As sine of the angle $56^{\circ} 15'$ comp. arith.	-	-	0.080154
Is to the radius	-	-	10.000000
So is the base 49.9 miles	-	-	<u>1.697997</u>
To the hypotenuse 60 miles	-	-	+1.773151

For the PERPENDICULAR.

As the sine of the angle $56^{\circ} 15'$ comp. arith.	-	-	0.080154
Is to the sine complement $33^{\circ} 45'$	-	-	9.744739
So is the base 49.9 miles	-	-	<u>1.697997</u>
To the perpendicular 33.3	-	-	+1.522890

By G U N T E R's Scale.

The extent from $56^{\circ} 15'$ to 90° in the line of sines, will reach from 49.9 to 60 on the line of numbers; for the hypotenuse, and the extent from $56^{\circ} 15'$, in the line of sines, will reach from 49.9 to 33.3 on the line of numbers for the perpendicular.

C A S E IV. and V.

Are exactly the same, only changing the names of the base and perpendicular.

Given the hypotenuse 60 miles, and base 49.9 miles; required the angles and perpendicular.

As the hypotenuse 60 miles complement arithmetic	8.221849
Is to the base 49.9 miles	-
So is the radius	-
To the sine of the angle $56^{\circ} 15'$	-
	<u>10.000000</u>
	+9.919846

By G U N T E R's Scale.

The extent in the line of numbers from 60, to 49.9, will reach in the line of sines, from 90° to $56^{\circ} 15'$; and when the angle is found, the other side, may be found by *Case* 1. or 3.

CASE

C A S E VI.

Given the base 49.9 miles, and perpendicular 33.3; required the angles and hypotenuse:

Making the perpendicular radius, the base will be the tangent, and the proportion will be,

As the perpendicular 33.3 miles complement arithmetical	8.477110
Is to the base 49.9	1.697997
So is the radius, or tangent of 45°	10.000000
To the tangent of the angle $56^{\circ}, 15'$	20.175107

In this case, the first figure in the logarithm of 33.3 is a cypher; therefore the next to it must be subtracted from 10, and all the rest from 9 as before, to find the complement arithmetical; and when the three are added, the characteristick will be 20, the characteristick of the radius being subtracted, there will remain 10; and this found in the table of artificial sines, tangents and secants, against it is $56^{\circ} 15'$ the tangent of the angle required.

By G U N T E R's Scale.

The extent in the line of numbers from 33.3 to 49.9; will reach in the line of tangents, from 45° to a point in the same line, which is either 33° , $45'$ or $56^{\circ}, 15'$; and to know which of the two it is; I find the extent in the line of numbers, is from a less to a greater, and therefore it must be so in the line of tangents; so the point must be more than $45^{\circ} 0'$, and of consequence must be $56^{\circ}, 15'$.

These are all the cases in right angled triangles. We might here likewise shew how to solve all the cases in oblique triangles, but as the whole business of navigation may be performed without them, we shall apply the doctrine of right angled triangles to navigation.

Now as navigation is a science which teaches us how to direct a ship's way from one port to another; it will from thence follow, that the situation, and distances of the places must be known. Our first business then shall be to shew how this may be attained. In order to this, it will be absolutely necessary to understand the principles of geography, which shall be the subject of the next chapter.

C H A P. II. *Of GEOGRAPHY.*

S E C T. I. *Of SURVEYING of LAND.*

Geography is that science by which we learn how to lay down, either all the places in any particular country, called a chart, or map; or all the habitable parts of the world upon a globe. The first is commonly called surveying of land, and the latter is what is generally understood by geography; we shall explain both.

The chief design of surveying, is to lay down all the places, in any particular piece of ground, upon paper; by which means, their distances, and positions, from one another, may be had, with as much certainty, as if they were to be measured on the very spot of ground where they are situated.

Let it be required to lay down all the places in the plane *ABDE*, viz. *F, G, H, I, K, L, M*.

The instruments necessary for this purpose, are, a well graduated circle, with an index, and sights, to take angles by; and a chain, line, or staff, to measure distances by.

Assume, (*See Plate 10.*) at any convenient distance from one another, any two points *B* and *D*, from whence, all the points in the plane, may be seen, place the circle with its center over the point *B*, and the index right over the diameter; move the circle, upon a pin provided for that purpose, till the point *D* may be seen thro' the sights, and fasten the circle in that position; then move the index, till all the points, may be seen successively, thro' the sights, noting the degrees cut by the index, corresponding to each point; then move the instrument, from the point *B*, to the point *D*, placing it so, that the index being laid upon the diameter, the point *B* may be seen thro' the sights; and then fasten the instrument, with its center right over the point *D*; move the index, till all the points, successively, may be seen thro' the sights, noting the degrees cut by the index, as before; then measure the distance betwixt *B* and *D*, which suppose 40 fathoms, or yards, &c.

To lay this down upon paper, take 40 from any scale of equal parts, which lay off upon any strait line from *b* to *d*, at the point *b*, make the same number of angles, equal to those taken in the field; do the same at the point *d*, the intersections of the lines drawn from *b*, with their

D d

corre-

corresponding lines drawn from d , will give the points f, g, h, i, k, l, m ; and if we measure their distances from one another, by the same scale that the distance of the points b and d was taken from, we shall have the true stance of the points in the field.

For the triangles DFB , and dfb , are equiangular, by construction; therefore $bd : bf :: BD : BF$, but bd contains as many equal parts of the scale, as BD does fathoms; therefore bf must contain as many equal parts of the scale as BF does fathoms.

As these are all the places that are supposed can be seen from the points B and D , the adjacent places cannot be laid down without altering the two stations: Let the next two stations be K and M , from whence taking the angles as before; all the places that can be seen from K and M , may be laid down in the same manner as those seen from B and D . We may proceed in the same manner to lay down all the places in an island, or country, by continuing to alter our station to places already laid down, 'till we have gone over the whole; this gives us the nearest distance betwixt any two places, but by the intervention of valleys, it will be impossible to go the nearest way in a mountainous country.

Another method is by actually travelling over the country, in a direct line, and measuring the distances by a wheel provided for that purpose. There must also be a contrivance to keep in a direct line, which may be done by setting a stake, at such a distance from the place we set out from, that it may from thence be distinctly seen; we may then, by the help of the sights, set several intermediate stakes in a strait line between them. After we have travelled to the farthest stake, we may then place another stake at such a distance, that from it, at least two of the former stakes may be seen, by which it may be set in a direct line with them; and placing several intermediate stakes betwixt these two last ones, we may by them, produce the line to another stake, and proceed thro' the whole extent of the country. We may likewise keep in a direct line by the mariner's compass, of which, we shall only here remark, that wherever it is carried to, the needle will always point to the north, or its variation may be found. We do not propose this method as the most expeditious, neither is it strictly true, or practicable, unless upon a plane; but as it agrees exactly with the plain sea chart, we shall insert it.

Being now provided with a wheel, and mariner's compass, with sights properly fitted to it, let us set out from the point C , directly as the needle points, setting a stake at every mile, till we arrive at the point A , so from C to A , is due north: in travelling along, if we see any places either to the right or left, thro' the sights placed at angles to the needle, they will

will be either due east, or due west of us; we must measure their distance from the line CA, and also the distance we are then from the point C, both which must be noted in a book provided for that purpose. When we arrive at A, the sights being at right angles with the needle, we may set a stake at X, and move directly to it, setting stakes at every mile as we go, and when we arrive at X, we may travel directly south again to S, measuring the distances of all the places from the line XS, as soon as they can be seen thro' the sights; we may proceed in the same manner, first going north or south, then east or west, till we have gone over the whole country to be laid down. The lines AC and XS, will be so nearly parallel to each other, that they may without any sensible error be taken as such.

Now in order to lay down this in a map (*See Plate 10.*) let the extent from south to north be 90 miles, and likewise that from east to west 90 miles; draw the line AC, and perpendicular to it, the lines AE and CF; draw also the line EF parallel to AC; so these four lines will limit the map, let each be graduated into 90 equal parts, and at every tenth draw lines parallel to AC, and also to EF; so the whole map will be divided into nine equal squares.

We may now lay down all the places in the map from the notes taken off in the field, as for instance, if it were required to lay down the point Y. I find by my notes, I travelled $28\frac{1}{2}$ miles due north from C, and $14\frac{1}{2}$ miles due east before I arrived at Y; therefore lay a ruler across from $28\frac{1}{2}$ upon the line AC, to $28\frac{1}{2}$ on the line EF; and take $14\frac{1}{2}$ with a pair of compasses, which lay off by the edge of the ruler, from the line AC, and this will give the point Y.

It is very plain that no place can be laid down in the map, unless the distance and position of it, with respect to some other place be known; and when this cannot be measured by reason of its being inaccessible by the intervention of seas, or otherwise we must have recourse to celestial observation; and tho' two places be so remote, that they cannot be seen from one another; yet we may by observing the sun, or some star, at both places, find how far the one is to the northward, or southward of the other; we must also find some way to know how far the one is to the eastward, or westward of the other, and when those two are found, their distances and situation may with certainty be found by trigonometry.

That we may comprehend, the manner, by which the situation, and distances of places, have been found, we must explain some principles of geography, which science, consists, in giving a true description, of all the habitable parts of the whole world, as was before observed.

S E C T. II.

Of the G L O B E.

THE first thing we shall observe is, that this earth, and sea together, is supposed to compose a globe; the first geographers found out this, by observing, that in whatsoever place of the earth, they were; their sight was always terminated, by a circle, unless intercepted by hills or otherwise, and the observer in the center of it; the firmament at the same time, forming the half of a concave sphere over his head, and tho' he moved his situation, for thousands of miles, he still found himself in the center of a circle, and the point over his head continue at the same distance, which could not be, if the earth was flat; for supposing the observer placed at C, in the center of the circle H Z O N; (*Plate 11.*) then Z would be the point in the firmament over his head, but if he moved upon the diameter, from O to G, he would then be under the point D, which is much nearer to him, than the point Z was, when in C; but if the earth be allowed to be round, and the observer at I; Z will be the point over his head, but when he has moved to G, the point H will be over him, and at the same distance from him, that the point Z was, when in C: Upon this account, they resolved, to describe the earth, not upon a plane, but upon a globe: For which purpose, an artificial one was made, to represent the natural one. As the earth, was supposed a solid globe, the firmament, which every where surrounded it, was supposed to be a concave sphere, in which the sun and stars seemed to move, for they were continually shifting their situation with respect to us, so that either the sun and stars, or the earth must be in a continual motion, and tho' it is certain that the earth has the motion, we shall suppose the sun, moon and stars to move round, and the earth immoveable in the center, being apparently so to our senses.

They likewise found, that the sun, by his daily motion from east to west, described an arch of a circle, ascending one half, and descending the other, the stars, also describing arches by their motion; amongst them, they observed one, seemingly not to change its situation, but always at or near the same distance, from the point over the observer's head, while he continued his station, or moved either east or west: But when he moved, either south, or north, the star appeared either nearer to, or further from, that point.

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They likewise found that the observer might move so far to the southward till the star would just disappear, and the sun, which at his first setting out, was a great distance from the point over his head at noon, was now right over him; from thence, they supposed the heavens, with all the stars, sun and moon, to be carried round the earth once in 24 hours, two points only being immoveable, one near the aforesaid star, and the other diametrically opposite to it; by which motion, every point in the heavens, excepting those two, would by every revolution, describe a circle.

GEOGRAPHICAL DEFINITIONS.

Def. 1. A globe, or sphere, is a solid body, which has a point within it, equally distant from every point of its surface; and may be conceived to be formed by the revolution of a semi-circle, round the diameter, which is supposed to continue immoveable. (As was observed in the first part).

Def. 2. The axis of the globe, is that line passing thro' the center, round which the whole globe is moved, and may be termed a diameter.

Def. 3. The poles, are the two points in the globe's surface, through which the axis is supposed to pass; these have no motion, one is called the north, and the other the south pole.

There are two globes used; upon one, all the kingdoms of the earth are described, this is called the terrestrial, or terraqueous globe, containing all the land, and water: The other is called the celestial, containing all the constellations; and tho' the firmament be supposed a concave, they may be duly represented on the convex superficies of a globe; and supposing it transparent, and the observer in the center, they would appear to him, in the same manner as they really are in the heavens; the same circles are described upon both, and are either great, or small; the planes of the great circles go thro' the center of the earth, and their diameters are the same with the diameter of the globe; the planes of the small circles do not pass thro' the center, they are parallel to the plane of some great circle, their diameters are less than the diameter of the globe, and continually decrease, according to their distance from the plane of the great circle, to which they are parallel.

Def. 4. Meridians, are great circles, intersecting one another in both poles, of which there may be an infinite number.

Def. 5. The equator, or equinoctial line, is a great circle, cutting all the meridians at right angles exactly in the middle, being equally distant from both poles.

Def. 6. The ecliptick, is a great circle crossing the equinoctial in two
opposite

opposite points in such a manner, that the planes of those circles form an angle of $23^{\circ} 30'$.

Def. 7. The horizon, is that circle which terminates the sight; supposing the observer at sea, he finds himself in the center, and the sea and sky uniting: It is plain, if he alters his station, this circle will change likewise; so that it cannot be described upon the superficies of the globe; it is called the visible, or sensible horizon.

Def. 8. The zenith, is that point in the heavens, which is right over the observer's head; the opposite point in the other hemisphere, is called the nadir; these two points vary continually as the observer changes his station.

In order to represent the horizon, and zenith by the globe, the axis is fitted in a brass circle, in which the axis turns round, this circle then will represent any meridian, because any place upon the superficies of the globe, may be brought right under it; this, with the globe within it, is placed in a broad wooden circle, the inside of it is the same diameter, with the inside of the brazen meridian; so the half of the globe will always be under, and the other half over this circle; and the meridian may be so moved in the notches, as to bring any part of it to this wooden circle; which is therefore called the real, or rational horizon, and is always parallel to the visible, before described, as in the artificial globes they are both the same, because the eye may be so placed as to see the whole half of the globe at one view; by the help of this circle, we may represent the horizon of any place, for it is only turning the globe upon its axis, 'till the place is under the brazen meridian; and then moving the meridian in the notches, 'till the point is 90 degrees distant from the horizon; for that point will be the zenith, and is equally distant from every point in the horizon.

Def. 9. Azimuth circles, are supposed to pass thro' the zenith, and nadir points, all divided into two equal parts by the horizon; these cannot be described upon the globe, but are represented by a quadrant of altitude, which is a thin piece of brass, that may be screwed to any part of the brazen meridian; and when this is brought to the zenith, the other end will move round upon the horizon; all these five circles, here defined, bisect each other, and the globe into two equal parts.

Def. 10. Parallel circles, are such as divide the globe into two unequal parts, and are drawn parallel to some great circle; those parallel to the horizon are called parallels of altitude or almucanters, but are not described on the globe: Those parallel to the equator, on the terrestrial globe, are called parallels of latitude, and are actually drawn thro' every tenth degree of the meridian.

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The equinoctial is divided into 360 degrees, and a meridian drawn thro' every tenth : One of these meridians is graduated, every quarter of it into 90 degrees ; beginning at the equinoctial, and ending at each pole, each pole being 90 degrees distant from the equinoctial.

Tho' there are no more meridians, nor parallels, actually drawn upon the globe ; there may be an infinite number of both ; for every place on the earth's surface, is supposed to be at the intersection of a meridian, and a parallel of latitude, which point may be found by the brazen meridian ; it is by these intersections, that the situations of places are determined.

Def. 11. Latitude of a place, is an arch of the meridian, intercepted between the place, and the equinoctial ; and is either south, or north, according as it lies to the southward or northward of the equinoctial.

Difference of latitude, is an arch of the meridian, intercepted between the parallels of two places.

Def. 12. Longitude of a place, is an arch of the equinoctial, intercepted betwixt the graduated meridian, and a meridian drawn thro' the place, and sometimes is accounted quite round the globe, and at other times it is accounted both ways, east and west.

Difference of longitude, is an arch of the equinoctial, intercepted betwixt the meridians, of two places.

Def. 13. Departure, is the distance of any place from the meridian, taken upon the parallel of latitude of the place, and therefore is always less than the difference of longitude ; it is sometimes called the meridional distance ; of which in another place.

Def. 14. Declination, is an arch of the meridian, intercepted betwixt the sun, or star, and the equinoctial.

Def. 15. Tropicks, are two circles, parallel to the equinoctial $23^{\circ} 30'$ distant from it ; that to the northward, is called the tropick of cancer ; that to the southward, the tropick of capricorn, these two limit the ecliptick.

Def. 16. Polar circles, are $23^{\circ} 30'$ distant from the pole, that at the north, is called the arctic, and that at the south antarctic.

From these definitions, the following inferences may be deduced.

Inf. 1. Every point on the earth's surface, has a corresponding point in the heavens for its zenith, from which, if a line be drawn thro' the point assumed on the earth's surface, it will pass to the earth's center : And the latitude of any place, is always equal to the distance of the zenith from the equinoctial.

Inf. 2. Those inhabitants of the earth, who have the pole in their zenith, have the equinoctial in their horizon, and are in 90 degrees of latitude ;

titude; and those who have the poles in the horizon, have the equinoctial in their zenith, and are in no latitude.

Inf. 3. The elevation of the pole, above the horizon, is equal to the latitude of the place. (*Plate* 11. *Fig.* 1.)

D E M O N S T R A T I O N.

Let P represent the pole, Z the zenith, $\mathcal{A}E$ Q the equinoctial, H O the horizon; the arch $\mathcal{A}E$ Z, is the latitude of the place by *Inf.* 1. if to the arch Z P, be added the arch Z $\mathcal{A}E$, their sum will be 90 degrees, because the pole is 90 degrees from the equinoctial; but if to the same arch Z P, be added the arch P O; their sum will also be 90 degrees, because the zenith is 90 degrees from the horizon; therefore the arch P O, is equal to the arch $\mathcal{A}E$ Z.

The globe being thus prepared, with the aforesaid circles delineated upon it, their next business was to find the latitudes and longitudes of places which they effected in the following manner, by celestial observations.

As to the latitude, it is plain, that if there was a star in the pole, there would be no more required, but to take its altitude or height above the horizon, with a good instrument, which would be the latitude of the place; But as there is no star in the pole, they were forced to take the least and greatest altitude of any star near the pole, which by making an entire revolution round the pole in 24 hours, would be twice in the observer's meridian: Suppose then its least altitude 48 degrees, and greatest 52 degrees, the difference betwixt these is 4 degrees, the half of which must be the star's distance from the pole; and this being added to the least, or subtracted from the greatest, gives the latitude; for in the first, the pole is 2 degrees higher above the horizon than the star, which must therefore be added to the altitude 48, which gives 50 the latitude, but when the star is 52 degrees of altitude, it will then be elevated 2 degrees above the pole; which being subtracted from the altitude, there remains 50, the latitude as before. This way of finding the latitude would require 12 hours difference betwixt the two observations, which therefore cannot be done at sea, because a ship in that time, may alter her latitude considerably; neither is it practicable when the pole is near the horizon: But after they had found the declination of some stars, or their distances from the pole; it was then enough to find one of the altitudes, and when below the pole, the distance of the star from the pole, added to the altitude, gives the latitude; but if above the pole, it must be subtracted from the altitude, the remainder is the latitude; one exam-

ple will be sufficient to illustrate this: Let the star be at 10 degrees from the pole, and b O, the altitude, 40 degrees; the arch P O is the latitude by *Inf.* 3. but this is the sum of the star's altitude, and distance from the pole, which must therefore be 50 degrees the latitude of the place.

But if the star is in r , then r O, the altitude, is 60 degrees, and P O the latitude as before; therefore subtracting the stars distance from the pole, *viz.* 10 degrees, from the altitude 60, there will remain 50, the latitude as before. But as these observations are made in the night, they cannot be depended upon at sea; we shall therefore shew how to find the latitude by the sun's zenith distance, or altitude when in the meridian of the observer.

If the sun was always in the equinoctial, his distance from the zenith, would be the latitude of the place, by *Inf.* 1. but that it is not so, is evident, because then the sun would always rise and set in the same points of the horizon, and his meridian altitude, would be the same every day to an observer while he continues in the same place; therefore we must likewise know the declination at the same time that we have the zenith distance.

The geographers, by a daily observation of the sun's greatest altitude, which always happens when he is upon the meridian of the place, that is at mid-day, found that an observer, who was above $23^{\circ} 30'$ distant from the line, had the sun approaching his zenith, for one half of the year, and the other half receding from it, and after his nearest approach, it was a whole year before he came back to the same place; they likewise found the greatest distance from the zenith was just 47 degrees, more than the nearest approach; and from thence his greatest distance from the line was $23^{\circ} 30'$, and by keeping an exact account of his zenith distance every day, they formed tables of his daily declination, or distance from the line; so that we have now by inspection, the sun's declination on any day of the month.

The table being thus made, the latitude will always be found, either by adding the zenith distance and declination together, or subtracting the one from the other, all the various cases that can happen, shall be explained in the next section; and we shall now proceed to shew how they found the longitude.

To attain this, they considered that the sun, and stars in the space of 24 hours, return'd near to the same points in the heavens, in which they were at that time the day before, and consequently moved 360 degrees in the 24 hours, which in one hour would make 15 degrees; so that if two places were 15 degrees to the eastward or westward of one

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another,

another, those in the easternmost place, would see the sun, a whole hour before those in the westernmost place; and these last would see the sun an hour after it disappeared to the others.

Now if the whole earth, should be, at the same instant of time, deprived of the light of the sun, by the intervention of some opaque body, it would happen just at the sun's rising in some places, whereas in other places the sun would be upon the meridian, and in other places 15 degrees from the meridian; in short it would happen at different times of the day, to all the inhabitants of the enlightened hemisphere, that lie under different meridians; and therefore, if the hour were exactly known at all those places, from thence their difference of longitude might with certainty be found, for the difference of the hours multiplied by 15, will give the degrees; as supposing three places C, B, A, to those at B, let it be noon; to those at A, let it be 10 in the forenoon; and at C, 3 in the afternoon; C would be the easternmost, for the sun passed the meridian of that place, and got to the meridian of B, in three hours, which makes 45 degrees difference of longitude that B is to the westward of C, but then the sun must move two hours more to come to the meridian of A, it being only 10 in the forenoon at that place, which makes 30 degrees difference of longitude, A is to the westward of B, and 75 degrees to the westward of C.

As such cases of total darkness seldom happen, the longitudes of places are not much to be depended upon except where good observations, of the sun, moon, or other heavenly bodies, have been made; sometimes the longitude has been found by actual mensuration, or by good sea journals; and by frequently going to the same ports at last, tables of the latitudes and longitudes of the most remarkable places in the whole habitable world have been made; and by these tables, all the places may be laid down, according to their latitudes and longitudes upon the globe; as suppose it were required to lay down a place in 10 degrees east longitude, and 20 degrees north latitude, or which is the same thing, to find a place upon the globe, in that latitude and longitude. Move the globe upon its axis till the brazen meridian cuts the 10th degree upon the equator, on the east side of the first meridian; then look for the 20th degree upon the brazen meridian, and right under that is the place required.

S E C T. III.

The Description and Use of the ANAL E M M A.

IT would be foreign to our design to shew how to solve all the geographical problems by the globe; it will be of greater use in navigation, to shew how to delineate all the circles of the sphere upon a plane, which may be done several ways; but as we have already explained the principles of the orthographick projection of solids, we shall now make use of that way. (*Plate 12.*)

It will be very easy to conceive that if the globe were cut by a plane passing thro' the center; the section would be a circle, and when this plane passes thro' the poles, the circle will be a meridian, as in the plate; PÆSQ, may represent the meridian of the place, P the north, and S the south pole; PS the axis; ÆQ the equinoctial; for tho' this is only the diameter of it, yet because the plane of the equinoctial is perpendicular to the plane of the meridian, it will be represented by its diameter, and for the same reason all the parallels of latitude, which upon the globe, are circles parallel to the equinoctial, will in this projection, be represented by their respective diameters, and drawn parallel to the line ÆQ; each quarter of the meridian is divided into 90 degrees, beginning at the equinoctial, which will be all equal; but the degrees of the equinoctial, tho' upon the globe they are all equal, will here be unequal, and are represented by their respective sines, beginning at the center: PS will represent a meridian described upon the globe, at right angles to the meridian, thro' which the globe is supposed to be cut; every other meridian will be represented by an elipsis, and may be drawn thro' every 10th degree of the equinoctial. The most expeditious way of describing them, is by finding the points in every parallel of latitude, thro' which the meridians will pass, which may be done by a sector, or as readily without it; thus, First divide the equinoctial into degrees, which being only a line of sines, will be the same as the construction of the line of sines on the plain scale; and because there are as many degrees in every parallel, as in the equinoctial, they must likewise be divided into the same number of parts; and in the same proportion that the line ÆQ is divided into: Now let it be required to divide the parallel of 23° 30': Draw the line ECK, which will represent the ecliptick, divide it into degrees, by fixing one point of the compasses in

C, and with the other transfer all the divisions of the equinoctial to this line, and thro' each division, draw a line parallel to the line P S, to intersect the parallel of $23^{\circ} 30'$, which will divide it into the same number of parts, and in the same proportion with the equinoctial; for E C m , is a right angled triangle, of which E C, the radius, or semi-diameter of the globe, is the hypotenuse; E m the radius of the parallel, or sine complement of the latitude, is the base; and C m , the sine of the latitude, is the perpendicular. Now the lines drawn thro' the several divisions of the line E C, will all be parallel to the perpendicular C m ; and therefore constitute so many right angled triangles, all similar to E C m . Therefore E C : E m :: E o : E n , that is as the radius is to the sine comp. of the latitude, so is a degree, or any part of the equinoctial, to a degree, or proportional part of the parallel. After the same manner all the other parallels may be divided: and the meridians drawn as in the plate; there will be no occasion to draw lines from the divisions of E C to the parallel, only lay a ruler parallel to P S, and mark its intersection with the parallel. After the meridians and parallels are all drawn, the places may be laid down according to their latitudes and longitudes, in the same manner as on the globe, but their distances cannot be measured so easily; we shall therefore shew how to do this without laying down the places, and also how to solve all the geographical problems necessary in navigation, by scale and compasses; and by this plate, with as much certainty as with the globe itself.

The meridians, the equinoctial, with its parallels, and the ecliptick being thus delineated upon brass, wood, or pasteboard, are made to move about the center, within the circle Z H N O; which may represent a meridian in the heavens; Z is the zenith, and because the meridian Z H N O, passes thro' the zenith, Z H, and Z O will be azimuth circles of 90 degrees each; they are graduated both ways, to shew the altitude and zenith distance. H O is the horizon, the meridian's drawn thro' each 15th degree, are the hour circles, and because the sun is the same distance from the meridian of the place at any hour in the afternoon, that he is in the forenoon, at the hour which is as much before as the other is after 12; the meridian of 11 in the forenoon, will be that of 1 in the afternoon, as they are numbered in the plate. When the pole is in the zenith, the meridians will all be azimuths, and the equinoctial will be the horizon.

The observer is always in that point of the circle P Æ S Q, which is directly under Z, the points of the compass are marked on the horizon; and the sun's place in the ecliptick to every fifth day of the month,

is

is marked on the line EK: This projection is called the analemma; when the north pole is above the horizon, the right hand quarter, viz. ZCO, will be east, and the hours before six in the morning, and after six in the evening, will be on the right hand side of the meridian of six, which will always be represented by the line PS; the hours from six in the morning to noon, and from noon to six in the evening, will be in the left hand quarter, viz. ZCH, which is the west: When the south pole is elevated above the horizon, that which was east, will now be west, and the forenoon hours, will be the afternoon hours; the sun will rise in the left, and set in the right hand quarters. Tho' in this plate, the heaven's and earth coincide, yet in reality, they are at such a distance, that the earth is accounted only a point; so that in all observations, the observer is supposed at the center of the earth.

Geographical Problems solved by the Analemma, and by Scale and Compasses.

P R O B. I.

Given the sun's declination and zenith distance at mid-day, or meridian altitude; to find the latitude of the place.

This admits of three varieties.

C A S E I.

The latitude and declination both north, and the sun to the northward of the observer, or both south, and the sun to the southward of the observer.

Note. This can only happen to those within the tropicks.

Rule. Subtract the zenith distance from the declination, the remainder will be the latitude.

E X A M P L E.

Declination	20° 00' north	} Latitude north.
Zenith distance	10° 00' north	
		10° 00'

But if the declination and latitude were both south, and the sun to the southward of the observer, the latitude would be 10 degrees south.

To delineate this by scale and compasses.

Describe the circle ZHNO, from Z set off towards O, the given zenith distance 10° 0' to ⊕, and from ⊕ set off 20° 0' the given declination to

α , then αZ is the latitude, by *Inf* 1. or from $\oplus$ set off the complement of the declination $70^{\circ} 0'$ to p , then pO is the latitude by *Inf* 3. By the analemma, set $20^{\circ} 0'$ from $\mathcal{A}$ towards P , to $10^{\circ} 0'$ from Z counted towards O , then P will be against $10^{\circ} 0'$ from O , the required latitude.

C A S E II.

The latitude and declination both north, and sun south, or both south, and sun north.

Rule. Add the declination to the zenith distance; their sum will be the latitude of the place.

E X A M P L E.

Declination	$20' 00''$ north	} Latitude.
Zenith distance	$30' 00''$ south	
		$50' 00''$

But if the declination were south, and the sun north, the latitude would be $50^{\circ} 00'$ south.

By scale and compasses, set off the given zenith distance $30^{\circ} 00'$ from Z to 30 towards H , and the given declination from 30 to $\mathcal{A}$; $Z\mathcal{A}$ will be the latitude.

By the analemma, set $20^{\circ} 00'$ counted from $\mathcal{A}$ towards P , to $30^{\circ} 00'$ counted from Z towards H ; P will be against $50^{\circ} 00'$ accounted from O , the required latitude.

C A S E III.

The latitude and declination of different names, that is the one north, and the other south.

Rule. Subtract the declination (which in this case, will always be the least) from the zenith distance; the remainder will be the latitude.

E X A M P L E.

Declination	$15^{\circ} 00'$ south	} Latitude.
Zenith distance	$65^{\circ} 00'$ south	
		$50^{\circ} 00'$

But if the declination was north, the latitude would be south.

By scale and compasses, describe the circle $ZHNO$, from Z set off the given zenith distance 65 degrees towards H , if in the north latitude, but towards O , if in south latitude, from 65 , set off the given declination 15 to $\mathcal{A}$; then $\mathcal{A}Z$ is the required latitude.

By

By the analemma, set $50^{\circ} 00'$ counted from Æ towards S, to $65^{\circ} 00'$ from Z towards H, and P will be against 50° the latitude.

It is by this problem, that the latitude is found at sea, when the sun's zenith distance is taken at noon; for which we may take the following general rule. If the sun and the equinoctial be both on the same side of the observer, consider which is nearest the zenith; and if it be the sun, the declination and zenith distance must be added; but if it be the equinoctial, the declination must be subtracted from the zenith distance,

Those that live within the tropicks, will sometimes have the sun on one side, and the equinoctial on the other side of their zenith, and then the zenith distance must be subtracted from the declination.

Those that are so remote from the equinoctial, that the sun performs the diurnal revolution above the horizon, will have him twice in their meridian every 24 hours: In this case, instead of the declination, take the complement of it, or the sun's distance from the pole; when the observation is at 12 at night, or more properly when the sun is nearest the horizon, the latitude will be found in the same manner, as if the altitude of a star was taken, which was sufficiently explained in the last section: But when the observation is made at noon, it will be the same as in the second case of this problem.

P R O B. II.

Given the latitude and declination, to find the sun's amplitude, and the hour of his rising or setting.

C A S E I.

In order to solve this, it must be observed, that the sun by his apparent diurnal motion, describes circles nearly parallel to the equinoctial, which may be called parallels of declination, being the same as the parallels of latitude betwixt the two tropicks. The sun performs his revolution in 24 hours thro' all the meridians, and when in the meridian of the place, it is either mid-day, or mid-night; in the analemma, this meridian is that which limits the projection; the sun will be all that day somewhere in the parallel of declination, and if a meridian be drawn thro' the point where the parallel of declination intersects the horizon, it will give the hour of his rising or setting, and the degrees of the horizon intercepted between the sun, and east or west point of the horizon, is the sun's amplitude.

This

This being premised, the following general rule will solve all the varieties of this problem by the analemma.

Rule. Set the pole to the given latitude, and we have the amplitudes, and hour of the sun's rising and setting, for any day in the year in that latitude, by inspection.

For the day of the month being given, the declination may be found by the table, or by looking for the sun's place in the ecliptick, on the analemma; the degree of the horizon cut by the parallel of declination is the required amplitude, and the meridian passing thro' the sun, when in the horizon, gives the hour.

By scale and compasses, describe the circle Z H N O as before, Let the given latitude be $50^{\circ} 00'$ north, and declination $23^{\circ} 30'$ north. Set off by the line of chords $50^{\circ} 00'$ from O to P, and draw the line P S, and at right angles to it, the line $\text{Æ} Q$, which will represent the equinoctial; draw the parallel of latitude $23^{\circ} 30'$, to intersect the horizon in x ; thro' x draw a meridian, to intersect the equinoctial in y . Cx is the amplitude measured on the line of sines, and Cy measured on the same line, gives the degrees and minutes of the equinoctial before or after six, which converted into time, gives the hour of the sun's rising or setting.

E X A M P L E.

In $50^{\circ} 00'$ north latitude.

Decl. north.	Amp.	Sun rise.	Sun set.
Deg.	Deg. M.	H. M.	H. M.
5	7 48	5 36	6 24
10	15 40	5 11	6 49
15	23 45	4 46	7 14
20	32 08	4 17	7 43
33 30	38 20	3 55	8 05

The amplitudes will be the same in south declination, but the hours of the sun's rising, will be the same with the hours of his setting, when the declination was north.

P R O B. III.

Given the latitude, sun's altitude and declination, to find the azimuth and hour.

By the analemma, set the pole to the latitude, lay a ruler, or strait slip of paper, across the given altitude; this will intersect the given parallel of declination. The meridian passing thro' this point of intersection, gives

gives the hour; at this point make a mark with a pencil upon the ruler, then move the pole to the zenith, so as not to move the ruler; the meridian that passes thro' the pencil mark, will be the azimuth circle passing thro' the sun, and it will cut the horizon in the degrees and minutes of the required azimuth. It will be proper to take the distance with a pair of compasses, between the sun, and the line T O, or T H; in case, the ruler should slip by moving the pole to the zenith.

By scale and compasses; let the given latitude be $50^{\circ} 0'$, declination $23^{\circ} 30'$ both north, and altitude $20^{\circ} 0'$. Describe the circle Z H N O, and draw the equinoctial parallel of declination, and axis as before. Draw also the parallel of altitude $a l t q$ by setting off $20^{\circ} 0'$ of the line of chords from O to q , and from H to a ; this will intersect the parallel of declination in t ; thro' which draw an azimuth circle, and a meridian, and these two will give the things required. There will be no occasion to draw the whole circles, only to find the intersection of the azimuth circle with the horizon, and of the meridian with the equinoctial, which is only dividing the horizon in the same proportion with the parallel of altitude; and the equinoctial in the same proportion with the parallel of declination: Thus, for the azimuth, draw the radius C q , and C $l q$, will be a right angled triangle; thro' t draw $t f$ parallel to C l ; transfer C f to the horizon in s , and draw $s v$ parallel to C N; so $v O$ will be the azimuth. Or it may be done thus, with $l q$, the radius of the parallel of altitude, describe from the center C, an arch, $i g$ produce the line $t f$ to g ; thro' g draw a line from C, which will give the point v as before. For the hour, draw the line $t i$ parallel to P S, to intersect C E in 1 , transfer C 1 to the equinoctial in 2 ; so shall C 2 , measured on the line of sines, and converted into time, give the hour after 6.

Those are the problems that are absolutely necessary to be known at sea; the first for finding the latitude, the last two for finding the variation of the compass, of which in its proper place.

P R O B. IV.

Given the latitude and longitude of two places; to find their distance and angle of position.

The nearest distance between any two places will be in a plane passing thro' the two places, and the center of the earth, which upon the surface of the earth, will be an arch of a great circle; and the angle which this circle makes with the meridian of each place, is the angle of position, which will not be the same in both places.

By the analemma, Let the two places be A and B; A in $50^{\circ} 0'$, B

F f

in

in $13^{\circ} 30'$, both north latitude; their difference of longitude $52^{\circ} 58'$; B the westmost: First set the pole to the latitude of A, then look for $37^{\circ} 2'$ (the complement of the difference of longitude) upon the equinoctial; the meridian passing thro' that point, will intersect the parallel of latitude of $13^{\circ} 30'$ in the point B; thro' B lay a ruler across parallel to the horizon; its intersection with the graduated meridian, will give the distance of B from A, counting the degrees from the zenith, and will be about $56^{\circ} 15'$.

For the angle of position from A to B. The ruler being laid across as before, mark the point B upon it; move the pole to the zenith, the meridian passing thro' the point, will be the azimuth circle required; but if it were required to find the angle of position from B to A, set the pole to the latitude of B, and proceed as before.

By scale and compasses describe the circle, and quarter it as before. Draw the axis and equinoctial to the latitude of A, also the parallel of latitude of $13^{\circ} 30'$: Now to find the point B in this parallel, it is only to draw a meridian, which shall make an angle of $52^{\circ} 58'$, with the meridian of A. To do this by the line of chords, set off $52^{\circ} 58'$ from AE , both ways to d and d' , from which two points, lay a ruler across to intersect the equinoctial in b , which will be the point in the equinoctial, thro' which the meridian of B must pass; or by the line of sines, take the complement of the difference of longitude $37^{\circ} 2'$, which set off from the center C, and this will give the point b ; thro' which draw a meridian as before directed, to intersect the parallel of latitude of $13^{\circ} 30'$ in the point B, and thro' this point draw LT , parallel to the horizon; ZL or ZT , measured on the line of chords, will give the distance nearly $56^{\circ} 15'$: Draw also the azimuth circle ZBG ; CG measured on the line of sines, nearly 21 degrees, will give the complement of the angle of position with the meridian of A; or thro' G, draw a line parallel to ZCN , to intersect the circle in D and F; HD or HF , measured on the line of chords, will be the angle of position, that the meridian of A makes with the great circle passing thro' A and B, and will be nearly 69 degrees; but to find the angle it makes with the meridian of B, we must draw the equinoctial and axis to the latitude of B, and proceed as before.

This problem can be of little use to the mariner, because it gives the nearest distance betwixt two places, and the compass, which is the only guide he has to conduct him from one port to another, leads him out of the direct road; for the path which the ship describes, will be a curve, making equal angles with all the meridians she crosses; whereas the arch of the great circle, which is the nearest distance, makes unequal angles with all the meridians; and therefore in practice, we cannot go the nearest way,

way, by the compass, unless the places lie under the same meridian, or under the equinoctial.

The compass is so well known that it needs no description, being only a card, upon which a circle is described; it is divided into 32 equal parts called points, each consisting of $11^{\circ} 15'$; these are subdivided into quarters, without which there is another circle divided into 360 degrees; a needle, touch'd with the load-stone, by which it acquires that surprising quality of pointing to the north pole, is fixed under the card, upon which there is a flour de luce, which will point out the meridian; for though it sometimes varies from it, we may discover the quantity by celestial observation of which in another place: The lines that the ship forms in steering by the compass, are called rumb; and that these make equal angles with all the meridians is evident, because the needle always lies in the meridian of the place, and the rumb all meet at the center of the circle upon the card. The needle is the diameter of this circle, so that moving the card can never alter any angles, that are made by the intersection of any two lines upon it.

It will be very difficult, if not impossible, to describe the rumb lines, either upon the globe, or upon any plane, where the meridians intersect in the pole, for they must be so described, that if a ship sail upon any rumb to any port, and then sail back the same distance upon the opposite rumb, she will then return back to the same port from which she sail'd; and that this is really true, where there is no variation or currents, or the ship is not forced out of her direct course by the sea, wind, or bad steerage, must be allowed; and agreeable to the nature of the triangle formed by the rumb lines, meridians, and parallels of latitude upon the globe; tho' strictly speaking, they cannot be called triangles, for the mathematicians have reduced all triangles either to plane or spherick, but the properties of neither agree to these. However as they have three sides, and three angles, a small distance may be allowed to be a strait line; we shall therefore consider them as right angled triangles.

Let there be two ports A and B, A in $50^{\circ} 0'$, B in $13^{\circ} 30'$, and the difference of longitude $52^{\circ} 58'$ as before, that is the distance of the two meridians upon the equinoctial in miles, will be 3178; but in the parallel of latitude of B, the distance of the meridians will be 3090 miles, and in the parallel of A 2043 miles; the difference of latitude betwixt A and B, is $36^{\circ} 30'$, or 2190 miles.

Now if a ship sail directly south 2190 miles from A, and then 3090 directly west, she will certainly arrive at B; and if she sails back again the same distance upon the opposite rumb, that is first 3090 miles east,

and then 2190 miles north, she will certainly arrive at A; this would make the whole distance sailed 5280 miles. But if in sailing from A, she first sails 2043 miles west, and then 2190 miles south, she would likewise arrive at B; and by sailing the same distances upon the opposite rumb, she would return back to A. Her distance sailed would be 4233 miles, but the nearest distance is 3376; supposing it possible to steer upon the arch of a great circle. The direct course by the compass from A to B, will be south $50^{\circ} 6'$ west, and the distance to be sailed in that course, will be 3414 miles, as shall be shewn in *Mercator's* sailing; what we are here to prove is, that if a ship sails from B, north $50^{\circ} 6'$ east 3414 miles, she will arrive at A. The only difficulty will be to reconcile this to the properties of a right angled triangle; for in sailing from B to A, the distance 3414, is the hypotenuse; the difference of latitude 2190, is the perpendicular; and the meridian distance in the parallel of latitude of A, viz. 2043 will be the base. Now in sailing back from A to B, upon the opposite rumb, the distance, difference of latitude, and angle, will be the same as before, from whence it may be argued, that the base will be the same as before, viz. 2043; so that when the ship arrives in the parallel of latitude of B, she will be 1047 miles to the eastward of it.

But it must be considered that the whole difference of longitude to be run down from B to A, is $52^{\circ} 58'$, which cannot be done by sailing only 2043 east, unless it be all in the parallel of A. Now in sailing directly upon one rumb, there is no easting made in the parallel of A, and of consequence, all the easting that is necessary to be made, in order to run down the longitude, must be made before she arrives at the parallel of latitude of A; and the nearer to B the easting is made, the more it will require to run down the longitude: Again, neither the whole easting, nor strictly speaking, any part of it, can be said to be run down in any parallel of latitude betwixt B and A; for if she sail from B, in the direct course towards A any assignable distance, she must make a small part of it nothing, and a small part of it easting; and if she sail back upon the opposite rumb, she will make the same westings, betwixt the same parallels she made the eastings in; and because the same eastings, are made in the same parallels of latitude that the westings are made; the difference of longitude in both, will be the same; and the ship in sailing back upon the opposite rumb, will certainly arrive at B.

Or let us suppose 2190 parallels of latitude actually described betwixt A and B, likewise 2190 meridians. Now in sailing from B to A, instead of sailing the direct course, let us sail first directly east till we come to the first of these meridians; then due north again to the first parallel, and due east

east to the second meridian; and so proceed sailing, first north, and then east, till the last north course brings us to the parallel of A, and then an east course will bring us to A. By this means she forms 2190 small right angled triangles; the perpendiculars will all be equal, *viz.* one mile each, but the bases will all be unequal, still decreasing the nearer we come to A. In sailing back, if we go upon the opposite rumb, we must first sail as much west in the parallel of A, as we did east before; and then south to the next parallel, in which we must sail as much west as we did east before to come at the next meridian, and so proceed sailing first south and then west, till we come to B; where it must be observed, that in every parallel of latitude, we make as much easting as we did westing, which will infallibly bring us back to the same port.

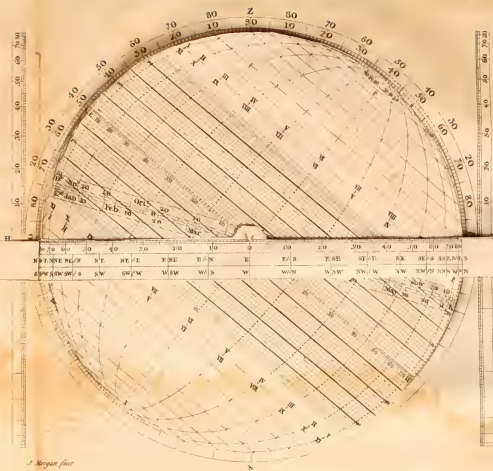
Now let us suppose an infinite number of parallels of latitude, and an infinite number of meridians; and let us steer an infinite number of courses, first north, and then east, we shall have an infinite number of small triangles, and their perpendiculars and bases being infinitely small, both may be said to vanish, and leave nothing but the hypotenuse, which upon the globe, is represented by a curve, making equal angles with all the meridians, being the path a ship describes, which is led by the direction of the compass from B to A; and in sailing back from A to B, the triangles will be the same as before, and so quite vanish, leaving only the distance, which will certainly bring the ship back to B.

We shall not here examine how these rumb lines are described upon the globe, for as we observed before, though in theory they may be conceived to be drawn, yet it will be scarce possible to draw them true, with inclin'd meridians; this shews the necessity of a projection of the sphere upon a plane, where all the meridians may be parallel to each other; in this case the rumb lines will all be strait lines. How to construct such a projection, shall be the subject of the next chapter.

We shall only here remark that in navigation, the departure, and meridian distance, may be considered as one thing, *viz.* The whole easting, or westing that a ship must make in steering upon a direct course from one port to another; and this will always be equal to the sum of all the departures made every 24 hours, but will not agree with the common definition of departure and meridian distance, as in *Def.* 10. for by that, we can assign no proper departure betwixt any two places, that are not in the same parallel of latitude; for let the two places be A and B, the distance betwixt their meridians, in the parallel of A will be 2043 miles, and in the parallel of B 3090 so that neither of these can properly be called the meridian distance, or departure betwixt these two places; but if we sail
in

in a direct course, viz. south $50^{\circ} 6'$ west from A 3414 miles; we shall make 2619 westing when we arrive at A, which we shall call the departure, or meridian distance betwixt A and B, and in sailing back from B, the course will be north $50^{\circ} 6'$ east; the distance 3414, and the easting 2619, the same as the westing before.

From this we may infer that the same course and distance, whether we steer from or towards the equator, will always give the same difference of latitude and departure; which is the reason that several experienced artists keep their account by their meridian distance, without regarding their longitude; for we must not suppose, as some have alledged, that they are ignorant of the cause why they make more westing in going towards the equator, than they make easting in returning back, for they well know that (outward bound) they get into the parallel of their port, long before they run down their difference of longitude, which therefore must require more departure than in coming home; because that it is possible they may have 6 or 8 degrees of longitude to run down in the parallel of $13^{\circ} 30'$ when sailing to B; and the same number of degrees of longitude to run down in the parallel of $50^{\circ} 0'$, when sailing from B to A. So they do not return near the opposite rumb to that on which they sailed out; but as they constantly steer the same courses out every voyage, as near as the wind will permit, in the course of many voyages, they find the meridian distance nearly the same every voyage outward bound; and if in several voyages they can steer the same courses home, that they steered home the preceding voyages, they will find their departures nearly the same as in their former voyages, tho' always less than the departures out, for the reasons before assigned: But it happens so rarely, that they can keep the same courses two voyages; that very few have any regard to the meridian distance, but keep their account by the difference of longitude, which will always be the same out and home, whatever courses they steer.



C H A P. III.

Of P L A I N S A I L I N G.

S E C T. I.

The CONSTRUCTION and USE of SEACHARTS.

The LONG-LINE, and HALF MINUTE GLASS.

PLAIN sailing is that where the plain chart is used, all the various cases of which will be exactly the same with those of right angled triangles; so that we are only to shew how those triangles are formed upon the chart; and the names of the sides.

In this chart the meridians are strait lines drawn parallel to one another; the equinoctial, and all the parallels of latitude are likewise strait lines parallel to one another; by this means the rumbs will be strait lines, and make equal angles with all the meridians, but then all the parallels of latitude will be equal to the equinoctial, which is a very great error, at any considerable distance from the equinoctial: it is called the plain chart, and constructed in the same manner as that of surveying land in *Plate 10*. All the lines drawn parallel to A B, may be called meridians, and those parallel to B E, parallels of latitude; which being all equal to the equinoctial, will make the departure and difference of longitude the very same thing, though in the parallel of latitude of 60 degrees, it is only one half of the difference of longitude; and if nearer the pole, the error will still be greater, so that this chart must be very erroneous.

The only true sea chart is *Mercator's*, which retaining the parallelism of the meridians, will occasion the degrees of longitude in any parallel, to be equal to the same degrees in the equinoctial, as in the plain chart; but to remedy this, the degrees of the meridian in this chart, are enlarged in the same proportion that the degrees of the parallels of latitude are. Before we shew the construction of this chart, we shall shew all the uses of the plain chart; for as there are two sea charts, from hence arises the division of navigation in two parts, *viz.* plain, and *Mercator's* sailing; we shall here treat of the first.

Every place is supposed to have a meridian, and parallel of latitude, and if actually drawn, and likewise a strait line from any place to another, that differ both in latitude and longitude; we shall then have a right angled

angled triangle, whereof the meridian will be the perpendicular, the departure the base; and the distance the hypotenuse: Let the two places be A and Z, A *a* or Z *z*, will be the perpendicular; *a* Z or A *z*, the base; and A Z, the distance upon the rumb line; and that A *a* is the difference of latitude, and *a* Z the departure; betwixt A and Z, is evident by the definition of those terms: The angle that is formed by the rumb line, and the meridian, is called the course, which corresponds to the angle opposite to the base in trigonometry.

As in trigonometry there are six cases, so there are the same in plain sailing, and the solutions the same in both, only changing the names of the parts. The hypotenuse, is called the distance; the perpendicular is called the difference of latitude; the base is called the departure; and the angle opposite to the base, is called the course.

Now here, as in trigonometry, there are four things, any two of which being given, the other two may be found.

The mariner has two things given, *viz.* the course and distance; the former by the compass, and the latter by the log-line and half minute glass.

To the end of this line there is fastened a piece of wood, with as much lead at the lower end as will serve to make it swim upright in the water: It is divided into knots, the distance betwixt the knots must be exactly the 120th part of a mile; and there must be a sufficient quantity of line betwixt the log, and the mark from which the line begins to be dived; so that when the mark is at the ship's stern, the log may be clear of the eddy of the ship; and then the half minute glass is turned, and the line vered of the reel, which is stopped as soon as the glass is out: this will give the exact distance the ship has sailed in the half minute; and if she continues at the same rate for a whole hour, her distance run in the hour is also known; for she will go as many miles in the hour, as there are knots run out in the half minute; this is the only method they have to measure the distance, and may be liable to great errors unless the line be very carefully divided, and the glass actually half a minute.

S E C T. II.

The Resolution of the six Cases of PLAIN SAILING.

THE most expeditious way, and which is always used in practice, is by the table of difference of latitude and departure.

This table gives, by inspection, the difference of latitude, and departure, to any course, for any distance less than 100 miles.

In the uppermost rank, are placed the courses from 1 degree to 45, including points and quarter points; and in the lower rank, the courses from 45 to 90; each course is divided into two columns; under lat. is the difference of latitude; and under dep. is the departure; and under dist. is the distance corresponding to them; but when the course is more than 45 degrees, it will be found in the lower rank; and the difference of latitude over lat. and the departure over dep.

We shall work an example in each case, by the logarithms, by scale and compasses, and by these tables which will sufficiently explain their nature: We shall likewise shew how to delineate them by the line of rumbs.

C A S E I.

Given the course and distance, to find the difference of latitude and departure.

E X A M P L E.

A ship sails SW b W 60 miles. I demand the latitude come to, and departure.

This is exactly the same with the first case of trigonometry, and the triangle delineated in the same manner, only we make use of the line of rumbs to set off the angle by.

The line of rumbs is constructed in the same manner as the line of chords, from a quarter of a circle, divided into 8 equal parts, for points of the compass, and those sub-divided into quarters; the use of this line is to set off the course, which is always given in points of the compass, and not in degrees; as in this example the course is five points from the meridian; therefore upon the line R T, from the point R, describe an arch with the chord of 60 degrees, upon which set off five points taken

G g

from

from the line of rumb; this might be taken off the line of chords, tho' not so exactly, because of the odd minutes; being $56^{\circ} 15'$, so that in all the cases in navigation, we shall make use of the line of rumb, observing that the arch must be described by the chord of 60 degrees, to which that line of rumb is adapted. We shall give no further directions for delineating the following examples, only refer to the like cases in trigonometry. In order to work them by the logarithms, the points must be turned into degrees, for which there is a table to find them by inspection; 5 points is $56^{\circ} 15'$, then it will be,

As the radius or sine of 90	-	-	10.000000
Is to the distance 60 miles	-	-	1.778151
So is the sine of the course $56^{\circ} 15'$	-	-	9.919846
To the departure 49.9 miles	-	-	+1.697997
As radius	-	-	10.000000
Is to the distance 60 miles	-	-	1.778151
So is S. C. of the course $33^{\circ} 45'$	-	-	9.744739
To difference of latitude 23.3 miles	-	-	+1.522880

By the table of difference of latitude and departure, the course is more than four points, therefore it will be found in the bottom, that is 5 points; and in the column over lat. is 33.3, for the difference of latitude; and 49.9 in the column over dep. for the departure, both right against 60 in the column dist.

By extending on *Gunter's* scale, if the course be turned into degrees, it will be just the same as was in right angled triangles.

But there are two lines on *Gunter's* scale by which it may be done, one is marked S R, and the other T R; that is the sines and tangents of the rumb; and as the sine of 90 degrees, and the tangent of 45 degrees are equal to the radius, so 8 points is for the radius on the line sine rumb, and 4 points for the radius on the line tangent rumb.

For the departure R : S 5 points : : 60 : 50 nearly.

For the difference of R : S 3 points : : 60 : 33.3.

The extent from 8 points to 5 points on the line S R, will be the same as from 90 to $56^{\circ} 15'$ on the logarithmick line of sines, and will reach from 60 to 50, on the line of numbers for the departure; the extent from 8 points to 3 points, will be the same as from 90 to $33^{\circ} 45'$ on the logarithmick sines, and will reach from 60 to 33.3 on the line of numbers for the difference of latitude.

CASE II.

Given course and difference of latitude, to find the distance and departure.

EXAMPLE.

A ship sails SE b S, till she alters her latitude 50 miles; I demand her distance and departure.

This is exactly the same triangle with the preceding, but what was the difference latitude before, is now the departure; the proportion will be,

As sine comp. of the course, viz. $56^{\circ} 15'$	<u>9.919846</u>	
Is to the difference of latitude 50 miles	1.697997	
So is the radius or sine of 90	10.000000	
To the distance 60 miles	<u>1.778151</u>	
As sine comp. of course or of $56^{\circ} 15'$	<u>9.919846</u>	
Is to the difference of latitude 50 miles	1.697997	} 11.442736
So is the sine of the course $33^{\circ} 45'$	<u>9.744739</u>	
To the departure 33.3 miles	1.522890	
	1.522890	

By the table of difference of latitude and departure, the course is now three points, find it in the upper rank, and under lat. find 50 the given difference of latitude; corresponding thereto under dif. is 60, and under dep. is 33.3.

By *Gunter's* scale, S of 5 points : sine of 8 points :: 50 : 60; the extent from 5 points to 8 points, on the line S R, will reach from 50 to 60 on the line of numbers and gives the distance.

For the departure S of 5 points : sine 3 points :: 50 : 53.3.

CASE III.

Given course and departure to find the difference of latitude and distance.

EXAMPLE.

A ship sails NW by W till she gets 50 miles to the westward; I demand the distance and difference of latitude; this is a case that can scarce ever happen, and is the same with the preceding, only calling what was difference of latitude in the former, in this the departure, and what was departure in the former, is now difference of latitude; and then the operations will, in all respects, be the same as before.

G g 2

CASE

C A S E IV.

Given the distance and difference of latitude; I demand the course and departure.

E X A M P L E.

A ship sails betwixt the south and west 60 miles, and finds by a good observation, she has made 50 miles southing; I demand the course and departure, delineate the triangle as in *Case 4.* right angled triangles; the proportion is

As the distance 60 miles	-	-	-	<u>1.778151</u>
Is to difference of latitude 50 miles	-	-	-	1.697997
So is radius or sine of 90	-	-	-	<u>10.000000</u>
				11.697997
				<u>9.919846</u>

To the S. C. of the course $56^{\circ} 15'$

So the course will be S. W. b S. the departure will be found by *Case 1.* or 2.

By the table of difference of latitude and departure, turn over till you find 50 in the difference of latitude column, and 60 in the distance column; and because I find 50 in the column that has lat. at the top, the course will be there also, *viz.* 3 points, that is S. W. b S. and in the departure column I find 33.3 westing.

By *Gunter's* scale; the extent from 60 to 50, on the line of numbers, will reach from 8 points to 5 points on the line S R.

C A S E V.

Given distance and departure, to find the course and difference of latitude.

E X A M P L E.

A ship sails betwixt the north and east 60 miles, till her departure is 50 miles; I demand her course and difference of latitude. This is exactly the same with the preceding, and scarce can happen, so we shall proceed to the next.

C A S E VI.

Given difference of latitude and departure, to find the course and distance.

E X A M P L E.

Two islands P and T, T in $32^{\circ} 41'$, and P in $20^{\circ} 53'$ both north latitude;

titude; P lieth to the westward of T $6^{\circ} 45'$. I demand their bearing and distance, or which is the same thing, what course must be steer'd from P to T, and how many leagues distant; delineate the triangle as in *Case 6*—trigonometry, and find the difference of latitude by subtraction, which turn to leagues off 20 to one degree, as in the operation.

		Departure.
A	32 41	6 45 by 3 15
B	20 53	<u>20</u>
	11 48	120
	<u>20</u>	15
	220 leagues	135 departure
	16 for 48 miles	
	<u>236</u> diff. of lati. in leagues	

The proportion is,

As the difference of latitude 236 miles	2.372912
Is to the departure 135 miles	<u>2.13334</u>
	10.00000
So is the radius or tangent of 45	<u>12.130334</u>
To the tangent of the course $29^{\circ} 46'$	9.757422

By the table of difference of latitude and departure, these numbers are too large to be found in the table, therefore take the quarter of each, so 59 will be for the difference of latitude, and $33\frac{1}{4}$, the departure; find these in their proper columns, and corresponding thereto in the distance column, will be 68 or 69, which multiplied by 4, will give the whole distance; and because the difference of latitude is greater than the departure, the course will be found to be near 30 degrees, and taking 68.5, which is the nearest distance corresponding to the difference of latitude and departure; in the tables, the whole distance will be 274 leagues, and the course from P to T, will be S. S. W. $\frac{1}{4}$ W 2 degrees westerly.

By *Gunter's* scale; the extent on the line of numbers from 236 to 135, will reach from the tangent of 45 to 30, or 60 degrees on the line of tangents: Now to know which of these two will be the angle, observe which is greatest, the difference of latitude or departure; for if the difference of latitude be greatest, as in this example, the angle will be less than 45 degrees, but if the departure be greatest, the angle will be more than 45 degrees.

These are all various cases in plain sailing, which being well understood, it will be easy to give a true solution to the following questions, by
the

the table of difference of latitude and departure: For it will be too tedious to work them by the logarithms, or construct them geometrically; therefore in practice we always use the tables, and for the proof, let them be performed by *Gunter's scale*.

A ship in $22^{\circ} 51'$ north latitude, sails N. N. $\frac{1}{2}$ W. 83 leagues the latitude come to, and departure from the meridian is required.

A ship in $0^{\circ} 56'$ north latitude, sails S b W $\frac{1}{2}$ W, till by a good observation she is 1.13 south latitude; her distance and departure is required.

A ship in $36^{\circ} 40'$ south latitude, sails betwixt the N and E, till she gets into $34^{\circ} 50'$ south latitude, and finds by the log, she has run 63 leagues; the course and departure is required.

Two islands A and B, A in $2^{\circ} 2'$ north latitude; B in $5^{\circ} 16'$ south latitude; B lies 250 leagues to the westward of A; the course and distance is required.

S E C T. III.

Of working a Traverse, or reducing various Courses into One.

BEing now provided with a good sea chart, to find the bearing and distance of any two places, and with a compass to direct the course, also with a log line, and half minute glass, to know how far we have advanced towards our port; we need only keep an exact account of the distances, by setting down every day at noon, the number of miles sailed the preceding 24 hours, in a book provided for that purpose: Now if at any time we want to know our distance from the port we are bound to, it is only collecting the several distances we have sailed every 24 hours, and subtracting the sum from the whole distance: the remainder will be the distance we are from our port: But as it is impossible to keep a ship in her direct course, by reason of contrary winds, the intervention of lands, or various other accidents, which forces her out of the direct course; it will be absolutely necessary to keep an exact account of every particular course and distance, sailed in 24 hours; and then reduce all these different courses and distances into one: This is what is called a traverse, and is only so many different questions of the first case of plain sailing; for after we find the differences of latitude and departure, to every particular course, if they are in the same quarter of the compass, we may collect all the differences of latitude into one; which will be the whole difference

rence of latitude, from the first port sailed from, and the sum of all the departures will be the whole departure; and having the difference of latitude and departure, the course and distance is found by the last case of plain sailing. If some of the differences of latitude be northerly, and some southerly, add the several northings together, and also the several southings; if they are equal, the ship has not altered her latitude, if unequal, subtract the less from the greater, the remainder will be the whole difference of latitude; do the same by the departure when there are eastings and westings. The whole art of navigation depends upon keeping a correct and distinct account of these various courses, and reducing them to one: For which purpose it will be proper to make a table of six columns, as the following:

Course.	Distance.	North.	South.	East.	West.
S. E.	38	0.0	26.9	26.9	0.0
S. E. ½ E.	42	0.0	23.3	34.9	0.0
E. S. E.	25	0.0	9.6	23.1	0.0
E.	30	0.0	0.0	30.0	0.0
E. N. E.	15	5.7	0.0	13.9	0.0
N. W.	24	17.0	0.0	0.0	17.0
W. S. W.	17	0.0	6.5	0.0	15.7
		22.7	66.3	128.8	32.7
Course good:	Distance.				
S. 66° E.	106		43.6	96.1	

In the first are the several courses; in the second their corresponding distances; then by the table of difference of latitude and departure, find a difference of latitude and departure to every course and distance, and set them down in their proper columns, sum up all the northings, is 22.7, which subtract from 66.3, the sum of the southings; there remains 43.6 miles of south difference of latitude; the sum of the eastings is 128.8, from which subtract 32.7, the sum of the westings; there remains 96.1 miles east departure. As this number exceeds the limits of the table, take half of it 48; take also half the difference of latitude 21.8; the nearest I can find to these two in the tables, is 21.6 and 48.4 in the column over 66 degrees, and in the distance column is 53, which doubled, makes 106 for the whole distance.

By this way of keeping an account, we may at any time, know the course and distance to the port sailed from, which suppose in $30^{\circ} 30'$ north latitude, and likewise the course and distance to the port bound to, which suppose in $29^{\circ} 18'$ north latitude, and 72 leagues to the eastward: Let the port sailed from be A, and the port bound to B; the difference

of latitude will be 24 leagues, and the departure 72; the course will be nearly E. S. E. $\frac{1}{2}$ E. about 77 leagues distant; but after steering the several courses, as in the table, the ship arrives at C. I find by the preceding calculation, the whole difference of latitude made from A to C, is 43.6 miles southing; and the whole departure 96.1 miles easting; therefore subtracting the difference of latitude made from A to C, from the whole that was to be made from A to B; the remainder will be the difference of latitude yet to be made, and subtracting the departure made, from the whole that was to be made at first setting out, the remainder will be what is yet to be made: And so having the difference of latitude and departure given, by them the course and distance is found. See the operation.

Difference of latitude from A to B	72 miles
Difference of latitude from A to C	<u>43.6 miles.</u>
Difference of latitude from C to B	28.4 miles is $9\frac{1}{2}$ leagues.
Departure from A to B	216.0 miles.
Departure from A to C	<u>96.1 miles.</u>
Departure to be made from C to B	119.9 miles is 40 leagues.

The nearest I find to those two in the table of difference of latitude and departure are 39.9, and 9.2; so the course will be S. 77° W. 41 leagues distant.

By this it is evident, that it is first absolutely necessary to know the whole difference of latitude and departure, that is to be made before we set out. Secondly, we must keep an exact account of the various courses and distances steered, by which we may at any time know how much of our difference of latitude is made, and consequently by subtraction, we may always tell what difference of latitude and departure we have to make; all this may be done without a chart, by a table of latitude and longitude of places.

The geometrical construction of a traverse would be too tedious for practice, but as there are several small islands, rocks, and lands, laid down in a chart, that are not in the tables of latitude and longitude; it will not be improper, every day at noon, to mark the place the ship is in upon the chart; by this means we may have a view, not only of our port, but also of any rocks or sands; this is especially necessary when we come near to the land, we shall therefore shew how this is to be done.

SECT.

S E C T. IV.

To prick the P L A I N C H A R T. (Plate X).

THIS is to lay down the place the ship is in at any time, or to find the bearing and distance of any two places upon the chart.

Suppose a ship bound from A to O; required the course and distance.

There are several compasses, or rumb lines upon the chart, by which the course may be readily found; thus, lay the strait edge of a ruler so that it may touch the two places A and O, then take a pair of compasses, and placing one foot in the center of any compass, open the compasses till the other foot touches the edge of the ruler; then sliding the compasses with one foot by the edge of the ruler, the other foot being perpendicular to it, will trace out the rumb line on which the ship must steer: Now as the rumbs are only drawn to the whole points, it may happen, as in this case, that the foot of the compasses will not fall exactly in the rumb, but we may easily estimate the quarters; so the course from A to O, will be S. E. b S $\frac{1}{4}$ E. A C, is the whole difference of latitude; suppose 90 leagues, C O the whole departure $66\frac{2}{3}$ leagues. Now it is certain if we sail 90 leagues south, and then $66\frac{2}{3}$ leagues east, we shall arrive at O; but I find by my account, that I have made only 14 leagues southing, and 35 leagues easting: Now I want to know what place I am in, that I may steer a direct course for O. To find this, provide two pair of compasses: In one take the difference of latitude 14 leagues, which set off in the line A C from A to *e*; with the other pair take the departure 35, which set off from C to *r* in the line C F; from *r* with the difference of latitude, describe an arch, and from *e*, with the departure describe another arch, to cut the former in *s*, which is the point the ship is in at the time the calculation is made, and may be done every day at noon, or if need be at any other time. The next day at noon, after reducing the various courses to one, I find I have made, in that time, 42 leagues southing, and 28 leagues easting; I want to know the place the ship is in: If the point *s*, where the ship was the day before, be at the intersection of a meridian, and parallel, we may find it by the same method we found the point *s* the day before; if it is not, take with one pair of compasses, the nearest distance of the point *s*, to any parallel; with another pair of compasses take the departure 28, with which describe an arch from *s* as center; then slide the first pair of

H h

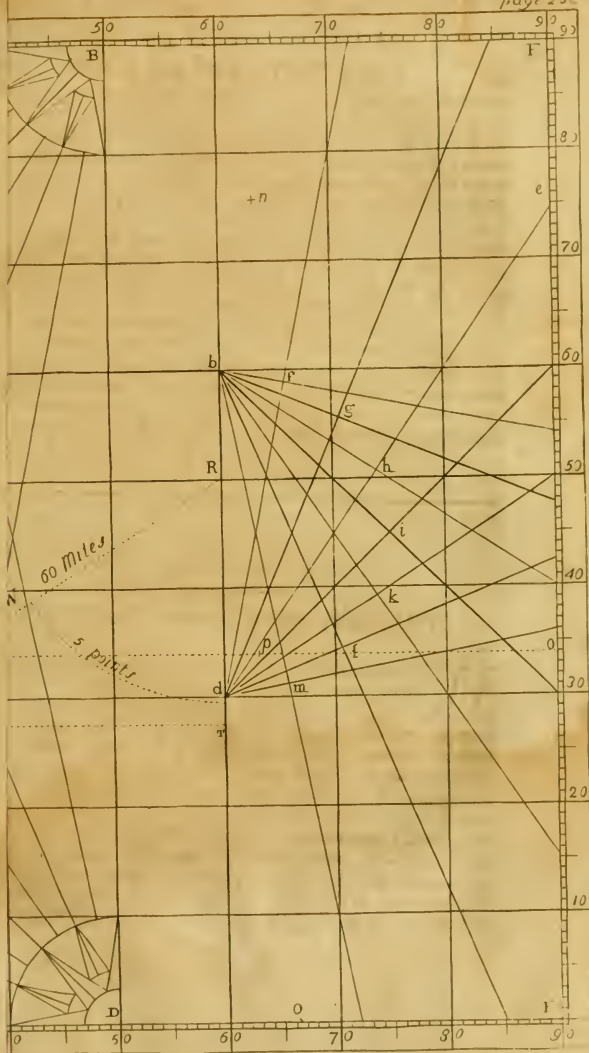
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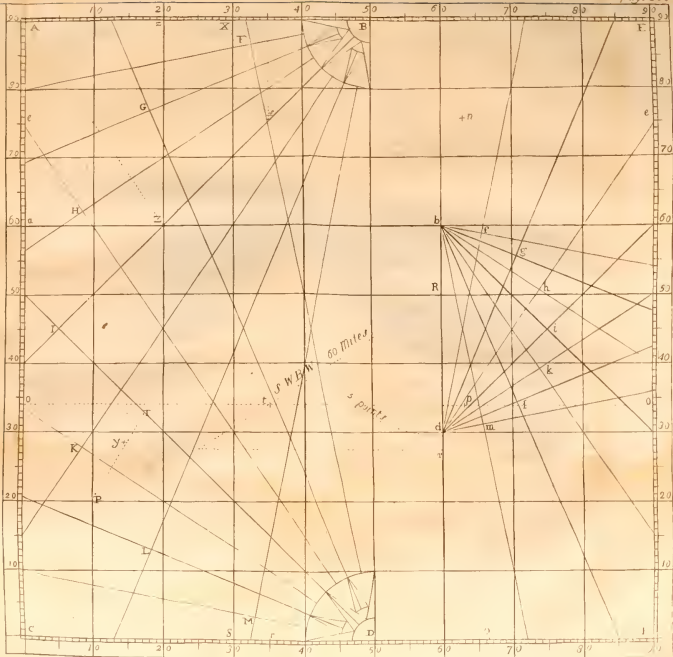
compasses with one foot in the parallel, and the other perpendicular to it, till it cut the arch in n ; again with one pair of compasses, take the nearest distance of the point n to a meridian, and with the other pair take the difference of latitude 42 , and describe an arch from the point n as center, and move the first pair upon the meridian, till it cut the arch in p , which is the place the ship is in. After the same manner we may find the place the ship is in every day at noon: When the chart is not very large, it may be done thus; let the ship be in s as before; first lay a ruler across the chart, let the edge be parallel to some east or west line, and passing through s , mark the two points e and o , where the ruler cuts the two lines that limit the chart, then take the difference of latitude 42 , and set it from e and o to o and o , and lay the ruler across by these two points. It is plain the ship must be some where in the line oo ; to find which, take with a pair of compasses, the nearest distance of the point s , to any meridian; observe where that meridian cuts the edge of the ruler; from this set off the distance in the compasses by the edge of the ruler to t ; then taking the easting 28 , it will reach from t to p , which is the place the ship is in.

C H A P. IV.

Of Mercator, middle Latitude, and parallel Sailing.

WE have now fully explained all the varieties of sailing by the plain chart, but it is subject to very great errors, for by making all the parallels of latitude equal to the equinoctial; the difference of longitude betwixt any two places will always be equal to the departure, whereas in fact the difference of longitude, may be sometimes double, or treble, and always more than the departure, except when the two places are upon the equinoctial. To illustrate this, let there be two islands A in $32^{\circ} 41'$, and B in $20^{\circ} 53'$, both north latitude, and let their difference of longitude be 7.15 , or 145 leagues, that is to say, if there be a meridian drawn thro' A , and one thro' B ; their greatest distance will be upon the equinoctial, *viz.* 145 leagues, but these meridians continually approach one another, till at last they meet in the pole, therefore their distance in the parallel of $20^{\circ} 53'$, will be less than 145 leagues; let it then be 135 leagues, and their distance





stance in the parallel of $32^{\circ} 41'$, will be still less than their distance in the parallel of $20^{\circ} 53'$, which suppose 121 leagues; the difference of latitude from A to B, is 236 leagues, so if a ship at A sails 236 leagues due north, and then 135 leagues due east, she will certainly arrive at B; again if she sails from B 236 leagues due north, and then 121 leagues due west, she will arrive at A, but by the principles of the plain chart, she must sail 135 leagues, which makes an error of 14 leagues; and if the two places were more to the northward, the error would still be greater.

S E C T. I.

Of the Principles of MERCATOR's Chart.

TO remedy this error, a new chart must be made, in which the degrees of any parallel, shall have the same proportion to the degrees of the meridian, that they actually have upon the globe; but this must not be, by inclining the meridians, because the rumb lines would make unequal angles with them: The meridians then must continue parallel to one another; this will make a degree in any parallel, equal to a degree in the equinoctial, which is a monstrous error; for a degree in the parallel of 60° , is but half a degree in the equinoctial: Therefore in this chart, the degree of the meridian that lies betwixt $59^{\circ} 30'$, and $60^{\circ} 30'$, must be double the degree that lies betwixt the equinoctial, and the first degree upon the meridian, that is supposing the parallel of latitude of one degree to be 60 miles distant from the equinoctial; then the distance betwixt the parallel of $59^{\circ} 30'$, and the parallel of $60^{\circ} 30'$ must be 120 miles: This will occasion the meridian to be unequally graduated, and suppose a parallel of latitude be drawn thro' every degree of the meridian, the distances betwixt these parallels will be unequal; that betwixt the first and the equinoctial will be the least, but it increaseth the further the parallel is removed from the equinoctial, in the same proportion that the parallel is less than the equinoctial.

In order then to graduate the meridian, let us suppose a degree of the equinoctial, or of the meridian, which are both great circles, to be 60 miles; the parallel of 1 degree of latitude, is very near as great as the equinoctial, and so the first degree of the meridian will be 60 miles; in like manner the parallels of 2, of 3, of 4, of 5, are so near the equinoctial,

tial, that the distances betwixt them may be all equal, that is 60 miles; but then the parallels begin to decrease very perceptibly, and therefore the distance betwixt the parallel of 5 and 6 will be 61, and the distance betwixt the parallel of 6 and 7, will be a little more, and still increasing; so the only difficulty will be to graduate the meridian. The first thing to be done, is to calculate how many miles will make one degree in any parallel of latitude, for which take this proportion:

As the radius, is to the sine complement of the latitude,—so is 60 the miles in a degree upon the equinoctial, to the number of miles that will be contained in a degree in that parallel, or which is the same thing.

R : S. C. latitude :: difference of longitude : departure.

DEMONSTRATION. (*Plate 11. Fig. 1.*)

Let ÆZ be the latitude, then will ZP be the complement; Zz the radius of the parallel is the sine of the complement angle PCZ . Now the degrees of one circle are to the degrees of another, as the radius of the one is to the radius of the other, as was proved in part first. Therefore ÆC , the radius of the equinoctial is to Zz , the radius of the parallel, or sine complement of the latitude, as a degree of the equinoctial, or difference of longitude, is to a degree of the parallel or departure.

By this calculation it will be found that a degree upon the parallel of 60 degrees, will contain but 30 miles; for the complement of 60 is 30, and the sine of 30 degrees, is half the chord of 60 degrees, or half the radius; so the diameter of the parallel of 60, is half the diameter of the equinoctial.

Tho' this proportion be true, it would be a great labour to make tables by it, which Mr *Wright* has effected by another method; for he considering that the radius is to the sine complement as the secant is to the radius, calculated a table of meridional parts, by a continual addition of the secants. In this chart the sine complement is equal to the radius, because the meridians are parallel to each other; so the degrees of the parallel are all enlarged beyond their due measure, and therefore the degrees of the meridian must be enlarged in the same proportion, which will be as the secant of the parallel is to the radius; to prove this, let CB be 60 miles, equal a degree of the equinoctial (*Plate 11. Fig. 3.*) with this, as radius, describe a circle, and draw the parallel FE ; it is plain DE would be a degree in that parallel; but upon the chart it is DG . In the triangles DEC , and ACB , the angles ACB , and DEC , are equal being alternates, to the two parallels DG , and CB ; the angles at D and B , are right, there-
fore

fore the triangles are similar, and $CE : DE :: CA : CB$, but in *Mercator's* chart DE is enlarged till it is equal to CE ; therefore to keep the same proportion, CB must be enlarged till it is equal to CA , that is if CB be a degree of the meridian at the equinoctial, CA must be a degree of the meridian at the parallel FE ; by this means, it will be enlarged in the same proportion that a degree of a parallel is enlarged: Now let DE be the parallel of 30° , if 60 miles be a degree of the equinoctial; then 51.96 should be a degree in this parallel, as in the following operation; but it is enlarged to 60, therefore the degree of the meridian, that lies betwixt the parallel of $29^\circ 30'$, and the parallel of $30^\circ 30'$, must be enlarged in the same proportion; that is $51.96 : 60 :: 60 : 69.28$; for $60 \times 60 \div 51.96 = 69.28$. (See the operation by the logarithms). The complement of the parallels is 60 degrees; therefore

As radius	10.000000	} 60 miles	1.778151
Is to sine of 60°	9.937531		1.778151
So is 60 miles in the equator	1.778151		3 556302
To 52 miles in the par.	+ 1.715082	52 miles	1.715682
		69.28	1.840620

So a degree in the meridian at the parallel of 30 degrees of latitude must be 69.28 miles, and that this is equal to the secant of 30° degrees, will appear by the following operation.

As radius	10.000000
Is to the secant of 30°	10.062469
So is 60 miles	1.778151
To 69.28 miles	+ 1.840620

By the same manner, we may find the number of miles that will make a degree of the meridian in any latitude; but Mr *Wright* has constructed tables of meridional parts in such an excellent manner, that the meridian by them may be very easily graduated.

In this table, we find by inspection, the number of miles that any parallel of latitude is distant from the equinoctial; so the parallel of $29^\circ 30'$, will be 1854 miles from the equinoctial, and the parallel of $30^\circ 30'$ will be 1923; the difference is 69, which must therefore be a degree of the meridian at that parallel.

The manner Mr *Wright* calculated this table, was by a continual addition of the secants, which he calculated even to minutes, with the utmost care and exactness; he assumed 60 equal parts for a degree of the equinoctial, and therefore the first five degrees of the meridian would be

one

one of those equal parts each, because if the radius of a circle be 60 equal parts, the secants of 1, 2, 3, 4, 5 degrees, will be so near equal to the radius, that they may be accounted 60 each, so the sum of those secants will be 300 equal parts; these are called meridional parts, and shew the distance of the parallel of 5 degrees from the equinoctial: The sum of all the secants from the equinoctial to the parallel of $29^{\circ} 30'$, is 1854, which is the distance of that parallel from the equinoctial: Now if it be required to find the distance of the parallel of $30^{\circ} 30'$ from the equinoctial; find the secant of 30° , which will be 69, and this added to 1854, makes 1923, which exactly agrees with the meridional parts in the tables, corresponding to $30^{\circ} 30'$ latitude. Having thus shewn the construction of the table of meridional parts, we shall now shew its use in making Mercator's chart.

S E C T. II.

To make MERCATOR'S Chart. (Plate 13).

THIS may be made to contain all the known parts of the whole world, which is called a general chart, or to contain only a part of it, which is called a particular chart; and as they are both projected in the same manner, we shall only shew how to make one from the latitude of 30 degrees to the latitude of 60 degrees, and to contain 44 degrees of longitude. Draw the line A B, to represent the parallel of 30 degrees, which make 2640 equal parts, being the miles in 44 degrees, upon the equinoctial, or in any parallel of latitude; then draw the perpendiculars C A and D B, to represent two meridians; and to find the parallel of 60, look for it in the table of meridional parts, from which subtract the meridional parts of the parallel of 30; the remainder will give the distance betwixt these two parallels, as in the following operation it is 2640, which set off by the same scale of equal parts,

$$\begin{array}{r} 60-4528 \\ 30-1888 \\ \hline 2640 \end{array}$$

from A to C, and from B to D, and draw the line C D, to represent the parallel of 60 degrees, thus the chart is limited; then draw another meridian any where at pleasure, to meet the parallels A B and C D.

This

This is called the first meridian, where the longitude is supposed to begin; from which graduate the parallel into degrees both ways, and at every tenth draw a meridian, as in the draught where they are numbered 10°, 20°, &c. on each side of the first meridian; then graduate the first meridian into degrees; and at every tenth draw a parallel numbered 30, 40, 50, 60. Now to find the number of equal parts, or which is the same thing of equinoctial miles betwixt these parallels, proceed thus:

Lat.	Mer. parts.	Diff.
60	4528	} 2640
<u>30</u>	<u>1888</u>	
50	3475	} 1587
<u>30</u>	<u>1888</u>	
40	2623	} 735
<u>30</u>	<u>1888</u>	

In the first column are the degrees of latitude; in the second, the corresponding meridional parts; and in the third, the distance of each parallel from the parallel of 30 degrees. We must proceed in the same manner, to find the distance of each degree from the parallel of 30, or from one another, and where the scale will permit, the degrees may be divided into minutes; the meridians A C, and B D, may be graduated, also the parallels A B, and C D, and the places laid down according to their true latitudes and longitudes; or if the places be actually laid down their true latitudes and longitudes may be found, and also their bearings and distances by the following problems.

P R O B. I.

Having the latitude and longitude of a place; to find it upon the chart.

E X A M P L E I.

I want to find the lizard, which by the tables, is in 50 degrees north latitude, and in 0 longitude.

Look on the graduated meridian for 50 degrees of latitude, where in this chart, there is actually a parallel drawn, and because the lizard has no longitude, it will be at the intersection of the first meridian with that parallel.

E X A M P L E II.

I want to find the island of *Madeira* in the chart, whose latitude is 32° 17' north, and longitude 12° 8' west.

Look on the graduated meridian for 32° 17'; then with a pair of compasses

passes take the nearest distance of that point to any parallel of latitude; find upon the graduated parallel $12^{\circ} 8'$, on the west side of the first meridian, and with another pair of compasses take its nearest distance from any meridian; move both compasses, keeping the foot of the first in the parallel of latitude, and the other in the meridian, till the other two feet of the compasses meet, which will be the place required; observing to move the compasses so that their points be parallel either to a meridian or parallel. Or it may be done as in the plain chart, by laying the strait edge of a ruler across the chart, parallel to some east and west line, to intersect the meridian in $32^{\circ} 17'$, the given latitude; then with a pair of compasses, take $12^{\circ} 8'$, the given longitude, which set off by the edge of the ruler, from the point where the ruler intersects the first meridian, this will give the place required.

P R O B. II.

To find the latitude and longitude of any place in the chart: This is only the reverse of the former.

Let the latitude and longitude of *Madeira* be required.

With a pair of compasses take its nearest distance to any parallel of latitude, which set off from the intersection of that parallel, with the graduated meridian and it will be found to be $32^{\circ} 17'$.

For the longitude, take its nearest distance from any meridian, which set off from that meridian upon the graduated parallel, and it will give $12^{\circ} 8'$.

P R O B. III.

To find the bearing and distance of any two places upon the chart.

The course is found in the same manner as upon the plain chart.

The distances in this chart cannot be measured, as on the plain chart, at once, by a scale of equal parts; for a degree of the meridian upon the globe, is at all places equal to a degree upon the equinoctial; and therefore if two places lie in the same meridian, their proper difference of latitude will be their true distance, as in the following.

E X A M P L E.

Required the distance from the *Lizard* in $50^{\circ} 0'$ north, and 0 longitude to an island in $32^{\circ} 17'$.

			<i>Lizard</i>	$50^{\circ} 0'$
	Lat. $50^{\circ} 0'$	3475	} An island in	$32^{\circ} 17'$
	$32^{\circ} 17'$	<u>2048</u>		
Meridional difference of latitude		1427		Pro. dif. of lat. $17^{\circ} 43'$
		miles		60 miles
				Distance $106\frac{2}{3}$ miles

Now tho' the real distance betwixt these two parallels upon the earth be 1063 miles, yet upon the chart it is 1427; so in this chart the distances are not truly set down, except the places lie under the equinoctial, but there is a certain method of finding the true distances, by these laid down in the chart; which admits of four different cases.

C A S E I.

If the two places lie under the same meridian, then the proper difference of latitude turned into miles, is the real distance, as in the aforesaid example.

C A S E II.

If both places lie under the equinoctial, then the distance is truly measured upon the equinoctial.

C A S E III.

To find the distance of two places lying in the same parallel of latitude.

In all *Mercator's* charts there is a direction how this is to be found, but when the places are at any considerable distance, it will be very erroneous, as will appear in the following example.

E X A M P L E.

Let the two places be in the parallel of $50^{\circ} 0'$, and their difference of longitude 42 degrees, which is 2520 miles, and is the real distance betwixt them upon the chart, when measured upon the equinoctial, or graduated parallel; but their true distance upon the parallel, is 1620 miles, which will make 42 degrees of the parallel of $50^{\circ} 0'$, tho' it will take 2520 miles to make 42 degrees upon the equinoctial, for $R : \text{fine of } 40 : : 2520 : 1620$; so the true distance in the parallel is 27 equinoctial degrees.

The chart directs to take the distance betwixt the two places with a pair of compasses, and apply it to the graduated meridian, in such a manner, that one foot may be as many degrees above, as the other is below the parallel; the degrees intercepted between the feet of the compasses, allowing 60 miles to one degree, will, according to their direction be the true distance in the parallel; so that 2520 miles taken upon the equinoctial, which is the real distance upon the chart, should reach in the graduated meridian from $36^{\circ} 30'$, to $63^{\circ} 30'$, the one being as much

above, as the other is below the parallel of $50^{\circ} 0'$; for the true distance in the parallel is 27 degrees, the half of which, $13^{\circ} 30'$, being added to 50, gives $63^{\circ} 30'$, and subtracted from 50, gives $36^{\circ} 30'$; but the distance betwixt these parallels upon the chart is 2617 equinoctial miles, as by the operation, which will occasion an error of 997 miles.

$$\begin{array}{r} 63^{\circ} 30' \\ 36^{\circ} 30' \\ \hline 4972 \text{ meridional parts} \\ 2355 \\ \hline 2617 \text{ difference.} \end{array}$$

To remedy this, instead of the former take the following method.

Rule. Open the compasses till one foot is half a degree upon the meridian below, and the other foot half a degree above the parallel of latitude; count how many times that extent is contained betwixt the two places, which will give the number of equinoctial degrees betwixt them, and multiplied by 60, it will give the true distance in miles betwixt these two places upon the earth, if taken in the parallel; which tho' not the nearest, as was before observed, yet is the shortest that can be made by steering upon one point of the compass: In the following operation we shall see how near this comes to the truth. A degree upon the graduated meridian at the parallel of $50^{\circ} 0'$, should be equal to the secant of 50 degrees, supposing the radius to be 60 miles; then $R : \sec. 50^{\circ} :: 60 : 93.34$.

	Logarithm.	93.34
		<u>27</u>
Sec. 50°	10.191932	65338
60 miles	<u>1.778151</u>	18668
93.34 miles	<u>+1.970083</u>	2520.18

C A S E IV.

When the two places differ both in latitude and longitude, to find their distance.

Rule. 1st. Lay the edge of a ruler to touch the two places.

2d. Take their proper diff. of latitude, with a pair of compasses, upon the equinoctial, apply this distance to the edge of the ruler, so that when one foot is placed close to the ruler, the other foot may just touch some east and west line, crossed by that edge of the ruler; and there stay the compasses, the distance by the ruler's edge, from the place where the compasses rested; to that place where the ruler crosseth the aforesaid east and west line measured on the equinoctial, gives the true distance.

E X A M P L E. (Plate XIII.)

Let the two places be the *Lizard* and *Madeira*.

<i>Lizard</i>	50° 0'
<i>Madeira</i>	32° 17'
Difference of latitude	17° 43'

The ruler being laid upon the two places, take the difference of latitude 17° 43' from the equinoctial, and when applied to the ruler as before directed, one foot of the compasses will be in the point *b*, when the other will just touch the parallel of 50 in the point *c*; so the distance from *b* to the *Lizard*, measured on the equinoctial, will be near 20 degrees, is 400 leagues, the true distance from the *Lizard* to *Madeira*; and the course S. S. W. almost half W; the distance would be the same, if the difference of latitude were taken from the point *L*, to touch the parallel of 60 in *H*; for then *LM* would be the distance.

It must be observed that by the distance is meant, that which is made on the rumb line from one place to another, which will not be the nearest.

P R O B. IV. (Plate XIII.)

The course and distance given, also the latitude failed from, to find the difference of latitude, departure, and difference of longitude.

The difference of latitude and departure are found exactly as in the plain chart, which we omitted there, judging it proper to insert it here.

Let the place failed from be *A*, in the latitude of 30° 0' north, and the course N. E. b N. 192 leagues.

Rule. First thro' *A*, draw a meridian and parallel of latitude. Secondly, With the chord of 60 degrees, describe an arch, on which set off three points, the given course; and draw the line *AH*, which make 192 leagues, the given distance. Thirdly, From *H* let fall to the line *AC*, the perpendicular *HF*. *AF* measured on the equinoctial, or graduated parallel, will be 160 leagues, the difference of latitude; and *HF* measured on the same, will be nearly 107 leagues, the departure.

This may be done without delineating the triangle thus: First, lay a ruler thro' the given place *A*, parallel to some N. E. b. N line. Secondly, Lay off 192 leagues by the edge of the ruler from *A* to *H*. Thirdly, Thro' *H* lay the ruler parallel to an east and west line to intersect the graduated meridian; the distance from the ruler to 30° in the graduated meridian, will be the difference of latitude, observing to measure it upon

the equinoctial, the distance must likewise be set off from A to H, in equinoctial leagues. To find the departure. First, With a pair of compasses take the nearest distance of the point A, to any meridian. Secondly, set off that distance by the edge of the ruler, (now passing thro' H, and parallel to an east and west line); from the point where the ruler intersects that meridian; the distance of that point from H, measured on the equinoctial, will be the departure.

To find the difference of longitude, it will necessary again to remark, that if a ship sail any determinate distance upon any particular rumb, the difference of latitude and departure, will always be the same in whatever latitude she is in; for if the place sailed from were in the parallel of 38° , or 46° , the course and distance the same as before; the difference of latitude would still be 160, and departure 107, as in the triangle K L M, or O P R.

Now tho' she alters her latitude equally in both places, it will not be so in respect of the longitude.

By the plain chart, when she sails from 30° , the point H at which she arrives, will be in 38° ; and when she sails from 38° , the point M at which she arrives, will be in the parallel of 46° . But in *Mercator's* chart, the point H is in the latitude $36^{\circ} 41'$, and the point M in the latitude of $44^{\circ} 0'$; so that neither of these points is the true place upon *Mercator's* chart that the ship arrives at. In order to find the true place, produce the line A H, to intersect the parallel of 38° in the point K, produce also the line A F to G; K will be the place come to; K G, the difference of longitude; H F, the departure; A F, the proper difference of latitude; A G, the meridian difference of latitude: But tho' K be the place the ship is in, A K is not the true distance, but it may be found by the preceding problem: When she sails from the parallel of 38° , the true place O, at which she arrives, is found by producing the line K M to O, in the parallel of $46^{\circ} 0'$: Produce also the line K L to N, then will N O be the difference of longitude; L M, the departure; L K, the proper difference of latitude; K N, the meridian difference of latitude. The difference of longitude in the first, will be 388 miles; in the last it will be 433, being both measured upon the equinoctial; this shews the necessity of knowing betwixt what latitudes any departure is made, before the difference of longitude can be found; and that in order thereto the first thing to be done, is to find the departure, as directed in plain sailing, and then the difference of longitude may be found as here directed; the reason of which will appear by carefully examining the principles by which *Mercator's* chart is constructed. For tho' in this chart, the degrees of the parallels of latitude are apparently equal to the degrees
of

of the equinoctial. Yet if they be measured as directed in *Case 3.* of the preceding problem, they will be found to retain the same proportion to the degrees of the equinoctial in this chart, that they actually do upon the globe. Now the distance of any two places in any parallel is their departure, and the distance of their meridians on the equinoctial, is their difference of longitude equal to their apparent distance in the parallel on the chart; but when a ship alters both her latitude and longitude, the departure cannot be said to be made either in the latitude sailed from, nor in that come, at was observed before.

Let us then suppose the departure to be made in that parallel of latitude that lies in the middle between the two, viz. in 34° . Now a degree of the enlarged meridian at this parallel is 72.4; for 2207.8 is the meridional parts to $34^{\circ} 30'$, and 2135.4, the meridional parts for $33^{\circ} 30'$; the difference is $72^{\circ} 4'$, which multiplied by $5^{\circ} 21'$ (the degrees in the departure) gives 387 miles, or 129 leagues, the difference of longitude.

This is what is called middle latitude sailing, which tho' not strictly true, because the distances betwixt the parallels of latitude on the meridian are not in a continued geometrical proportion, yet in a short run there can be no considerable error; we shall therefore in the next section work the problems of *Mercator's* sailing both by the meridional parts, and middle latitude.

There are two lines on *Gunter's* scale, one of equal parts marked E P, which may serve to graduate the equinoctial; to this is adapted another line marked *Mer.* which serves to graduate the meridian; and by these *Mercator's* chart may be constructed. Now to find the distance betwixt any two parallels by these two lines; as for instance, betwixt the parallel of 30, and the parallel of 60, extend from 30 to 60 on the line *Mer.* measure this on the line E P, is 44 degrees; makes 2640 miles.

S E C T. III.

CONTAINING THE VARIOUS CASES.

Of MERCATOR'S SAILING. (Plate XIII.)

HAVING in the former section shewn how to construct *Mercator's* chart, and the use of it in navigation; we shall now shew how to find all that is necessary for the mariner to know without the chart; this is what is called *Mercator's* sailing, which presupposeth the knowledge
of

of plain sailing: The only defect of which, is that by the plain chart, the difference of longitude cannot be found, and tho' *Mercator's* chart gives the difference of longitude; the departure must be first found by the plain chart.

PROB. I.

The latitude and longitude of two places being given to find their bearing and distance.

EXAMPLE. (Plate XIII.)

I demand the course and distance from the *Lizard* to *Madeira*.

	Latitude N.	Longitude W.	
<i>Lizard</i>	50° 0'	0° 0'	50° 0' 3474
<i>Madeira</i>	32 17	12 8	32 17 2048
	17 43	12 8	merid. dif. of lat. 1427
	60	60	
Proper differ. of lat.	1063	728	difference of longitude.

To delineate this. 1st. Find by the table of meridional parts the meridional difference of latitude 1427, which set off from *L* to *l*. 2d. At *l* draw a perpendicular to *Lm*, on which set off the difference of longitude 728, and draw the line *lm*. 3d. Set off the proper difference of latitude 1063, from *L* to *b*. 4th. Draw the line *bb* parallel to *ml*; *bb* will be the departure; *Lm* the distance; the angle *lLm*, the course.

The triangles *Llm*, and *Lbb*, are similar; therefore *Ll*:*lm*::*Lb*:*bb*, that is the meridional difference of latitude is to the difference of longitude, as the proper difference of latitude is to the departure.

As the meridional difference of latitude	1427	3.154424
Is to the proper difference of latitude	1063	3.026533
So is the difference of longitude	728	2.862131
		<u>5.888664</u>

To the departure	542.3	2.734240
------------------	-------	----------

By *Gunter's* scale, the extent from 1427 to 1063, will reach from 728 to 542 on the line of numbers.

The departure being thus found, the course and distance may be found by the 6th case of plain sailing; or without the departure, the course may be found by this proportion, as the meridional difference of latitude is to the difference of longitude, so is the radius to the tangent of the course. The extent from 1427 to 728, or from 1063 to 542.3; if taken on the line of numbers, will reach on the tangent line from 45 to 27° 2'.

See the Operation.

As

As meridional diff. of latitude 14727	3.154424	} Prop. diff. of lat. 3.026533
Is to the radius or tangent of 45	10.000000	
So is the difference of longitude 728	2.862131	
To tang. of course 27° 2' S.S.W. $\frac{1}{2}$ W.	9.707707	
		Radius 10.000000
		Departure 12.734240
		Tangent 27° 2' 9.707707

The course being thus found, the distance may be found by *Case 2. of Plain Sailing*. Thus S. C. of 27° 2' R: difference of latitude 1063 :: distance : 1191.

To find the departure by the middle latitude, add the two latitudes, the half of which subtract from 90, gives the complement of the middle latitude; then R : S. C. latitude :: difference of longitude : departure.

Lizard	50.0	} As radius 10.000000
Maderia	32.17	
Sum	82.17	
Half	41.8	
Complement	48.52	Is to fine 48.52 9.876899
		Diff. of long. 728 2.862131
		Departure 548.2 2.738920

By *Gunter's scale*; the extend from 90 to 49 on the line of fines, will reach from 728 to 548 on the line of numbers.

The difference betwixt the departure, by this and the preceding, is only 6 miles, which is so small, that it will make no difference in the distance and bearings.

The departure may be found in the table of difference of latitude and departure: Thus look for the complement of the middle latitude as if it were a given course, and for the difference of longitude as if it were the distance sailed on that course; the departure corresponding to that course and distance, will be the true departure required. Here the complement of the latitude is 49 nearest, which find in the table. But the difference of longitude exceeds the distance in the tables, therefore take 100 seven times, and then 28; now against 100 in distance column, is 75.7 in the departure column, which multiplied by 7, is 528.5; to which add 18.4, the departure corresponding to the distance 28, makes 546.9 for the whole departure.

The reason of this is, because, if in a right angled triangle, if the angle be made the complement of the latitude, the hypotenuse will be the radius, and the base the sine of the angle; but the hypotenuse may be called the distance, and the base the departure.

P R O B. II.

Both latitudes and departure given, to find the difference of longitude.

Rule.

Rule. Find the proper and meridional difference of latitude, as in the former, and the proportion will be, as the proper difference of latitude is to the meridional difference of latitude, so is the departure to the difference of longitude.

E X A M P L E. (Plate XIII.)

A ship in latitude N. $49^{\circ} 10'$, and $15^{\circ} 22'$ W. longitude, sails N. N. E. $\frac{1}{4}$ E. till by a good observation she is in the latitude of $52^{\circ} 40'$. I demand the longitude come to.

Latitude come to	52.40	3731 meridional parts
Latitude sailed from	49.10	3397
Prop. diff. of lat. 70 lea.	3.30	334 merid. dist. of lat. 111 leagues

The departure by the tables will be found to be nearly 42 leagues.

To delineate this. 1st. Make the line ad 70, and ab 111, by any scale of equal parts, and draw the perpendiculars bc , and de . 2d. Make de 42, and draw the line ae , which produce to c ; then ae will be the distance; de the departure; bc the difference of longitude; ad the proper difference of latitude; ab the meridional difference of latitude.

Note. This is by a larger scale than that by which the chart is made.

As 70 proper difference of latitude	1.845098
Is to 111 meridional distance of latitude	2.045323
So is the departure 42	1.623249
	<u>3.668572</u>
To difference of longitude 66.6 leagues	1.823474

By *Gunter's* scale; extend from $49^{\circ} 10'$ to $52^{\circ} 40'$ upon the meridional line: This upon the line EP , will be 5 degrees, and something more than a half, makes 334 miles, the meridional difference of latitude: The proper difference of latitude is 210; the departure is 126. Then

Prop. di. lat. Depar. Mer. di. lat. Dif. long.

210 : 126 :: 334 : 200 nearly.

By the middle latitude SC mid. : lat. : R : dep. : D longitude.

52° 40'	} As sine of 39° 5'	9.799651
49 10		10.000000
101 50 sum		11.623249
50 55 middle latitude		1.823598
39 5 comp. middle latitude		

By *Gunter's* scale; the extent from the side of 39 to 90, will reach from 42, on the line of numbers, to $66\frac{1}{2}$.

By

By the table of difference of latitude and departure find 39 degrees, and in the departure column look for 42.2, against which in the distance column is 67.

There are several other problems commonly inserted in *Mercator's sailing*, which we shall omit; for all that is necessary to be known, is the latitude and longitude of the place, which may be had every day at noon; the latitude sailed from the day before, the latitude come to at noon, and the departure made the last 24 hours, are found as directed in plain sailing, and the difference of longitude by *Prob. 2.* of this; and the latitude and longitude being found, the course and distance to the port bound to, are found by *Prob. 1.*

And as to what is called parallel sailing, it is in effect the same as middle latitude; for when a ship sails in any parallel of latitude, the latitude sailed from, and that come to, are the same, and of consequence, the parallel the ship sails in may be called the middle latitude, and the distance sailed may be called the departure, by which the difference of longitude may be found as in the preceding problem; one example will be sufficient to illustrate this.

- Suppose two islands in the parallel of $50^{\circ} 55'$, and their distance in that parallel 42 leagues; required the difference of longitude.

It is plain that the distance here is the departure, which being the same as in the last problem, the operation will be exactly the same as in that; and if the difference of longitude were given, suppose $3^{\circ} 20'$, their distance in the parallel may be found by *Prob. I.*

In order to construct this geometrically, let the globe be supposed to be cut thro' the plane of the equinoctial, and the meridians and parallels of latitude projected orthographically upon this plane; the equinoctial and parallels would be concentrick circles, of which the pole would be the center; the meridians would all be strait lines intersecting one another in the center, and so the radius of the equinoctial would be one quarter of the meridian. The radius of any parallel of latitude would be the sine of the complement of that parallel, or its distance from the pole, and the sine of the parallel would be that part of the meridian intercepted betwixt the parallel circle and the equinoctial; this being premised. 1st. With the chord of 60, or sine of 30, describe an arch, or circle as in (*Plate XI. Fig. II.*) 2d. From any point H, set off the given difference of longitude 67 leagues, to F, and draw the chord H F, and radii C F and C H. 3d. With the sine complement of the latitude, viz 39 degrees from the center C, describe the arch R M; the chord R M will be the departure required. For the triangles C H F and C R M are similar,
K k there-

therefore CH , the radius, is to HF , the difference of longitude, as CR , the sine complement, is to RM , the departure. If you have no line of lines, lay off $50^{\circ} 55'$, the given latitude, by the chords from H , both ways upon the equinoctial, a ruler laid across by these two points, will intersect the meridian CH in R , and CR will be the radius of the parallel.

Having the distance in the parallel to find the difference of longitude.

To delineate this, is only the reverse of the former. 1st. With the sine complement of the latitude from the center C , describe the arch RM , making the chord RM equal 42 leagues, their given distance in the parallel. 2d. With the sine of 90 , or chord of 60 , describe from the center C another arch, and produce the lines CR and CM to intersect that arch in the points H and F , so shall HF be the difference of longitude.

We have now explained the fundamental principles of navigation, and shewn how to solve all the problems, and various cases of plain, *Mercator*, middle latitude, and parallel sailing, that are necessary for keeping a reckoning; and as to great circle sailing, it may be said to be impracticable, at least by any sea chart; for the arch of a great circle makes unequal angles with all the meridians; and how to describe such a curve upon a chart, wherein all the meridians are parallel to each other, seems if not impossible, at least so difficult, that the benefit arising from thence would not compensate the labour; for, supposing it actually described, there must be a new invention to direct a ship in that curve, for it cannot well be affirmed that it could be done by the compass. We shall therefore omit this, and proceed to the application of what has been said to the actual keeping of a reckoning, which shall be shewn in the next chapter.

C H A P. V.

To find the Latitude and Variation of the Compass by celestial Observation, and how to keep a Reckoning at Sea.

S E C T. I.

To work an Observation, and how to find the Zenith distance by DAVIS's Quadrant. (Plate XI. Fig. IV. and V.)

THIS instrument consists of two arches both drawn from one center H; to construct which, upon the point H raise H Z perpendicular to H O; with the radius H S describe an arch S F, which make 30 degrees; with the radius H G, describe another arch G K, which make 60 degrees; number the great arch, beginning at its intersection with the line H O, to 30 upwards; number the little arch from its intersection with the line Z H, increasing down to 60; so that both together make 90 degrees.

It has three vanes, one fixed immoveably at H, with a slit in it, this is called the horizon vane; another is fitted to move upon the great arch, with a hole, which must be put to the observer's eye, thro' which, and the slit in the horizon vane, the horizon must be seen; this is called the sight vane; the third is fitted to move upon the small arch, it is called the shade vane, because the sun throws its shadow upon the horizon vane: These two vanes must be so placed, that the observer may see the shade exactly upon the upper side of the slit, at the same time that he sees the horizon thro' the slit, and counting the degrees upon both arches, their sum will be the zenith distance.

To prove this; from the center H describe the semi-circle AZM $\oplus$ O to represent an azimuth circle; A the horizon; Z the zenith; $\oplus$ the sun.

Place the sight vane at 0 degrees, on the great arch, and when the horizon A is seen thro' the slit at H, the perpendicular Z H, from the zenith, will cut the little arch at 0 degrees: Let the sun be at $\oplus$; it is plain the angle Z H $\oplus$, is his zenith distance, which measured by the little arch, is 40 degrees, being the place where the shade vane is placed: But because the arch will not admit of being divided into minutes, let the shade vane be placed at 25 degrees, the instrument must be moved till the line

H $\oplus$; cut the little arch at 25 degrees, and then the line H O will cut the great arch at 15 degrees: Produce the line H S to M; it is plain the angle Z H M, added to the angle M H $\oplus$, will be the sun's zenith distance; to measure the angle Z H M, draw the line H G perpendicular to H S, it will cut the great arch at 0 degrees; the sight vane must be placed at K, to see the horizon A, thro' the slit which will be 15 degrees on the great arch, but the angle G H O is equal to the angle Z H M, for the angles M H G and Z H O are equal, being both right. Therefore taking the angle M H O from both, the remainings Z H M and G H O will be equal; therefore the degrees on both arches being added together, will be the zenith distance, which being had, the latitude will be found by *Chap. II. Sect. III. Prob. I.*

The best instrument for taking the altitude at sea is *Hadley's* quadrant, but as there is a small pamphlet explaining the nature, use, and the theory on which that curious and useful instrument is founded, given gratis with it wherever it is sold; it will be needless to give a description of it here.

After the latitude is thus found by a good observation, if it agrees with the latitude by the account, it may be presumed that your longitude by account is true; but if there be any considerable difference, it may be feared there will likewise be an error in the longitude; to correct which there can be no certain rule, because it is uncertain whether the error is in the course or distance; for it must always be supposed, that the artist has given all the proper allowances in casting up the day's work, and frequently examined the log line and glasses, and likewise taken all opportunities of examining the current, and comparing this with his former journals: If after all this the observed latitude, and that by account do not agree, the only thing that can be done, is to let the longitude go as by his account, or make a remark what the longitude would have been, provided the error was in the course, and supposing the distance true; and likewise what the longitude would have been, were the error in the distance, and the course true; so that it may be presumed one of these three may chance to hit.

If a ship be sailing due north or south, her difference of latitude, and distance, will be the same; and if they differ by observation, it is likely the error is in the half minute glass, or log line, but if she sails due east, or due west, she does not alter her latitude, but if by the observation it is found she has made any difference of latitude, there certainly must be error in the course, which may be owing to the steerage, or the compass, for the needle does not always point out the meridian, but varies sometimes to the eastward, and sometimes to the westward of it; and this is what is called the variation of the compass, and must be found before the course can be corrected.

SECT.

S E C T. II.

To find the Variation of the C O M P A S S.

WHEN any heavenly object, as a star, or the sun, is in the horizon, the point of the compass that it bears upon may be had by sights fitted to the common compass, and by an azimuth compass the degrees may be found, that the object is distant from the east, or west points of the horizon; this is called the magnetick amplitude, and if this agrees with the true amplitude, there is no variation; so the true amplitude must be found either geometrically, or by calculation.

It was shewn in *Chap. II.* how to do this geometrically; and to do this by calculation, the following problem will serve.

P R O B. I.

The latitude of the place, and the sun's declination given to find the amplitude.

E X A M P L E.

Given	Latitude	28° 16' N.
	Declination	15 24 N.

The proportion is

As the sine of 61° 44' the comp. of the latitude	9.944854
Is to the radius	10.000000
So is the sine of 15° 24' the declination	+9.424156
To the sine of the amplitude 17° 33'	9.479302

By this it appears that the sun then rises E. b N. $\frac{1}{2}$ N. nearly, but if by the compass it bears E. $\frac{1}{2}$ N. there will be a point variation to know whether the variation be easterly or westerly, take this general rule.

When you are looking to the sun to take the amplitude, if the magnetick be to the right hand of the true, the variation is westerly, as in this case; and to rectify the course steered, there must be one point taken to the left hand, if the course steered be N. b E. the true course will be N. which shews that the north point of the compass is a point to the westward of the true north, but if the magnetick amplitude be to the left hand of the true, then is the variation easterly; and in correcting the course, the variation must be allowed to the right hand of the course steered.

DE-

DEMONSTRATION.

Of the proportion for finding the true amplitude see *Plate XI. Fig. I.*

Let $P\ O$ be the given latitude, $P\ Z$ is the complement; thro' P draw $P\ p$, parallel to the horizon $H\ O$, draw the parallel of declination $E\ X$, to intersect the horizon in X : $C\ X$, will be the sine of the amplitude; $C\ x$ the sine of the declination, and $P\ p$ the sine complement of the latitude; the triangles $P\ p\ C$, and $C\ x\ X$, are similar, for the angles at p and x are both right; the angle at X is equal to the angle $P\ C\ p$, being each of them the complement of the latitude; therefore $P\ p$ the sine complement of the latitude, is to $P\ C$ the radius, as $C\ x$ the sine of the declination, is to $C\ X$ the sine of the amplitude.

When the sun is risen any considerable height above the horizon, it will not be easy to find its true bearing by the compass, but, if it is within 10 or 12 degrees of the horizon, it may be had by an azimuth compass.

The sun's azimuth is an arch of the horizon, contained between the south or north points of the horizon; and an azimuth circle passing thro' the center of the sun to find this geometrically was shewn in *Chap. II.*

And to do it by calculation the following problem will serve.

P R O B. II.

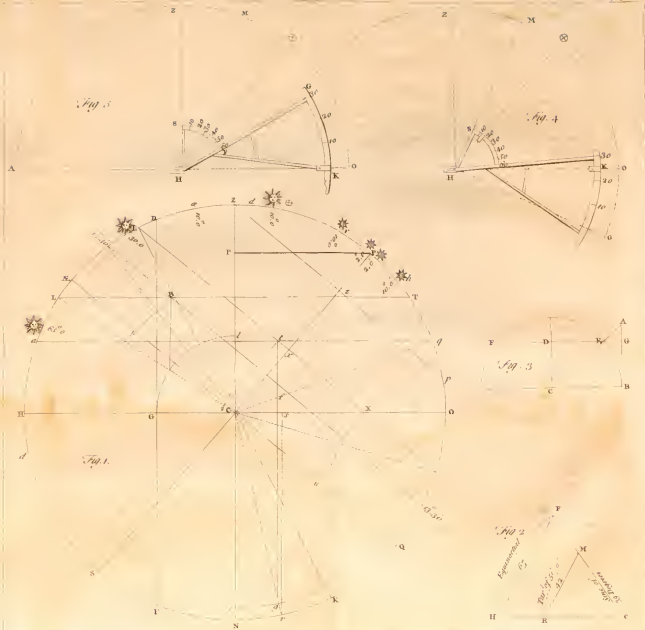
The latitude of the place, the sun's declination, and altitude being given, to find the azimuth.

E X A M P L E.

Latitude $28^{\circ} 16'$ N. declination $15^{\circ} 24'$ N. altitude $9^{\circ} 3'$.

Rule. 1st, Take the complement of the altitude; the complement of the latitude; the complement of the declination; and add these three together. 2dly, Take half this sum, from which subtract the complement of the declination. 3dly, Take the sine of this remainder, and the sine of the half sum from the logarithms. 4thly, Take the sines of the complement of the altitude, and the complement of the latitude from logarithms, and subtract each of them from the logarithm of the radius; the remainders will be the complements arithmetical of those sines.

Now these four must be added together, *viz.* the complement arithmetical of the sines of the complement of the altitude, and of the complement of latitude, and the sines of the half sum, and remainder. Half the sum of these four logarithms is the logarithm of the sine complement of half the azimuth required.



80° 30'	Comp. altitude comp. arithmetical of the sine	0.005997
61 44	Comp. latitude, comp. arithmetical of the sine	0.055146
74 36	Complement declination	
216 50	Sum of the three	
108 25	Half sum; from 180 remains 71° 35'.	Sine 9.977167
74 36	Complement declination	
33 49	Remainder	Sine 9.745494
	Sum of the four	19.783804
38° 46'	Is the comp. of 51° 14', of which the sine is $\frac{1}{2}$ the sum	9.891902
38 46		
77 32	The sun's azimuth from the north	

Note. If the declination be south, and latitude north, and the contrary, instead of taking the complement of the declination you must add 90 thereto, and proceed as before.

The demonstration of this problem depends upon the doctrine of spherick trigonometry; for here are three sides given, *viz.* the complement of the altitude, the complement of the latitude, and the sun's distance from the elevated pole; this last will be the complement of the declination, when the latitude and declination are both north, or both south; but if one be north, and the other south, 90 must be added to the declination; the three sides being given, the angles are found as in the preceding operation.

Having thus found the true azimuth, the magnetick is found by observation; the same directions as were given in the amplitudes, will serve to know whether the variation be easterly or westerly.

S E C T. III.

How to keep a Reckoning at Sea in order to know at any time the Latitude and Longitude the Ship is in, and the Course and Distance to any Port.

THE regular method of doing this, is by keeping an exact account of the various courses, and distances sailed in 24 hours; for which reason the log is hove every hour, and the distance and course set down in proper columns, in a book provided for that purpose, which is called the log-book, ruled and column'd as explained in the following pages.

In

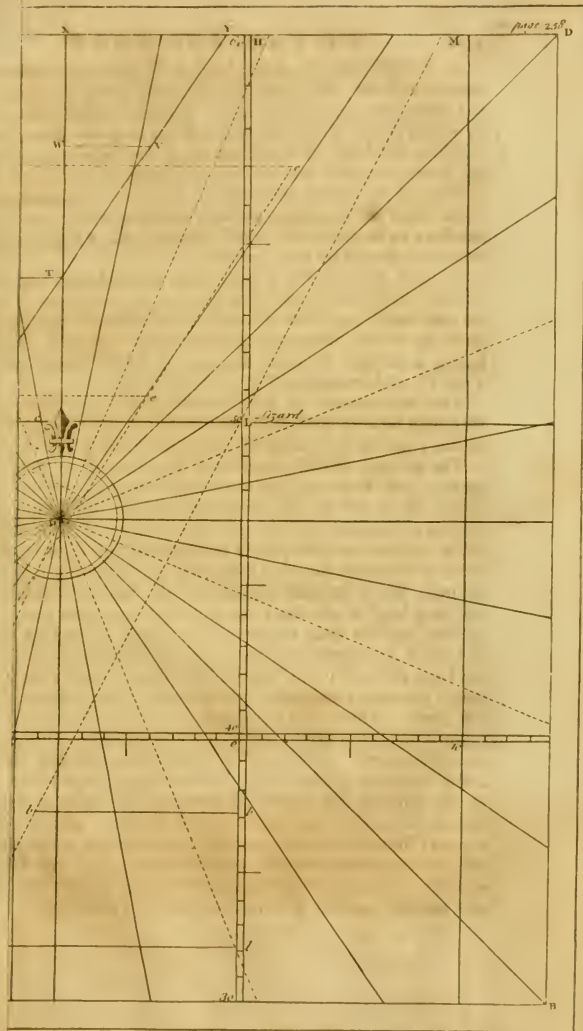
In the first column, at the head, is H for hours, under which are set down the hours; the second has K at the head, in which are set down the knots run out at every hour; the third column has F at the head, in which are set down the odd fathoms at every hour; the fourth has course at the head, in which are set down the courses corresponding to the hours; the fifth has winds at the head, in which are set down the shiftings of the winds; the sixth has remarks at the head, in which is set down what sail is carried, and the weather, which are two things very necessary to be observed; for if it blows hard a ship cannot make good the course steered by the compass.

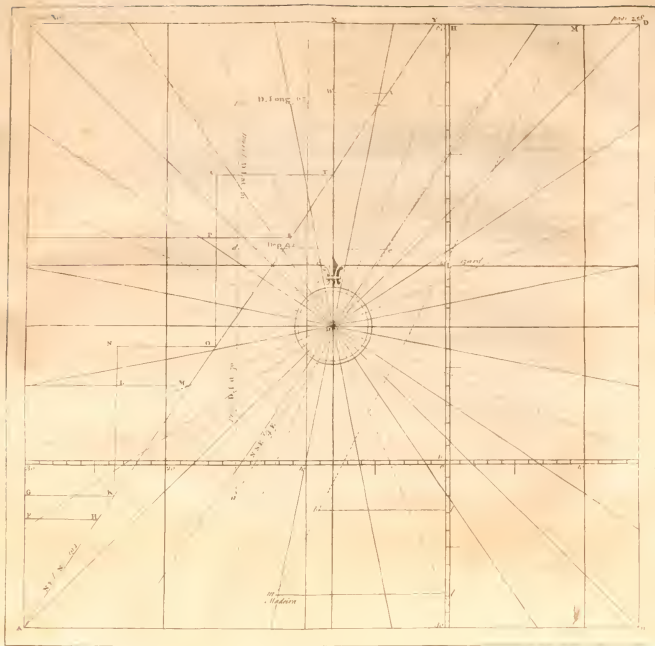
In some ships the log is hove once in two hours, and set down upon a board, from which every one that keeps an account, transcribe it into his own book, and then reduces all the different courses into one, and finds the whole difference of latitude and departure made the last 24 hours, as directed in plain sailing, and the difference of longitude by *Prob. II. of Mercator's*.

Tho' this account be kept by the greatest care and exactness, there will often be very great errors discovered in the latitude, by good observations; and in the longitude when any known land is made.

The occasion of these errors may be attributed to these four following causes. 1st. If the log line and glass be not duly proportioned to one another, there will be an error in the distance; for admitting the glass to be only 29 seconds, and the line right divided, the ship would be a head of the account the 30th part of the distance; for there is only an account taken of what she runs in 29 seconds, and what she runs in the 30th is omitted; but if the glass be true, and the line short divided, the account will be a head of the ship; for supposing the 120th part of a mile to be 50 foot, and the space between the knots to be only 45 foot, it is plain that every 50 leagues run by the account, is only 45; so that the ship must run the 10th part of the whole distance, more than by the account, before she makes the land. 2d. Is owing to the variation of the magnetic needle, which does not always point out the meridian; this will occasion a great error in the course, if the variation is not known, and allowed in the course. 3d. Is owing to the lee-way a ship make; for when a ship sails close by the wind, she does not make good the course steered by the compass, but falls to the leeward of it, more or less according to the sail she carries, and the height of the sea. 4th. Is when there is a current; for when that sets upon the same rumb on which the ship is steered, her real distance will be more than by the log; and if it sets upon the opposite rumb, it will be less; and if it sets athwart the ship's way, it will occasion an error both in the course and distance.

LOG-





H.	K.	F.	Courses.	Winds.	REMARKS, Saturday, May 27, 1738.
1					<i>Lizard</i> 50° 0' north latitude, and 0° 0' longitude.
2					
3					<i>Lizard</i> N. W. 4 leagues distance, fine moderate gale, and pleasant fair weather.
4	5	3	S. W.	N. b E.	
5	6	4		N.	
6	7			fresh gale	Handed all our small sails, and reefed both top-sails.
7	8			N. b W.	
8	7			N. N. W.	A great sea from the N. W.
9	6	3		N. W.	Double reef M.T.S. and handed. F.TS.
10	5	4		N.W.b.W.	
11	5			W.N. W.	
12	4	3			
1	4	4		W.	Drizzling rain.
2	4		S. S. W.		
3	5				Set fore top-sail.
4	5	4	S. W. b S.	W. b N.	Fine clear weather, out all reefs, and set the small sails.
5	5	3		N.W.b.N.	
6	5				
7	6				At noon clear weather, had a good observation.
8	7				
9	6	3			Zenith distance 25° 9'
10	6	4			Declination 22 47
11	6				Latitude come to 47 56
12	5				

Cours.	Diff.	S.	W.	Lati. by account	Lati. by observa.	Diff. of longi.	Longi. come to	Variation	Merid. distance.
S. 16 W.	124	118	33	48° 2'	47° 56'	51'	0° 51'	1½ point	<i>Lizard</i> 33 miles from the
Courses correc.							Latitude by observation		47° 56'
S. E. b E. ½ E.	12	6.2	10.3	0			Latitude sailed from		50° 00'
S. S. W. ½ W.	46	39.5	0.0	23.6					97 56
S. b W. ½ W.	62	58.4	0.0	20.9					
S. ½ E.	14	14.1	0.7	0.0			Middle latitude		48 58'
		118.2	11.0	44.5			C. middle latitude		41 02'
									90 00
							Degrees.	Dep.	Dif. of long.
							41	33.5	51

Lati. sailed from 50.00
 Difference of lat. 1 58
 Latitude come to 48 02

AFTER the log-book is thus copied, the next thing to be done is to reduce all the various courses into one, so that the whole difference of latitude, and the whole departure may be found; this is what

is called working a day's work. In order to which, it will be proper to make a traverse table, upon a slate or waste paper, as directed in plain sailing. But it must be observed the courses must be corrected, by allowing for variation and lee-way. Now, as our account is to begin from the *Lizard*, we must suppose the ship to sail S. E. from it, to the place she was in at 3, when it bore N. W. and because there is one point, and $\frac{1}{2}$ variation, the course corrected will be S. E. b E. $\frac{1}{4}$ E. The next course steered is S. W. allowing variation, makes S. S. W. $\frac{1}{4}$ W. but when the wind came to W. N. W. I allow 1 point lee-way, makes it S. b W. $\frac{3}{4}$ W. when the wind comes to west, she lies only S. S. W. and allowing for variation and lee-way, she makes only S. $\frac{1}{4}$ E. course; the last course steered is S. W. b S. the wind being large, makes no lee-way, allowing the variation; true course is S. b W. $\frac{1}{4}$ W. The courses being thus corrected, against the first set down 12 miles, the distance from the *Lizard*; against the next is 46, being what the ship has run by the log from 3 to 11; against the next is 62, being what she has run from 11 to 2; and from 5 to noon; against S. $\frac{1}{4}$ E. is 14 miles, what she sailed from 2 to 5: We may then find the proper difference of latitude and departure to each course, by the table; the sum of the southings is 118 miles, that is $1^{\circ} 58'$, and because the latitude is decreasing, this must be subtracted from the latitude of the *Lizard*, which makes the latitude by account $48^{\circ} 2'$, but by observation $47^{\circ} 56'$; the whole westing is 44.5, and easting is 11, so the departure is 33.5 west. And by this, to find the difference of longitude, add the latitude sailed from, and that come to, into one, and subtract half that sum from 90, the remainder is $41^{\circ} 2'$, the complement of the middle latitude. Again look in the table of difference of latitude and departure for 41° , and look for the departure 33.5, in the proper column, against which, in the distance column, is 51, the difference of longitude. To find the latitude by the observation, look for the declination corresponding to the day of the month, which is $22^{\circ} 47'$, added to the zenith distance $25^{\circ} 9'$, makes $47^{\circ} 56'$. After finding the whole difference of latitude and departure, because the numbers exceed those in the tables, I take half of each, which I find in the column corresponding to 16° against 62 distance, which doubled, makes 124, as in the operation under the log; the like process must be used every day at noon.

H.

SECT. III. Log on board the SEA-HORSE, Anno Dom. 1738. 261

H.	K.	F.	Courses.	Winds	REMARKS, Sunday, May 28, 1738.
1	7		S.W.b.W.	N. E.	Moderate gales and fine pleasant weather.
2	6				
3	5				
4	5				
5	4		S. W.		Little wind with small drizzling rain.
6	3				Tried the current
7	2				Found it S. S. W. one mile in an hour.
8	1				
9					
10				Calm	Fresh gales, handed all our small sails.
11					At noon clear, zenith distance $22^{\circ} 48'$
12	1			S. E.	Declination $22^{\circ} 53'$
1	2			S. S. E.	Latitude by observation $45^{\circ} 41'$
2	4		S. S. W.		At sun rising, by a good observation,
3	5	3			amplitude $23^{\circ} 23'$, in order to find
4	6	4			the true amplitude, find what difference
5	8				of latitude the ship made from
6	9				sun rising till noon.
7	9				Course corrected S. b W. distance 70,
8	9				difference of latitude $68.7'$
9	9				Latitude at noon $45^{\circ} 41'$
10	9				Diffe. of lati. since sun rising $14.9'$
11	9				Latitude at sun rising $46^{\circ} 50'$
12	9				Complement of the latitude $43^{\circ} 10'$

Course.	Dif.	S.	W.	Lati. by account	Lati. by observa.	Diffe. of longi.	Longi. come to	Variati- on	Meridio. distance from the
S 18° W	144	136	45	$45^{\circ} 40'$	$45^{\circ} 41'$	$1^{\circ} 2'$	$1^{\circ} 53'$	1 point	$1^{\circ} 18'$

Courses corrected.	Dif.	S.	W.	Sine of $43^{\circ} 10'$ comp. lati.	9.83513
S. W.	23	16.3	16.3	Is to radius	10.00000
S. W. b S.	13	10.8	7.2	Sine of declination $22^{\circ} 53'$	9.58979
S. b W.	87	85.3	17.0	Sine of amplitude $34^{\circ} 38'$	9.75460
Current S. b W.	24	23.5	4.7	Magnetick	23 23
Cour. good S. 18° W.	144	135.9	45.2	Variation	11 15

Latitude yesterday at noon $47^{\circ} 56'$

This day $45^{\circ} 41'$

$23^{\circ} 37'$

Middle latitude $46^{\circ} 48'$

Comp. of middle latitude $43^{\circ} 12'$

Degrees. Depart. Dif. of long.

43 45.3 62

L 12

H.

H.	K.	F.	Courses.	Winds.	REMARKS, Monday, May 29, 1738.
1	8		S. W.	N. b. E.	Fresh gales and cloudy most part of these 24 hours.
2	7	3			Found the sun's azimuth by a good observation $75^{\circ} 30'$ after the sun's rising; his altitude was then $9^{\circ} 30'$, the true amplitude is $67^{\circ} 4'$, which makes the variation $8^{\circ} 26'$, as by the following operation.
3	6	4			
4	6				
5	5	4			
6	5	3		N.	
7	6				
8	6		S. W. b. W.	N. N. W.	
9	6				
10	5	4			Com. alti. $80^{\circ} 30'$ } Ar. com. 0.005997
11	5	3			Com. lat. $45^{\circ} 34'$ } Of fines 0.146262
12	6				Com. dec. $67^{\circ} 2'$
1	6	4	The latitude at the time of observation of the azimuth $44^{\circ} 26'$		Sum $193^{\circ} 6'$ 0.152259
2	8				Half sum $96^{\circ} 33'$
3	7				Supple. $83^{\circ} 27'$
4	7	4		$\frac{1}{2}$ sum less	Com. dec. $29^{\circ} 31'$ } their fines } 9.997156
5	7	3			Sum of the logarithms 19.841977
6	8				Half 9.920988
7	9				Sine com. half azim. $56^{\circ} 28'$
8	8	3			Complement $33^{\circ} 32'$
9	8				True azimuth from the N. $67^{\circ} 4'$
10	8				West variation $8^{\circ} 26'$
11	8		Cloudy, no observati.		Magnetick azimuth $75^{\circ} 30'$
12	8				

Course.	Dif.	S.	W.	Lati. by account	Lati. by observ.	Diffe. of longi.	Longi. come to on.	Variation.	Veridic. distance from the Lizard.
S. W.	168	118	118	$43^{\circ} 43'$		$2^{\circ} 46'$	$4^{\circ} 39'$	$\frac{1}{2}$ point.	$3^{\circ} 16'$

Courses corrected.	Dif.	S.	W.	Points.	$\frac{1}{2}$ Depart.	$\frac{1}{2}$ Dif. of long.
S. W. b. S. $\frac{1}{4}$ W.	45	36.1	26.8			83
S. W. $\frac{1}{4}$ W.	123	82.6	91.2	4	59	2
S. W. dist.	168	118.7	118.0			
						Dif. of long. 166

Latitude yesterday at noon	$45^{\circ} 41'$
Diffe. of lati. these 24 hours	$1^{\circ} 58'$
Latitude come to	$43^{\circ} 43'$
Latitude failed from	$45^{\circ} 41'$
	$89^{\circ} 24'$

Middle latitude	$44^{\circ} 42'$
Comp. of middle latitude	$45^{\circ} 18'$

The preceding three days will be sufficient to shew the manner of taking off and working the log, as the operations for finding the latitude by the zenith distance, and the variation of the compass by an amplitude and azimuth, are there set down at large. We have also shewn how to correct the course, by allowing for lee-way and variation; and how to account for a current. After working each day's work in the log-book, they may from thence be transferred into a journal; the form of which is hereunto annexed.

Journal of a Voyage, intended by God's assistance, in the Ship Sea-Horse, from London to Jamaica, under the Command of A. B. in the Year 1738.

Weeks day.	Months day.	Course made good.	Dist. in miles.	Latitude by account.	Latitude by observation.	Longitude.	Meridional distance from the Lizard.	Variation.	Winds.
	May								Yesterday at 3 P. W. Lizard, N. W. 4 leagues, the first part fresh gales and a great sea, the latter moderate and clear, N. to W.
h	27	S. 16° W.	124	48° 2'	47° 56'	0° 51'	0° 33'	1½ point	N. E. to S. E. fresh gales, and clear the latter part.
⊙	28	S. 18° W.	144	45° 40'	45° 41'	1° 53'	1° 18'	By am. 11° 15'	N. E. E. to N. N. W. fresh gales and cloudy.
Ⓜ	29	S. W.	168	43° 43'		4° 39'	3° 16'	By azi. 8° 26'	

The following characters are generally used to express the days of the week.

⊙ Sunday; Ⓜ Monday; ♂ Tuesday; ♀ Wednesday; ♄ Thursday; ♀ Friday; ♄ Saturday.

S E C T. IV.

Of the Moon's Age, and Time of High Water.

FROM what has been said, it is plain, that if the account of the Journal be true, the ship will arrive at her designed port, by steering such a course as the Journal directs, and in order to sail into the harbour, if in a tide way, the mariner should know what time it will be high water; but as this is governed by the moon, it follows, that to attain this, the first thing to be done is, to find her age.

If the months all contained an equal number of days, and the change of the moon was always on the last day of every month, the day of the moon would then be the same with the day of the month, and we should have exactly twelve compleat moons every year; but it has been found by a long series of good observations, that every year contains twelve compleat moons and eleven days more, very nearly, so that if the moon happens to change any year the last day of *December*, it will be eleven days past the change on the last day of the *December* following; and twenty-two days after the change, the succeeding year, and the third year it will be thirty-three days; but as there are but 29½ days from the change of the moon till it changes again, it is plain that in three years time, which contain 36 months, we shall have 37 compleat moons, and three days more; so that the moon will be, on the last day of *December*, in the third year, 3 days after the change; and on the fourth it will be 11 days more, that is, the last day of *December* will be 14 days after the change. Now it will be very easy to find the moon's age any day of the month provided the moon's age be known the last day of the preceding year, our first business then shall be to shew how this is found.

As every year contains 12 moons and 11 days, every three years will contain one whole moon more than months, and three days more, so that 18 years will contain six whole moons more than months, and 18 days more; that is to say, if the moon changes on the last day of *December*, it will be in 18 years afterwards, 18 days past the change on the last day of *December*; and the year following, viz. the 19th year, it will be 11 days more, which makes 29 days, and this being a whole moon except half a day, we shall have new moon some time of the last day of *December*; so that at the expiration of 19 years the new and full moons happen on the same day of the same month they did 19 years before that.

This revolution of 19 years is called the lunar cycle, or the *Metonic* cycle, from its author *Meton* the *Atbenian*. The new moon, or the change,

was

was on the last day of *December*, two years before the birth of our Saviour, so that every nineteenth year from that time the moon changed on the last day of *December*, and the year of our Saviour's birth was the second year of the cycle. Hence it is manifest, that if we add 1 to any year since our Saviour's birth, and divide the sum by 19, the quotient will shew how many cycles have past since its first commencement, and the remainder will shew what year of the cycle that is, which is called the golden number, or prime for that year; it finds the the age of the moon on the last day of the preceding year, or the number of days past since the new moon; this number is called the epact, and since the epact of the first year is 11, of the second 22, of the thire 3, and so on constantly increasing by 11, as was before observed, it is evident that to find the epact for any year we must multiply the golden number by 11, the product if less than 30 will be the epact for that year, if it exceed 30 divide it by 30, and the remainder will be the epact.

E X A M P L E I.

Required the Epact for the Year 1730.

First find the golden number by the preceding rule.

To the year 1730

add

divide by 19)1731(91

golden number $\frac{1729}{2}$

golden number

multiplied by

product is the epact $\frac{11}{22}$

E X A M P L E II.

Required the Epact for the Year 1744.

1744

1

19)1745(91

1729

16 golden number

16

11

30)176(5

150

26 epact

Now as the epact expresses the age of the moon on the last day of *December*, it is plain that if the moon changes on that day in any year, it will change on the 20th of *December* the next year, because on the last day of *December* it will be 11 days past the change; but if this 20th day be called the 31st, as was the case in the year 1753, when the style was altered, it will make an alteration in the epact of 11 days: Therefore to find the epact since the commencement of the new style, we must divide

divide the year without adding 1 to it, by 19, the remainder will be the golden number, which multiplied by 11 will give the epact as before.

EXAMPLE III.

Required the Epact for the Year 1754.

19)1754(92

171

44

38

6 golden number

golden number 6.

multiplied by 11

product 3066(2

60

epact 6

It is plain if the moon changes on the last day of any year, the day of the moon will be the same with the day of the month in *January* following till the change; and because the time betwixt one new moon and the next is 29 days and an half, the 30th day of *January* will be the first day of the next moon, and the first day of *February* will be the third day of the moon, and consequently if we add two to any day of the month in *February* that year, it will give us the day of the moon; now as in common years *February* has but 28 days, if to this, 2 be added, it will make 30, which is half a day more than the moon contains; so that it will change on the last day of *February*, and therefore the first day of *March* will always be the same day of the moon that the first day of *January* is, and the first day of *April*, the same as, the first day of *February* (except in leap years, when one day more must be added to the first day of *March*), for if the moon changes on the last day of *February*, it will change again before the 30th of *March*, and so the first day of *April* will be the 3d day of the moon, and the 27th day of *April* the 29th day of the moon; so the 28th day of *April* will be the first day of the moon, and the first day of *May* will be the 4th day of the moon.

Now, though the moon does not change the last day of *December*, the epact gives the age of it on that day, and therefore if the epact be added to the day of the month in *January*, the sum will be moon's age; but in *February* we must add 2 to the epact and day of the month, to find the moon's age; in *March* again, except in leap years, the epact and day of the month will give the moon's age. Hence this general rule will serve to find the moon's age on any day of the month.

Rule. Add the epact to the day of the month, and the number for that month, the sum if less than 30 is the moon's age, if it exceeds 30 take 30 from it, and the remainder will give the moon's age.

The

The following numbers must be added in the months to which they correspond.

<i>January</i>	0	<i>April</i>	2	<i>July</i>	5	<i>October</i>	8
<i>February</i>	2	<i>May</i>	3	<i>August</i>	6	<i>November</i>	10
<i>March</i>	0	<i>June</i>	4	<i>Sept.</i>	8	<i>December</i>	10

E X A M P L E I.

Required the moon's age the 24th day of *January* 1754.

The epact by the preceeding rules will be found to be 6, the number for the month is 0, now we have only 6 to add to the 24, which makes 30, which being half a day more than a whole moon, the moon changes some time that day.

E X A M P L E II.

Required the moon's age the 26th of *April* 1754, to 26 add 2 for the month, and the epact 6, the sum is 34, from which subtracting 30, the remainder is 4, the moon's age.

After finding the moon's age we may thereby find the time of high water from the following principles.

It has been observed that when it is high water in any port, the moon will always be on the same point of the compass; and as the moon in 24 hours moves through all the points of the compass, it is plain she must take 45 minutes in moving from any point to the next; for 32, the points of the compass, multiplied by 45, gives 1440, the minutes in 24 hours; hence, if it is high water in one port at 12, and the moon then on the south point of the compass, and high water in another port, when the moon is on the S. S. W. point, it will be 1 hour and 30 minutes after 12 when it is high water at this last place.

Now, if the moon and the sun were always on the meridian at the same time, the moon would always come to the same point of the compass at the same hour of the day, and of consequence it would be always high water at the same hour; but since the moon comes to the meridian with the sun only on the day of the change, which happening only once in 30 days, it will from thence follow that the difference of the time of her coming to the meridian from the day of the change, or any day of her age, to the next day, will be 48 minutes; for, 30 the days in the moon, multiplied by 48 gives 1440, the minutes in 24 hours, at which time the moon will again come to the meridian with the sun, and will then be on the south side of the compass; the first day after the change it will be 48 minutes after 12, before the moon comes to the meridian, or south point

point of the compass; the second day 1 hour 36 minutes; and as it is thus 48 minutes every day later in coming to the south point of the compass, it will be so with respect to any other point, which is the reason that it is high water in any port 48 minutes later every day than it was the preceding.

From what has been said, it is manifest, that before the time of high water can be found on any day of the moon, two things must be known; first, on what point of the compass the moon will be every day at high water; secondly, at what time the moon will come to the meridian on that day. As for the first, which is called the flowing, it must only be had by experience, and for the second, which is called the moon's southing, it will always be found by multiplying the moon's age by 48, and dividing the product by 60, the quotient will give the hour, and the remainder the minutes the moon is on that day later of coming to the meridian than the sun. Now these two being given, if to the hours and minutes of southing we add the hours and minutes corresponding to the flowing, that is, to the point of compass on which the moon is at high water, the sum is the time of high water on that day.

E X A M P L E.

Required the time of high water at *London-Bridge, Feb. 27, 1754*, the flowing S. W.

Because when the sun is on the south point of the compass it will be 12 hours, that is, either noon or midnight, it will be 3 hours after noon when the moon is on the S.W. point of the compass; for in 45 minutes she moves from one point to the next; S.W. is 4 points from the south, and 4 times 45 is 180 minutes, which is three hours. The age of the moon on that is, 3 multiplied by 48 is 144, divided by 60 is 2 hours and 24 minutes, the moon's southing that day, to which adding 3 hours the flowing, the sum will be 5 hours and 24 minutes, the time of high water required.

F I N I S.

THE
ELEMENTS
OF
NAVAL ARCHITECTURE:
OR, A
PRACTICAL TREATISE
ON
SHIP-BUILDING.
LATELY PUBLISHED AT PARIS.

By M. DUHAMEL du MONCEAU,
Inspector General of the Marine to his most Christian Majesty, Member of the Royal Academy of Sciences at Paris, and Fellow of the Royal Society at London.

CAREFULLY ABRIDGED
By MUNGO MURRAY.

L O N D O N:

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M,DCC,LIV.

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A

PRACTICAL TREATISE

ON

SHIP-BUILDING.

C H A P. I.

General Proportions for Building.

NAVAL Architecture may be divided into three principal parts.

- I. To give the ship such a figure or exterior form, as may suit the service she is designed for.
- II. To find the true form of all the pieces of timber that shall be necessary to compose such a solid.

- III. To make proper accommodations for guns, ammunition, provisions, and apartments for all the officers, and likewise for the cargo.

We shall at present only treat of the first of these, namely the exterior figure, and consider it first, as it regards the bottom, that is, the part which lies under water, and may be called the quick-work ; or secondly, the part which is above water, and may be called the dead-work.

In order to give a proper figure to the bottom, all the qualities which are necessary to make a ship answer the service for which she is design'd, should be considered. A ship of war should carry her lower tier of guns four or five feet out of the water. A ship for the merchants service should stow the cargo well, and both of them should be made to go well, carry a good sail, steer well, and lie too easily in the sea.

Some

Some eminent geometricians have endeavoured to find the form of a solid which may best answer all these qualities, and meet with the least resistance in dividing the fluid through which it is to pass; but have not been able to reduce their theory to practice by reason of the different positions a ship is obliged to be in when under sail. The ship-builders despairing to establish this point by mathematical rules, have applied themselves wholly to their own observations and experience, which may indeed supply the deficiencies of art, but though they may thereby discover that a ship has several bad qualities, it will not be easy to determine where the fault lies; for it may be owing to the rigging; and though the fault be not there, yet they cannot be certain in what particular part of the body it is. If their observations be assisted by principles drawn from theory, it will conduce very much to attain their end.

As there have been several ships built which have seemed to answer all the services for which they have been designed, some builders have made it their principal study to copy ships which have gained the applause of the seamen. This method they very improperly call the principal rule which should be observed in building. Now, as the bodies of ships are very different from one another, so there are, by this means, as many different methods used; some chusing one, and some another for a standard. But it must be observed, that even though it were possible to find such a body as should give intire satisfaction, and have all the good qualities that should be necessary to answer the services proposed, yet this could by no means be established as a standard by which other ships of different dimensions may be built. For admitting we have a first rate of 100 guns, which by experience has been found to be a very good ship in all respects, yet we should find ourselves very much deceived, if we should build a ship of 20 guns by making all the parts have the same proportion to one another that they have in that of 100 guns.

The first thing to be done in order to lay down the draught of a ship is to determine the length, which should be either on the lower gun deck, or at the load-water line; for there must be great care taken that there is a sufficient space betwixt the ports. This will oblige us first to fix the number and dimensions of the ports, the distance of the aftermost port from the transom, and of the foremost from the stem, and the distance betwixt the ports. This article may be determined by the following tables:

A Table

A Table of the Number of Ports on each Side of a Ship, according to the Number of Guns, and the Weight of the Shot.

A Ship of 112 Guns.			A Ship of 102 Guns.			A Ship of 74 Guns.		
Decks	Ports	Shot	Decks	Ports	Shot	Decks	Ports	Shot
1	15	48 or 36	1	14	36	1	13	36
2	16	24	2	15	18		14	
3	15	12	3	14	12	2	14	18
Quarter	5	8	Quarter Forecastle Poop	13	6	Quarter Forecastle Poop	15	
Forecastle	3	8					8	8
Poop	2	4					8	
						2	4	
A Ship of 64 Guns.			A Ship of 58 Guns.			The Tiger.		
Decks	Ports	Shot	Decks	Ports	Shot	Decks	Ports	Shot
1	12	24	1	12	18	1	11	18
	13		2	13	12	2	12	8
2	13	12	Quarter Forecastle	4	4	Quarter	3	6
	14					Forecastle	2	4
Quarter	7	6						
Forecastle	5							
A Ship of 50 Guns.			A Frigate of 46 Guns.			A Frigate of 32 Guns.		
Decks	Ports	Shot	Decks	Ports	Shot	Decks	Ports	Shot
1	11	18	1	11	12	1	10	6
2	12	12	2	12	8	Quarter	6	4
Quarter	2	4				Forecastle		
A Frigate of 32 Guns.			A Frigate of 32 Guns.			A Frigate of 28 Guns.		
Decks	Ports	Shot	Decks	Ports	Shot	Decks	Ports	Shot
1	4	8	1	10	8	1	3	8
2	10	6	2	6	6	2	10	4
Quarter	2	4				Quarter	1	4
Forecastle								
A Frigate of 24 Guns.			A Frigate of 22 Guns.			Frigates of 20 Guns have		
Decks	Ports	Shot	Decks	Ports	Shot	10 Ports on each side		
1	10	6	1	9	6	on one Deck. Shot 6 lb		
Quarter	2	4	Quarter	2	4			

Vessels of 16 guns have 8 ports on one deck, the guns to carry 4 lb. shot.
Vessels of 12 guns have 6 ports on one deck, the guns to carry 4 lb. shot.

A Table of the Dimensions of the Ports and Height of their Sells, according to the Weight of the Shot.

Shot lb.	Hei. and brea. of ports.						Height of the ports		Sells		
	f.	in.	f.	in.	f.	in.	1st Deck	2d Deck	3d Deck	Quarter-Deck and Forecastle	
48	2	9	3	0	or 3	2	f. in.	f. in.			
36	2	8	3	0	or 3	1	2	2			
24	2	5	2	8	or 2	10	2	1	2	0	
18	2	4	2	7	or 2	8	1	11	1	9	
12	2	2	2	4	or 2	6	1	10	1	8	
8	1	9	1	11	or 2	3	1	8	1	6	1 5
6	1	6	1	8	or 1	11	1	7	1	6	1 5
4	1	4	1	6	or 1	9	1	5			1 3

A Table of the Number, Dimensions and Distances betwixt the Ports on the lower Deck; also the Distance betwixt the foremost Port and the Stem, betwixt the Aftmoft and the Sternpoft.

Ships Names.	N ^o of Ports		Breadth of Ports		Dist. betw. Ports	Foremoft from Stem		Attmoft from Poft		Length on L. Deck
Amiable	13	2	8		7 6	13	4	9	0	147
Invincible	13	2	8		7 4	12	4	9	0	144
Achilles	12	2	8		7 8	18	2	10	6	145
Toulouse	12	2	8		7 6	17	4	9	2	141
Ardent, 64 guns	12	2	8		7 6	17	0	9	2	140 8
Fleurion, 64 guns	12	2	8		7 8	18	10	10	6	145 8
Dauphin Royal 74 guns	13	2	10		7 7	18	2	10	0	156

Note, An inch *French* measure is equal to $1\frac{1}{8}$ inch *English*, and is divided into twelve parts called lines, which are divided into twelve parts, called points.

The next thing to be done is to establish the breadth by the midship beam; the builders are pretty much divided in proportioning this to the length. Most of them conform to dimensions taken from ships of the same burthen, and designed for the same service.

After these two dimensions are determined, the depth of the hold must be fixed, which in most ships is half the breadth; but the form of the body should be considered; for a flat floor will require less hold than a sharp one. The distances likewise between the decks must be determined. The following table may be very useful towards ascertaining the three aforesaid dimensions,

A Table

A Table of the Length, Breadth, and Depth in Hold of the following Ships.

Ships Names	Guns	Length at	Breadth	Depth in	
		load-water line		the Hold	
		feet in.	f. in.	f. in.	
Monarque	74	165	43	20	6
Intrepide	74	165	43	20	6
Alcide	64	149	40 6	19	4
Renommée	30	120	31 8	15	7
Palme	12	85	22 6	10	5
Soleil Royal	80	182	48	23	
Formidable	80	178	44 10	21	10
Tonnant	80	168	46	23	
Sceptre	74	165	43	20	6
Superbe	74	153 6	42 8	21	
Esperance	74	154	42	21	
Magnifique	74	165	43	20	6
Northumberland	68	149	40	20	
Lis	64	149	40	19	
Hercule	64	149	40 6	19	4
Protee	64	150	40 6	19	4
Illustre	64	150	40 8	20	
Opinatre	64	150	40 4	19	5
Dragon	64	149	40	19	
Leopard	64	146	39 6	18	6
St Laurent	60	145	39 4	18	8
Amphion	50	145	39	18	
Amazon	44	118			
Brillant	50	135	35		
Arc-en-Ciel	50	135	37	17	9
Tigre	52	131	37	17	
Alcion	50	132	35 4	18	
Aquilon	46	127	34	17	
Junon	46	136	36 6	16	6
Favorite	36	127	33	14	
Anglesea	32	121 8	33 6	16	
Serenne	30	118	31 8	15	9
Emeraude	28	118	31 8	16	
Galatée	24	110	29	14	6
Mutine	24	110	29	14	6
Cumberland	24	102	26	13	
Marshal Saxe	22	100	27	14	
Anemone	12	84	22	9	

Ships Names	Guns	Length	Breadth	Depth
		feet		f. in.
Amarante	12	84	22	9
Elizabeth	64	143	38 4	18
Brave	80	172	44	21
Florissant	74	165	45	22 6
Couronne	74	167	44	22 7
Hardi	64	149	40 6	20 9
Aigle	50	144	39	19 6
Hermione	26	126	33 8	13 8
Jufte	70	151	42	21
Triponne	26	114	31 8	
Panthere	20	108	28 6	
Badine	6	66	18 4	

We may then proceed to fix the length of the keel, which will oblige us to determine the rake of the stem and post, for which the builders have given us no invariable rule, they being very much divided in their opinions ; for where some have given a rake of 18 or 20 feet, others have given none at all. The height of the stem and wing-transom must also be determined, which may be regulated by the decks.

The difference betwixt the draught of water abaft and that afore, should likewise be considered ; for though some imagine that when a ship is loaded her keel should be parallel to the surface of the water, yet in many cases it will be found necessary that the keel abaft should be deeper in the water than it is afore. This will give the rudder more power, and thereby contribute to make a ship steer well ; but this difference of the draught of water is intirely arbitrary ; for in large ships some have given five, whereas others have given but three, or even two feet of difference. Though I could not procure the true difference of the draught of water of many ships of war, yet I am assured that the following are pretty exact.

The Difference of the Draught of Water in the following Ships.

	feet	in.		feet	in.
Northumberland	1	2	Panthere	1	4
Auguste	1	6	Couronne	2	1
Aloze	1	0	Triponne	2	
Hermione	2	0	Renommée	1	4
Amazon	1	6	Tigre	3	2
Badine	0	10	Intrepide	2	3
Palme	1	4	Alcide	2	0

The

The length of the wing transom must also be determined; some make it $\frac{2}{3}$ of the main breadth; but this is likewise arbitrary, the broader a ship is abaft, the more room there will be for accommodations for the officers; but this will be disadvantageous to her sailing upon a wind.

The following Examples will be sufficient to fix the Length of the Wing transom for any Ship,

- For a ship of 110 guns, $\frac{2}{3}$ of the main breadth, and 3 lines more to every foot.
 102 guns, $\frac{2}{3}$ of the main breadth, and 8 inches more.
 82 guns, $\frac{2}{3}$ of ditto.
 74 guns, 7 inches, 9 lines for every foot in breadth.
 62 guns, 7 inches, 8 lines for ditto.
 56 guns, 7 inches, 7 lines, 3 points for ditto.
 50 guns, 7 inches, 6 lines, 6 points for ditto.
 46 guns, 7 inches, 6 lines for ditto.
 32 guns, 7 inches, $5\frac{1}{2}$ lines for ditto.
 For a frigate of 22 guns, 7 inches 4 lines.
 12 guns, 7 inches.

Some, without regarding these proportions, make the wing transoms of the first and second rates two thirds of the breadth, and for all the rest one foot less.

After these dimensions are determined, the timbers may be considered which form the sides of the ship. A frame of timbers is composed of one floor timber, two or three futtocks and a top timber on each side: All these being united together, and secured by cross-bars, form a circular inclosure, that which incloses the greatest space is called the midship frame: The curve of this frame is inverted at the lower part, so that the floor timber will be somewhat hollow in the middle, whereby the ends will form a very obtuse angle; but this angle decreases the farther the frames are removed from the midships, in such a manner, that the foremost and aftermost will become very sharp, and form a very acute angle. These floor timbers are called crutches.

The builders seem to agree nearly as to the length of the midship floor timber, making it generally half the length of the main beam; but they differ very much about the rising of it, some chusing a flat and others a sharp floor. And if we consider the advantages and disadvantages that attend the one and the other, we shall not be much surprized to find them so much divided upon this article; for it is certain, the more rising a ship has, she will hold the better wind, but then this will occasion her to draw more water, which will be sometimes attended with very great inconveniencies.

A Table

A Table of the rising of the Midship Floor Timbers.

Guns	f.	in.	lines	} to every foot in length.	Guns	f.	in.	lines	} to every foot in length.
110	0	0	10		56	0	1	4	
102	0	0	10 $\frac{1}{2}$		32	0	1	4	
86	0	1	0		28	0	1	4	
74	0	1	0		22	0	1	6	
62	0	1	0 $\frac{1}{2}$		16	0	1	6	

Note, *What we have here rendered the rising of the floor timbers, the author calls the Aculement, and makes a distinction betwixt it and the rising, which we shall see when we come to form the frames.*

They differ as much in determining the station of the midship frame, some placing it before, others at the middle of the ship; others again have two floor timbers of equal length, and rising, one of which is placed exactly in the middle, or the breadth of the timber before the middle, and the other at a proper distance before it. Those who place it before alledge that if a ship is full forward, after she has once opened a column of water, she will afterwards meet with no resistance, and the water will easily unite abaft, and by that means force the ship a-head, and have more power on the rudder the farther it is from the centre of gravity; and besides this comes nearest the form of fishes, which should seem to be the most advantageous for dividing fluids.

Those who would have it placed in midships say, that by that means the water-lines forward will be easier, and of consequence properer for dividing fluids; and that there will be space enough betwixt it and the rudder for forming very fair water-lines, so that the water will easily unite at the rudder; and besides it will be easier by this means to balance the fore body and after body; and in general the building will by this means be very much facilitated: so that, in my opinion, it will be properest to place it very near the middle, though it is the general practice to place it before it.

After the rising of the midship floor timber is determined, we may then proceed to fix the height of the rising line of the floor abaft on the post, and afore upon the stem.

Now, as all ships are narrower abaft and afore, than in midships, the other floor timbers will of consequence be shorter and have a greater rising, which will be still increasing till it ends on the post and stem. There are several different methods used by the builders to settle the height of this line. Some imagine, that by narrowing the floor abaft, which will occasion the rising line to be high upon the post, the ship will

will thereby steer better, and besides, the water which is opened by the midship frame will then have a greater pressure upon the after part of the ship, and thereby contribute to her sailing: Yet these arguments are of very little weight; for if we only consider the steerage, it is certain, that the higher the rising line is carried abaft, and the narrower a ship is, the water will have the easier passage, and more power upon the rudder. But then we shall thereby run the risk of falling into two great inconveniencies; for by this means we take away the buttock, which is the only thing we have to support all the weight of the after part of the ship; neither shall we be able to give a proper balance betwixt the fore and after part, and when the fore and after parts are not duly balanced, it will occasion a ship to pitch very hard, and be in danger of being frequently pooped by the sea when it runs high. To prevent these inconveniencies it will be proper to give all ships, especially the large sort, a full buttock. As to the height of the rising line afore, it should be determined by the form of the water lines; but before this can be done, the timbers must be formed.

Note, What we have rendered the rising line of the floor, our author calls les façons, which, he says, is the increase of the acculement, the extreme points of which upon the perpendicular of the stem and post, are now to be determined.

The height of the lower deck is the next thing to be considered: It is determined in midships by the depth of the hold, and some builders make it no higher at the stem; but they raise it abaft more than it is in midships, as much as the load-water mark abaft exceeds that afore. As to the height betwixt decks, it is altogether arbitrary, and must be determined by the rate of the ship, and the service that she is designed for.

We come now to consider the upper works, or all that is above water, called the dead-work: And here the ship must be narrower, so that all the weight that lies above the load-water line will thereby be brought nearer the middle of the ship; by which means she will strain less by working the guns, and the main sail will be easier trimmed when the shrouds do not spread so much. But though these advantages are gained by narrowing a ship above water, great care must be taken not to narrow her too much, for there must be sufficient room upon the upper deck for the guns to recoil. The security of the masts should likewise be considered, which requires sufficient breadth to spread the shrouds, though this may be assisted by enlarging the breadth of the channels.

C H A P. II.

Of the Scantlings and Dimensions of the principal Pieces of Timber in a Ship.

ALTHOUGH it is not my intention, as I observed in the beginning of the last chapter, to treat of all the pieces that compose the ship, yet I think it necessary to say something of the principal pieces. I shall therefore in the following plate lay down each piece by itself, by which means we shall see the length of the scarphs, and in what manner they are to be joined together.

Explanation of Plate I.

I.

A. The keel in four pieces, to be well bolted together, and clinched.

II.

I. The fore foot, one end of which is scarphed to the fore end of the keel, of which it is a part, and the other end makes a part of the stem, to which it is scarphed.

III.

u u. Two pieces of dead wood, one afore and the other abaft, fayed upon the keel.

IV.

C C. The stem in two pieces, to be scarphed together.

V.

E E. The apron in two pieces, to be scarphed together, and fayed on the inside of the stem, to support the scarph of the stem; for which purpose the scarph of the apron must be clear from that of the stem.

VI.

- o. The stemson in two pieces, to support the scarph of the apron.
- o. The false post, which is fayed to the fore part of the post.

VII.

B. The stern post: It is tenanted into the keel, to which it is fastened with a knee.

D. The

VIII.

D. The back of the post, which is likewise tenanted into the keel and well bolted to the post; the design of it is to give sufficient breadth to the post, which seldom can be got broad enough in one piece.

IX.

F. The knee which fasteneth the post to the keel.

X.

N. The wing transom. It is fayed across the stern post, and bolted to the head of it: The fashion pieces are fastened to the ends of it; underneath this and parallel to it is the deck transom.

XI.

O O. Two transoms fastened to the stern post and fashion pieces, in the same manner as the wing transom.

XII.

P. The transom knee, which fasteneth it to the ship's side.

XIII.

Q. The fashion piece, of which there is one on each side: Their heels are fastened to the stern post at the height of the floor ribbands, and their heads are fastened to the wing transom.

XIV.

T. A floor timber. It is laid across the keel, to which it is fastened by a bolt through the middle.

XV.

K. The lower futtock.

XVI.

T T T T T. 2d, 3d, 4th futtocks and top timbers. These shew the proper length and scarf of the timbers in midships frame.

XVII.

U U. Riders. These are fayed in the inside of the ship, and consist of floor and futtock riders.

XVIII.

Z. The keelson. This is made of two or three large pieces of timber scarphed together in the same manner as the keel. It is placed over the middle of the floor timbers, and scored about an inch and a half down upon each of them.

XIX.

R, S. Breast-hooks. These are fayed in the inside to the stem, and to the bow on each side of it, to which they are fastened with proper bolts. There are generally four or five in the form of R in the hold, one in the form of S into which the lower deck planks are rabbited; there is one right under the hawse holes, and another under the second deck.

XX.

X, Y, Z. are thick planks which are fayed in the inside, and stretch fore and aft to strengthen the scarphings of the timbers.

XXI.

Z. are thick planks in the inside, called clamps, which support the ends of the beams.

XXII.

15, 15, 15, 15, 15, are the wales. They are planks broader and thicker than the rest, which are fastened to the outside of the ship in the wake of the decks. We shall have occasion in another place to show how they are laid down in a draught. As to the plank below the wale to the keel, and above it to the top of the side, we refer to the section of one half of the midship frame, as laid down in the plate.

XXIII.

d, d, d, d, d, d, are knees. These are crooked pieces of timber consisting of two arms which form an angle, either within or without a square, or exactly square; their use is to fasten any two pieces together, as the beams to the ship sides.

XXIV.

19. The rudder. This is joined to the stern post by the rudder irons, upon which it turns round in the googings which are fastened upon the stern post for that purpose. There is a mortise cut out of the head of it;

it, into which a long bar is fitted, called the tiller, by which the rudder is turned from one side to the other.

XXV.

23. The cat heads. These are two large pieces of square timber, one on each side of the bowsprit. They project out before the bow, in order to keep the anchor clear of the ship, which is hove up by a rope called the cat fall, that passes through shivers in the outer end of the cat-head: Their inner ends are fastened upon the forecastle.

XXVI.

m, m, i, i, i , are the several pieces which compose the knee of the head; the lower part m is fayed to the stem, the heel of it is scarphed to the head of the forefoot; it is fasten'd to the bows by two knees called cheeks, in the form of f , and to the stem by a knee call'd a standard, in the form of K .

XXVII.

Beams &c, $a, X Y$, are large pieces of timber which support the planks of each deck.

Having thus explained all the pieces in the plate, we shall in the following table give their scantlings.

The

C H A P. III.

A Method to lay down a 70 Gun Ship upon the Plane of Elevation.

THE Dimensions we have given of the principal parts of a ship of each class collected from the practice of different builders, which many have so great a regard to, as not to vary from them in the minutest article, we think only to be so far observed, as they shall produce such a form as the service the ship is designed for, shall require, agreeable to mathematical principles.

We shall now illustrate what has been said on that head by drawing a ship from these dimensions. But it will be first necessary to observe, that the builders make use of three different planes for one ship; 1st, the plane of elevation, in which the whole length is laid down according to a side view; 2d, The plane of the projection, which some call a vertical plane of the timbers, because it gives us an end view of the form of all the timbers, before the plank is put on. 3d, The horizontal plane, upon which are described all the curves that are formed by sections of the body parallel to the horizon, which must be considered as well as those vertical sections which form the curves of the timbers. We may likewise form the curves of the ribbands upon this plane, which will be of great use in proving whether the form we give the timbers will produce a fair side.

It is indifferent with which of these we begin, though that of the elevation seems most commodious. But first of all it will be very proper to draw out a list of all the dimensions of the vessel we are to build, so that we may have a view of the whole design.

This ship then is to have two tier of guns, so there must be two decks quite fore and aft, likewise a quarter deck as far as the main mast, a fore-castle 33 feet long, and a poop to the mizen mast.

There are to be 13 ports of a side on the lower deck, the guns to carry 24 lb. shot; 14 ports on each side upon the upper deck, the guns to carry 18 lb shot; on the quarter deck 4 guns, and on the fore-castle 2 guns of 8 lb. shot on each side, and 2 of 4 lb. on each side on the poop.

Ports

	feet	in.	l.
Ports on the lower deck fore and aft	2	10	
Distance betwixt the ports	7	9	
Aftermost port from the post	9	3	
Foremost from the stem	17	2	
Height of the fells, including the lower deck planks	2	5	
Ports up and down on the lower deck	2	7	
Distance from the upper side of the lower deck beam to the upper side of the upper deck beams	6	11	
Rising of the second deck abaft		11	
Second deck ports up and down	2	4	
Second deck ports fore and aft	2	6	
Height of the fells from the deck line	1	11	6
Distance betwixt the second deck and quarter deck from plank to plank	6	6	
Quarter deck ports up and down	1	10	
Quarter deck ports fore and aft	2		
Height of the fells	1	4	
Distance betwixt the quarter deck and poop	6	2	
Ports on the poop fore and aft	1	10	
Height of the fells	1	0	
Length from rabbit to rabbit on the gun deck	156	3	
Extreme breadth	42		
Depth in the hold below the plank	21	0	0
Rising of the lower deck abaft, not including the difference of the draught of water	2	11	6
Height of the stem	31	9	3
Height of the post	31	7	9
Rake of the stem	15	7	2
Rake of the post	3	1	5
Length by the keel	139	6	10
Depth of the keel	1	7	3
Length of the wing transom	27		
Length of the midship floor timber	21		
Rising of ditto	1	9	
Difference of the draught of water abaft more than afore	3	2	0
Height of the rising line of the floor abaft	13	6	0
Height of the rising line of the floor afore	5	7	5

I would advise young beginners in the art of drawing to conform exactly to these dimensions, which we have here given for an example, and

and observe all the particular directions which we shall give in laying down a ship of 70 guns; for they must begin by making themselves acquainted with the terms, and thereby gain a general idea of the whole design. After finishing this draught, they may then proceed to another of a different rate, and as we have given the principal dimensions of several good ships, they may chuse such a one as will best answer their design.

Plate II. Fig I.] 1st. Provide a scale of equal parts properly divided into feet and inches, adapted to the intended length of the draught, and draw the line *AB*, which make 156 feet 3 inches for the length of the gun deck, from the rabbet of the stem to that of the post.

To find the length on the gun deck, multiply 13, the } f. in. 1.
number of ports, by 2 f. 10 in. the dimensions of each port } 36 10 0
fore and aft, the product is

Again multiply 7 f. 9 in. the distance betwixt the ports, by } 93 0 0
12, the number of spaces, the product is

Aftmost port before the post

9 3 0

Foremost abaft the stem

17 2 0

Length on the gun deck

156 3 0

2dly, Draw the line *CD* equal and parallel to *AB*, let 21 feet, the half of the main breadth, be the distance betwixt them, and erect the perpendiculars *CF* and *DZ*.

3dly, Set off 3 feet 2 inches, the difference of the draught of water, from *B* to *G*, and draw the line *AG*, which will give the position of the lower side of the keel. From *A* set off 1 foot 7 inches 3 lines, the depth of the keel, as in the table of scantlings, to *K*, and draw the line *KI* parallel to *AG*, which will be the upper side of the keel.

4thly, Set off 1 foot 3 inches 3 lines, the breadth of the stem from *G* to *M*, and draw the dotted line *MN* parallel to *GZ*. From *G* set off 15 feet 7 inches 2 lines, the rake of the stem, to *O*.

5thly, Set up 31 feet 9 inches 3 lines, the height of the stem from *G* to *P*. With the radius *IP* describe the arch *PO*, which will be the fore side of the stem, and from the same center describe another arch within the former, which will give the inside of the stem, and another arch for the rabbet may be described four inches before the inside of the stem.

6thly, Set up 25 feet 1 inch from *K* to *L* for the height of the gun deck abaft, and 21 feet 6 inches from *I* to *e*, for the height afore.

7thly, Set up 2 feet 5 inches, the height of the port-bells from *L* to *a*, which will give the upper side of the wing transom; from which set up

D

2 feet

2 feet 7 inches, the height of the ports; also 1 foot for the depth, and 6 inches 9 lines for the round of the helm port transom, to the point F, which will be the height of the post; so KF will be 31 feet 7 inches 9 lines. From K set off 3 feet 1 inch 5 lines to *f*, for the rake of the post, and draw the line F*f* for the aft side of the post. From *f* to *b* set off the depth of the keel, and draw the line *bd* for the fore side of the post, making F*d* $\frac{3}{4}$ of *fb*, so shall *fO* be the whole length of the keel.

The builders are very much divided about assigning a proper place for the midship frame, for which the following method may be used:

Divide the line CD into two equal parts, then take 5 feet 6 inches 10 lines, that is $\frac{1}{12}$ part of 156 feet 3 inches, the length of the gun deck: Set off this before the middle of the line CD, which will give the point F the station of the midship frame. Set up 21 feet from F to Z, which will give the height of the gun deck at the midship frame. From the point Z set off 2 feet 7 inches 6 lines (the $\frac{1}{4}$ of the height of the gun deck at the midship frame,) through which point draw a line VT, parallel to CD, which will be the load-water line. Through the point F draw the line G*g*, parallel and equal to the load-water line, which will shew how much water the ship will draw abaft more than afore.

One of the frames is placed pretty near the cheffs tree, which is called the loof frame; to find its place, from the point of D set off $\frac{1}{4}$ of the line DC, and there draw a dotted line perpendicular to AB. Again divide the line FG into nine equal parts, and draw eight lines perpendicular to AB, which will station eight frames in the fore-body besides that of the loof.

There is in the after-body a frame to balance that of the loof in the fore-body; these two are of equal breadth in some points, and this will occasion the center of gravity of that part contained betwixt these two frames to be near the plane of the midship frame, which will keep the fore part and after part upon a balance. It must be as far abaft the middle of the line CD as that of the loof is before it.

The frames in the after-body are the same distance from one another as they are in the fore, which will occasion one more abaft than before; so there are nine abaft, besides that of the balance.

We shall in the next place lay down the deck lines, and first for the lower deck draw a fair curve through the points L Z *e*, and parallel to it draw another curve for the port-fells, which are 2 feet 5 inches above the deck line.

The

The aftermoſt port is 9 feet 3 inches before the poſt, which ſet off to *u*, and the ports are 2 feet 10 inches fore and aſt; which ſet off from *u* to *x*, the diſtance betwixt the ports is 7 feet 9 inches, which ſet off from *x* to *Y* for the aſt ſide of the ſecond port; from *Y* again ſet off 2 feet 10 inches, which will give the foreſide of the next. Proceed in the ſame manner till all the ports are ſpaced; ſo ſhall the foremoſt port be 17 feet 2 inches abaſt the rabbit of the ſtem. The height of the ports is 2 feet 7 inches; which ſet up from *u*, draw a curve parallel to the deck line, which will give the upper part of all the ports; after which theſe two lines may be wiped off the draught, which muſt be therefore drawn with a black lead pencil, and only the ports inked in.

Draw a line for the upper deck, which is 6 feet 11 inches above the lower from the midſhip frame forward, and 6 inches more abaſt. We may then draw a line for the port-ſells, and one for their height parallel to the deck line, and ſpace the ports ſo that they may be exactly over the middle of the diſtance betwixt the lower deck ports.

Before we can ſet off the height of the Quarter deck we muſt find the true place of the main maſt. The general rule is to take 4 lines for every foot the gun deck is in length, and ſet it off abaſt the middle, which will give the fore ſide of the maſt: now the length 156 feet 3 inches $\times 4$ lines = 625 = 4 feet 2 inches 1 line, which ſet off abaſt the middle of the line *C D*, and there erect a line perpendicular to the water-line, which will be the fore ſide of the maſt, and parallel to it draw a line for the middle, and one for the aſt ſide of the maſt, the diameter of which is 35 inches. Set off 6 feet 6 inches on the aſt ſide of the main maſt, for the height of the quarter deck afore, and 6 feet 10 inches for the height abaſt; and draw a line nearly parallel to that of the upper deck, which will be the line for the quarter deck. We may then ſpace the ports, ſo that they may be exactly over thoſe of the lower deck. The forecaſtle is 6 feet 6 inches high, at which diſtance draw a line parallel to the upper deck line, which will give the line for the forecaſtle deck. As to the length of this deck, it ends forward at the beak head, and is carried aſt diſcretionally, obſerving to leave room for the capſtan bars. In ſpacing the ports upon the forecaſtle, care muſt be taken that none be oppoſite to the fore maſt. Now to find the center of this maſt, take 15 feet 7 inches 2 lines, the tenth part of the whole length, which ſet off from the rabbit of the ſtem upon the lower deck abaſt, from which point ſet off 32 inches and 1 line, being the diameter of the maſt, through the middle of this draw a perpendicular line, as in the plate. The boltſprit generally makes an angle of 34 or 35 degrees with the load-water line.

The poop is pretty near parallel to the quarter deck; the distance betwixt them forward is 6 feet, an abaft 6 feet 3 inches. It ends about 18 inches before the mizen mast, the aft side of which is $\frac{2}{3}$ of the main breadth before the rabbit of the post upon the gun deck.

The counter is generally an arch passing from the upper side of the wing-transom to the lower side of the beam of the second deck. The rake of the lower counter is $\frac{2}{3}$ of an inch for every foot of the main breadth. The rake of the second counter is $\frac{2}{3}$ of the lower; its height above the deck is 3 feet 5 inches. The hollow of the counters is altogether arbitrary, inasmuch that some give none to the lower. The upright of the stern rakes 2 inches in a foot, as in the plate.

The beauty of a ship depends much upon giving the wales a proper hanging; for by them the sheer and drift rails are regulated, being all nearly parallel to one another, though they generally rise a little more abaft on account of the accommodations for the officers. It is this which makes a ship look airy and graceful in the water. There is no certain rule for laying them down; this is left entirely to the fancy and taste of the artist; but in placing the wales great care must be taken that they be wounded as little as possible by the ports; the foremost port on the gun deck must be $1\frac{1}{2}$ or 2 inches above, and the third port from abaft just touch the upper side of the upper strake of the main wales. The lower edge of the lower strake may glance with the edge of the water when loaded. There are two strakes of wales, and one strake between them of 15 inches broad each. The range of the deck should be considered in placing the wales, so that the scuppers may be in the strake betwixt the wales. The like caution must be used for the channel wales as may be seen in the plate, where they are all laid down, together with the sheer and drift rails; the rails, cheeks, and knee of the head are likewise laid down in the plate, and being for an ornament to the ship, are left to the fancy and taste of the builder. Though the knee may help a ship to hold a good wind, the fore part of it is generally one twelfth part of the length before the stem.

C H A P. IV.

To lay down the Frames upon the Plane of Projection.

HAVING thus explained all that is necessary to be delineated upon the plane of elevation, the next thing to be determined is the different breadths of the ship at any alligned points of the length, whereby we shall gain the forms of all the planes that are made by sections, perpendicular to the load-water line. The timbers that compose the body of a ship are supposed to have their planes in that position, and may be all delineated upon the plane of the projection; but as both sides of a ship are exactly the same, it will suffice to lay down the half of each, those of the fore-body on the right, and those of the after-body on the left hand. And whereas these planes diminish afore and aft, the planes of all the frames may be all delineated upon the plane of the midship one, which may be called the master frame. The first thing then necessary to be known is how to form this frame.

The mid-ship frame is that which is at the broadest part of the ship. The builders differ about the form of this frame, but there are several preliminary operations which are necessary to be observed in all the different methods used in forming it.

Preliminary Operations for forming the Midship Frame,

Plate III, Fig. I. and II. 1st. Draw the line A B to represent the upper side of the keel; it must be at least as long as the ship is broad. This line our author calls the line of *aculement*, because upon it the aculement of the midship floor timber terminates.

2^{dly}, Draw the line C D parallel and equal to A B, so that A C and B D may be equal to the rising of the midship floor timber. This line may be called the rising line, because it limits the height of the ends of the midship floor timber above the keel.

3^{dly}, Draw the line G H for the height of the lower deck parallel to the former, and below this draw a line to represent the load-water line, taking its distance below the deck line from the plane of elevation at the midship frame. Draw also the lines I K and L M, the one for the second deck, and the other for the sheer rail or top of the side in midships. The height of both are to be taken from the plane of elevation.

4^{thly},

4thly, Draw the line NO perpendicular to AB; this is called the middle line, and represents the middle line of the stem and post, dividing the whole ship into two equal parts; and parallel to NO draw the lines AL and BM, to limit the breadth; also a line for half the thickness of the stem, and one for half the thickness of the post. Draw the lines ax parallel to NO, dividing the lines OA and OB into two equal parts. Draw also the diagonal GB. These lines being drawn, we may proceed to form the midship frame by some of the following methods.

M E T H O D I.

To form a Midship Frame that shall be neither too sharp nor too flat.

Plate III. Fig. I.] 1st, Divide the line ax , which marks the head of the floor timber into three equal parts; set off one from a to b .

2d, Divide the line dB , the distance betwixt the load-water line and the upper side of the keel, into seven equal parts; set off one of these from d to e , and from e to m , and draw the diagonal aV , which divide into two equal parts in the point n . *Note, the diagonal aV is wiped out after finding the point n .*

3d, Describe an arch of a circle to pass through the points b and e , make the radius the whole length and half the length of the line Be , so the center A may be found by describing an arch with that radius from e , and one from b to intersect one another in A , we shall only make use of that part of this arch betwixt b and m . Now, to find the other arches md , la , an , nV , it must be observed, that in order to reconcile two arches, so as to make a fair curve, a strait line must pass through the centers of both, and through the points where they unite or touch one another; draw therefore the lines Am and Al , so shall k be the center of the arch md , and o the center of the arch la . Again through the center o draw the line ao , produce it to P , which will be the center of the arch an . Lastly, from P thro' n draw the line Ps , s will be the center of the inverted arch nV . *Note, the center s will be without the Plate.*

4th. To form the top timber set back the tenth part of the half breadth from K to S , upon the line of the second deck; describe an arch of a circle thro' the points d and K , taking $\frac{2}{3}$ of the whole breadth for the radius: Again, from the point M , upon the line LM , set back the fifth part of the whole breadth to g . Describe an arch of a circle through the points S and I , taking the diagonal GB for the radius. As this arch is inverted in respect of the arch dS , the center will be without the figure. This compleats the form of half the midship frame, and by the same operations we may find the other half.

It must be observed that there is no regard had to the round of the beam in setting off the deck line or depth of the hold. M E-

M E T H O D II.

To describe a Midship Frame of a circular Floor.

Plate III. Fig. II.] From the center G, the point where the middle line intersects the deck line, making the half breadth the radius describe the arch b, G, c, O : Let d be the head of the floor timber, and $d x$ the rising. Assume the point f , according to what round you propose to give to the second futtock, and describe the arch $d f$; the center may be found as directed in the preceding method. Divide the arch $c O$ into three equal parts; set off one from c to g , and from the center b ; describe the arch $d g$: there remains only the inverted arch $g Y$ to be described; the center may be found as before directed.

M E T H O D III.

To draw a Midship Frame which shall be very full.

Plate III. Fig. III.] 1st. Draw the rising and deck lines as before; let $d x$ be the rising.

2d. Make $d b$ the side of the square $d b a c$ equal to $C b$ the $\frac{1}{2}$ of the breadth.

3d. Inscribe the two quadrants $c e b$, and $c f b$ into the square.

4th. Divide the side $c a$ into a certain number of equal parts in the points O, N, M, L, a ; draw the lines $i L, b M$, &c. perpendicular to $a c$.

5. Divide the line $C G$, the depth of the hold after deducting the rising, into the same number of equal parts in the points E, F, I, K , and make the lines $E p, F q, I r, K s$, in the frame, equal to the lines $O t, N n, M e, L m$ in the square, describe a curve through the points G, p, q, r, s, b , and the remaining part of the frame may be described by the preceding methods.

M E T H O D IV.

To describe a Midship Frame for a very sharp Ship.

Plate III. Fig. IV.] Let the length of the floor timber be half the breadth as before, and the rising one fifth or one sixth of the whole length of the floor timber; lay this off from x to E , and describe a parabola through the points G, P, Q, E , of which the point G is the vertex, and $G C$ the axis. This method is extracted from M. Bouguer. The parabola may be formed by the following method: 1. Through the point E draw the line $T x$ perpendicular to $G C$, and the line $d E$ perpendicular to $A G$, and produce

produce the line CG to D . 2dly, Upon the line CD find the center of a semicircle that shall pass through the points T , d , and D , so shall GD be the parameter of the parabola, by which we may find any number of points through which the curve must pass: for instance, suppose it were required to find a point in the perpendicular XP , through which the curve must pass; upon the line GD find the center of a semicircle which shall pass through the points D and X ; this will intersect the line AG in b , make bP equal and parallel to GX , so shall P be the point required; in like manner the points a, Q, f may be found. The remainder of the curve from E to y will be composed of two arches, the one to reconcile with the parabola in the point E , and the other inverted to, pass through the point y ; the center of which may be found by any of the preceding methods. In order to find the center of that which joins with the parabola make TR equal to half the parameter GD , and draw the line ER , upon which find a point S for the center of the arch.

We might shew a great many more methods of describing this midship frame. It is very true that great care ought to be had in forming this frame, because upon it chiefly depends the form of all the other timbers; I say chiefly, but not altogether; for two ships may be similar as to their midship frames, and yet very different afore and abaft; and though the artists should make themselves acquainted with all the different ways of forming this frame, I should recommend that method to them which is the simplest and which gives them the most liberty to vary the form of it, according to every one's particular taste or fancy; and it is very possible there may be several other methods as easy and plain as those we have described. This frame being once formed, we may form all the rest upon the same plane. We shall in the next place shew the different methods used by the builders for that purpose.

The ancient builders not being acquainted with the methods of laying down their designs in a draught, found out a mechanic way of doing this only by help of the midship frame, which they might have formed by some of the preceding methods or any other contrivance of their own; and though this method is defective in several points, yet as it is an ingenious contrivance, we shall give it a place here.

METHOD I.

Of forming the Timbers by a Mould made to the Midship Frame; a rising Staff and overcast Staff.

[Plate III. Fig. VII.] 1st. Having formed the midship frame and set off its scantlings, make a mould to fit both outside and inside, which may be called the bend mould

2d.

2d. Draw the line Zx to limit the head of the floor timber at d ; let $d u$ be the rising, and draw the line au ; let t be the height of the rising line abaft, and draw the line dt to represent the floor heads, or floor ribband. Set off dx from d to H ; and from e , the head of the first futtock, to 6 , and divide each into six equal parts, being the number of frames from midships to the balance frame.

3d. Divide the line au into five equal parts, and set off two of them from a to S ; divide the line aS into the same proportion, that the part $A6$ of the base AC of the right angled triangle (*fig. 5.*) is divided into, and transfer these divisions to the bend mould, and let them be numbered $0, 1, 2, 3, 4, 5, 6$, which points will give the narrowing of the floor, as we shall shew, after constructing the triangle. We shall only remark, that the line aS , which is $\frac{2}{5}$ of au , is nearly the difference betwixt half the length of the midship floor timber, and half the length of the floor timber at the balance frame. But as this appears to be too much, we may take $\frac{1}{5}$ as in the figure, or any other quantity which shall be thought most convenient.

To construct the Triangle, Fig. 5.

Upon the line AC , drawn at pleasure, set off any distance from A to 1 , and double that distance from 1 to 2 , treble from 2 to 3 , and so on in the same progression till we have as many divisions on the line AC as we propose to have frames abaft the midship. Erect a perpendicular at A , which may be produced at pleasure, and from any point B draw lines to all the divisions of the base AC . Observe that though in the triangle we have drawn a line for every frame to the fashion piece, we shall only make use of six, there being so many to the balance frame. The triangle being thus constructed, apply the line aS to it, in such a manner, that it may be parallel to AC , and be contained betwixt the lines BA and $B6$, the lines drawn from the point B to the points $1, 2$, &c. will divide it into the required proportion.

To construct the Rising Staff, Fig. 5.

This staff K may be of the same breadth with the keel, and a little longer than at , the height of the rising of the floor. In order to graduate that staff, set off xu , the rising of the midship floor from K to 0 , and make oL equal to at ; apply the line oL to the triangle, so that it may be parallel to the base, and contained betwixt the lines AB and BC the

E

lines

lines from the point B to the several points in the base will divide it into the required proportion, which will give the rising of the floor.

Note, Our author calls x u the acculement, and u d the rising; the line u a will pass through the point where the inverted arch joins the floor sweep.

To construct the over cast Staff, Fig. 5.

That we may have a clear understanding of what is meant by *over-cast*, it will be proper to observe, that in forming the frames by the bend mould, when it is set to the narrowing of the floor, the head of the mould will come too far in at the deck; the mould must therefore be moved round upon the point which represents the floor ribband, till the head goes out to the proper breadth; this will occasion the lower part of the mould to rise a certain quantity, which is called the over-cast. In order to graduate this staff we must determine the difference betwixt the main breadth at the midship frame, and at the balance frame, which suppose D F, let this be placed parallel to the base, and contained betwixt the line B A and B 6; so shall the lines B 6, B 5, &c. divide it into the required proportion.

These are the instruments that are necessary for forming the after frames, those for the fore part are constructed in the same manner, only the graduations for these are but half the graduations of the former, for which reason there must be another bend mould graduated for the fore body.

Now, in order to form the frames by these instruments, place the bend mould upon the rising staff in such a manner that the middle line of the staff produced may pass through the narrowing of the floor upon the bend mould, expressed by the division corresponding to the frame to be formed; suppose frame 6, (*Fig. 7.*) the lower or strait part of it expressed by the dotted line in the figure being applied to the rising staff, till the middle line B a pass through the division 6 on the bend mould: mark by the edge of the rising staff the point 6, which expresses the rising of the floor at that frame. Set up the over cast (expressed by the space contained betwixt the points 5 and 6 upon the over cast staff) from the lower part of the bend mould to the point 6 upon the line B a; then keeping the point d immoveable, turn the bend mould upon this point till the lower part rise to the overcast at the point 6 upon the line B a, and when in this position we may describe the curve to the floor head, and then invert the bend mould, and placing the point 6 (betwixt d and H) to the point set off before to express the rising, turn the mould
till

till the strait part touch the curve before described, and then draw the lower part, which compleats the frame.

This is the method that is used when they mould the timbers, and it may likewise be used to lay them down upon a draught; for if the line *au* of the bend mould (*Fig. 8.*) be laid upon the line *AV*, we may, when in that position, describe the midship frame from the point *d* to the point *X*. In like manner we may describe all the rest of the frames, by giving each its proper over-cast and rising; as for instance, if it were required to describe frame 6, take the rising *K 6* upon the rising staff, and set it off from the point *B* to the point *a* upon the line *BG*, and draw a line through the point *a* parallel to *AV*, upon which laying the bend mould in such a manner that the point 6 which expresses the narrowing of the floor, shall be upon the point *a*; then will the point *d* be upon the point *RS*: set up the proper over-cast from *a* to 6, and keeping the point *d* immoveable, push up the bend-mould which at first was placed at the point *a*, till it be raised to the point 6, which will throw out the point *X* to the proper breadth at the deck. But because the deck is higher at timber 6 than at the midship frame. Take the distance betwixt *e* and 6, at the head of the futtock on the bend mould, and set it up from *x* to 6, and then inverting the bend mould, so that the point 6 betwixt *d* and *H* be at the point *X*, and the strait part of the mould touch the curve before described: we may then describe the lower part to the point *X*, which compleats the whole frame. The timbers for the fore body may be described by the same process as those of the after body, only making use of the bend mould, rising and overcast staff graduated for that purpose; but as we observed before, we cannot lay down any timbers by this method but those betwixt the midship and ballance frame.

The builders finding how very advantageous it would be for them to form all the timbers upon the plane of the projection, because they could then at one view see how they would compare one with another, have tried several expedients to perform this, of which I might instance ten or twelve, but shall content myself with explaining three, which may be sufficient for those purposes, in order to which I shall first shew another method of forming the midship frame, different from those we have shewn before.

Plate III. Fig. 6. 1st. Draw the rising deck and load-water lines, and set off the length of the floor timber as before.

2d. Take one fourth of the length of the floor timber, and set it off from *O* to *d*, upon which erect the perpendicular *dc*, and divide it into two equal parts in the point *e*.
3d.

3d. Describe an arch through the point a , the head of the floor timber, and the point e , taking for the radius the distance from the upper edge of the keel to the port-fells, or a little more or less, according to what round you propose to the floor head. This determines the rising of the floor timber, and with the radius Ol , half the length of the floor timber, describe the arch eY , which determines the *aculement* of the floor timber.

4th. At the point l , the middle of the line AO , erect the perpendicular lm ; and at the point n , the middle of the line Al , erect the perpendicular no ; erect also the perpendicular pq at the middle of the line AN ; and another rs , at the middle of the line Ap ; and lastly, another tu , at the middle of the line Ar .

5th. Take the distance ln , which set off on the line no from n to z ; and on the line pq , from p to g ; then taking the distance from a to g set that off from p to y ; again take the distance py , which set off from r to b , and the distance ba from r to F ; and lastly take the distance rF , which set off from t to E , and then the distance Ea from t to x , a curve passing through the point a, z, y, F, x, T , will form the midship frame under water. We may then set off half the thickness of the post and stem on each side of the middle line, and form the rest of the timbers; those for the fore body on the right, and for the after body to the left of the middle line.

Plate II. 1st. To lay down the post upon the plane of projection, take the difference of the draught of water abaft more than in midships, as marked on the plane of elevation (*Fig. 2.*) set off this from F to d (*Fig. 3.*) and draw the line de parallel to AB ; take also KF , the height from the plane of elevation, which set off from e to r , so shall the point r be the head of the post.

2d, to lay down the wing transom, take its height from the plane of elevation, which set up on the plane of projection to f , and draw the line gf perpendicular to the middle line, so gf represents the upper side of the wing transom without regarding the round up or the round aft. Take also the height of the rising line upon the post from the plane of elevation, which set off from e to G .

3d. To form the fashion piece; take upon the plane of the projection nG the height of the load-water line, above the rising line upon the post, which set off from n to o upon the water line; take also GP , the distance betwixt the rising line and lower deck, which set off from P to q upon the deck line, and describe a circle through the points f, q, o . There is a problem in geometry to find the center of this arch. *Note*, the point q may be taken further out or in, as you design a lank or full fashion piece.

Lastly,

Lastly, describe the arch $o G$; the radius of this arch may be the main half breadth; so shall f, q, o, G be the form of the fashion piece, which may be varied according to the fancy of the artist, by altering the centers.

Having thus formed the midship and after frames, we shall in the next place shew how to space the ribband lines, which are represented by the diagonals in the figure, but it will be proper to remark, that the ribbands are thin narrow planks which are made so, that they may easily be bent to the timbers. That which is nailed to the post at the height of the rising line, and to the midship frame, at the end of the rising of the floor timber, is called the floor ribband. That which answers to the wing transom and to the height of the lower deck, on the midship frame, is called the breadth ribband; all the rest betwixt these two are called intermediates.

From the Point H draw the line $H G$ for the floor ribband, and from the point T draw the curve T, E, q, p for the breadth ribband, and draw the two intermediates betwixt them, so that by them the curve of the midship frame and fashion piece may be divided into three equal parts.

Now, it is very plain, that if the ribbands had a proper form, and nail'd at the proper heights and positions, they would compose a kind of a model, by which the circular form of every timber might easily be discovered; but as we have only the extreme points of each given, we cannot from thence form such a curve as shall be necessary. We must therefore find a method to form some intermediate timbers betwixt the midship and after one, and thereby form the ribbands so that they shall make fair curves. There are some preliminary operations which are necessary towards performing this.

1st. *To construct an equilateral Triangle for the Progression of the Frames in the After-Body.*

Plate IV. Fig. 1. From the point M set off any distance to 1, upon any strait line, and from 1 to 2 treble that distance, from 2 to 3 five times that distance, from 3 to 4 seven times that distance, and proceed in that progression, increasing the spaces betwixt the figures by equal differences, *viz.* double the distance betwixt M and 1, till we have as many divisions less one as there are frames betwixt the midship and post, including that of the midship and post; and because there are nine frames the line must consist of ten divisions, from the point M to the point E . Let them be numbered 1, 2, 3, &c. make ME the base of an equilateral triangle SME , and draw the lines $S 1, S 2, \&c.$ observing to produce them all till the distance betwixt the lines SE and $S 9$, upon a line parallel to the base, be at least equal to the distance betwixt the frames in the plane of

of elevation. The line *S M* represents the midship frame, and the line *S E*, the post, and the nine intermediates, represent the nine frames betwixt the midship and post.

In order to give us a clear understanding of the use of this triangle, it will be necessary to remark, that the midship frame being that which incloseth the greatest space, and the aftermost that which incloseth the least, it will follow, that the intermediate frames will partake of the form of each ; but mostly of that to which they are nearest ; yet they will still retain a little of the form of each. Hence, when the intermediate frames are all formed, their curves will divide all the diagonals, drawn in the plane of projection, into as many parts as there are frames ; and all the methods the builders have invented serve only to divide them into such a proportion as shall produce the fairest curves.

Now, if the proportion pitched upon for that purpose, be as 1, 3, 5, 7, 9, &c. then they must all be divided into the same proportions as the base of the triangle is divided into ; and this may be performed very readily, only by taking the length of each diagonal from the plane of the projection, and applying it to the triangle in such a manner that it shall become the base of an equilateral triangle ; as for instance, to divide the first intermediate diagonal ; take the length of it in the plane of projection, (*Plate II. Fig. 3*) and set it off from the point *S* to *m* and *k* on the sides of the triangle *S M* and *S E* ; and draw the line *m k*, which being parallel to the base of the triangle, will be divided into the same proportion. In like manner all the rest of the diagonals may be divided ; but as the builders are not agreed as to the precise form of a ship's bottom, some chuse to divide the base of the triangle into another proportion ; others again in applying the diagonals to the triangle give them different inclinations to the line *M S*. It would be very proper to try several of these methods, by which means we might discover which would be most convenient ; and after all the diagonals are divided into as many points as there are frames, curves passing through these points will determine the form of all the frames from the midship to the post. It only remains to shew how to end each frame upon the post. It was before observed that the keel is not parallel to the surface of the water, so that it will be very easy to conceive that the height of each frame taken from the upper side of the keel, upon a perpendicular to the surface of the water, will always increase, the nearer the frame is to the stern post. Now *g K* is what the keel is deeper abaft than at the midship frame ; and to find how much any frame abaft exceeds that of the midship, suppose the first ; take the distance betwixt the line *g G* and *K I* at that frame, from the plane of

of elevation, (*Plate II.*) which set off from F towards *d*, (*Fig. 3.*) and at that point draw a line parallel to *de*, which will be the first frame upon the keel. In like manner we may draw lines parallel to *de*, for all the rest, as in the figure, which will determine their heights from the upper side of the keel to the surface of the water.

It must be observed, that the diagonals in the plane of the projection, which end on the fashion piece, must likewise end on the fashion piece on the plane of elevation; we must therefore draw the fashion piece on the plane of elevation. Thus, take the distance of the point G, in the plane of the projection, from the upper side of the keel, which set off upon the stern post in the plane of elevation to the point *b*; through *n*, the rabbit of the wing transom, draw the strait line *bn*, which will represent the fashion piece on the plane of elevation. Now as only the lowest diagonal ends upon the post, in the plane of projection, which in the plane of elevation ends at *b*, so the other diagonals that end upon the fashion piece, must likewise end on the fashion piece in the plane of elevation. Their height must therefore be transferred from the plane of the projection to that of the elevation; so the second diagonal will end at the point P, upon the fashion piece in the plane of elevation. In like manner all the rest may be transferred to the plane of elevation; and as the line that represents the fashion piece upon the plane of elevation rakes aft, this will occasion the line PS, which is perpendicular to the line that represents frame 9, to exceed the line *bM*. In the triangle, the line SM represents the midship frame and the line SE the post; that is, if the point where the ribband ends on the post, be equally distant from frame 9, that frame 9 is from frame 8. Now as *Mb* is longer than *ML*, we must draw the line SD without the triangle, which is to be used instead of the line SE, when we come to apply the diagonal HG to the triangle; for the point H must touch the line SM, and the point G the line SE. To find the point D, take *ML* from the plane of elevation, and apply it to the triangle, so that BC shall be equal to it; and parallel to ME; it must also be contained betwixt the line Sg and SE. Then take *bM* and set off from B, which will give the point D. In like manner the line SF must be used when we divide the diagonal MK; and to find the point F set off PS in the plane of elevation, from B to F in the triangle; and draw the line SF. In the same manner there must be lines drawn for every diagonal without the line SE; so the line SE is not used in dividing the diagonals. Let it be further observed, that in applying each diagonal to the triangle, it must not only be contained betwixt the line SM and the line corresponding to the diagonal, which is to be divided, but it must likewise form a certain angle with the line MS,

that

that is, with that part of it which is intercepted betwixt the diagonal and the point S. These which appear to me to be properest for that purpose are as follows: The first diagonal to make an angle of 60 degrees; the second $62\frac{1}{2}$, the third 68, the fourth 86, the fifth 65, the sixth 60 degrees; but the artists vary these angles according to the form they design to give to the timbers; nay, some draw them always parallel to the base of the triangle.

Our author then proceeds to the forebody, and forms a triangle, the base of which he divides in the same manner as that already described, by which he divides each diagonal. He likewise shews how to space the diagonals upon the stem; but as the artists leave us so much undetermined as to the angles that each diagonal is to make with the line SM, when they are applied to the triangle, it will be very difficult to apply this method to practice. So we presume it will be needless to say any more on that head, judging what has been already said sufficient to give our readers an idea of the principles on which the method is grounded; we shall proceed therefore to the next method he proposes.

To form the Timbers by a Quarter of a Circle, Plate IV. Fig. 2. 3.

1st. Form the midship frame, the fashion piece, the foremost timber, also the two balance frames, by some of the preceding methods. *Note, Those who make use of the following method of forming the rest of the timbers are supposed to be previously acquainted with the manner of forming the midship frame, &c.*

2d. Space all the diagonals for the ribbands as directed in the preceding method.

3d. From the center A with any radius describe a quarter of a circle, and divide it into so many equal parts, that there may be a point for each timber to be formed, and draw the radii A 1, A 2, &c. to A 9, so we shall have one for each frame.

4th. Take *ab* the first diagonal, which set off from the point A upon the line AC, to 1.

5th. Take *ac*, the distance upon the lower ribband, betwixt the post and balance frame, which in the plane of projection is the 6th frame, set off this distance upon a perpendicular erected upon the line AB, to intersect the radius A 6, in such a manner that the perpendicular G 1 shall be equal to *ac*.

6th. Produce the line CA to F, and upon this line find a point, which shall be the center of a circle whose circumference shall pass through the point 1, before marked upon the line AC, and the point 1, now marked upon the radius A 6; describe the arch through these two points to the point 1 on the line AB.

7th,

7th. Let fall perpendiculars to the line A B from the points where the arch 1, 1, 1 intersects the several radii. Transfer these perpendiculars to the line *a b*, which will divide the lower diagonal into the points through which the curves of the frames must pass. *Note, the perpendiculars are not drawn to avoid confusion.*

After the same manner all the other diagonals are graduated, first by taking the whole length of each diagonal, and setting them up on the line A C, from the point A to the points 5, 4, 3, 1, 2, and secondly, by taking the several distances upon each diagonal intercepted betwixt the after frame and the balance frame, and applying them severally to the radius A 6, in such a manner that they shall be contained betwixt the radius A 6 and the line A B, upon the perpendiculars let fall from the points 5, 4, 3, 1, 2. And thirdly by describing arches through the points in the line A C, to pass through the points of the same number upon the radius A 6, whose centers are in the line A F; the arches to be produced to intersect the line A B in the points 5, 4, 3, 1, 2, will intersect all the radii; the perpendiculars let fall from the intersections of the radii with the arch corresponding to each diagonal, will divide that diagonal into the points through which the curves of the frames must pass.

The diagonals for forming the frames in the fore body are divided into the points through which the curves must pass by the same operations, only observing that frame 4 is the balance frame for the fore body.

C H A P. V.

Of the Projections on the horizontal Planes, and of the Water and Ribband Lines on the Plane of Elevation, and that of the 1 projection.

WATER Lines are described upon a ship's bottom by the surface of the water into which she swims; that which determines how much is under water when she is loaded is called the load-water line. Now it is plain, that if a ship is lightened, she will rise higher out of the water; and if she be lightened so as to rise equally afore and abaft, the surface of the water will then form another water-line parallel to the load-water line. Again, if the ship is lightened more she will still rise higher, and if the same difference still continues betwixt the draught of water abaft and afore, we shall have another water line parallel to the two former;

so that by this means we may describe as many water lines as we please, all parallel to one another.

In order to form an idea how these lines are represented on the different planes, let us suppose a ship upon the stocks upon a level ground, and her keel in the same position, with respect to the horizon, that it is to be in the water when loaded; we may then describe several black lines upon the ship's bottom, which may be whitened for that purpose, all parallel to the horizon: These will all be water lines.

Now, if a spectator be removed at any considerable distance from the ship upon a line in the same direction with the keel, all these black lines which were drawn upon the ship's bottom, parallel to the horizon, and which are actually curves, will appear to him all strait lines, because he sees them all upon a plane formed by a section passing through the mid-ship frame perpendicular to the keel. Hence the water lines will be represented by strait lines upon the plane of the projection.

Again, if a spectator is removed at any considerable distance from the ship upon a line perpendicular to the keel, so as to see the whole length of the ship at one view, the water lines will then appear to him strait lines, because he sees them upon a plane erected perpendicular to the horizon upon the middle line of the keel. Hence the water lines will be represented by strait lines upon the plane of elevation.

But if the spectator be supposed to be placed underneath the middle of the ship at any considerable depth, in a line perpendicular to the level ground, he will then, viewing the ship's bottom upwards, discover the curvings of all the water lines. These curves are all projected upon a plane, which we must imagine to be formed by a section of the ship through the load-water line, and we are now to shew how these are formed:

To form the Water Lines upon the Horizontal Plane.

Let the water lines to be formed be represented in the plane of the projection by strait lines all parallel to one another. These will be represented by the strait lines in the plane of elevation. Suppose qr , st bx , and TV , all parallel to one another, and the same distance from the load-water line TV that the lines which represent them in the plane of the projection are from it. In order to form these upon the horizontal plane,

1st, Take half the thickness of the post from the plane of the projection, and lay it off on the horizontal plane from A to E , and through the point

point *E* draw the line *Es* parallel to *AB*, five or six feet long; lay off the same distance from *B* to *F*, and thro' the point *F* draw a line *FR* parallel to *AB*, five or six feet long.

2d. From the points where the water lines intersect the stern post upon the plane of elevation, let fall perpendiculars. In like manner let fall perpendiculars from the points where the water lines intersect the stem.

3d. Take upon the water lines, in the plane of the projection, the several distances intercepted betwixt the middle line and the curve of the midship frame, and lay them off from the line *AB* in the horizontal plane, upon the perpendicular that represents the midship frame. Take also from the plane of projection the several distances intercepted betwixt the middle line and the curvings of the other frames, and lay them off in the horizontal plane from the line *AB* upon the perpendiculars corresponding to their respective frames, both in the fore body and after body, and curves passing through all these points will give the true form of all the water lines. They end forward at the points where the perpendiculars intersect the line *FR*. The water lines abaft which end upon the post in the plane of elevation, will end where the perpendiculars intersect the line *Es* upon the horizontal plane. But the 3d and 4th water lines cannot end upon the post, by reason of the fashion pieces; and in order to find the points where these shall end, we must proceed in the following manner.

To find the point where the load-water line ends, let fall a perpendicular from the point *k*, where it intersects the fashion piece on the plane of elevation, to *N*. Take from the plane of the projection upon the line that represents the load-water lines, the distance betwixt the fashion piece and the mid-line; lay this off upon the horizontal plane from the line *AB* to the point *N*, which will end the load-water line upon the horizontal plane, from whence it may be drawn to *g*; so *gN* will be the flat of the Tuck; and to find the point *g* draw a line parallel to *kN* thro' the point where the line *TV* cuts the rabbit of the post, which will give the point *g*. We may after the same manner find the ends of the other water lines that do not go the stern post for a square tuck.

To form the Ribbands upon the Horizontal Plane.

We observed before, that the ribbands were thin planks nailed to all the frames from the post to the stem; and that when they are carried round, so as to make fair curves, the form of all the filling timbers may be by them determined. These filling timbers are to be placed betwixt

the frames, which were methodically laid down in the draught. We shall here further observe, that these ribbands will round two ways, one in a vertical, and one in an horizontal sense, occasioned by the nature of the form of the ship's body; for they will, in carrying them about, naturally fly higher abaft and before than they are in midships, which gives them a vertical curve, and the narrowing of the ship's breadth from the midships both ways gives them the horizontal curve; thence they will be represented by different lines on all the planes.

They are represented upon the plane of the projection by straight lines, all but the breadth ribband, which is usually represented by a curve; but upon the plane of elevation, and that of the horizon they will be represented by curves. The reason of these different appearances, arises from the different situations in which they are supposed to be viewed, as was observed in respect of the water lines.

Now in order to comprehend the relation betwixt these horizontal curves, and the lines that represent them upon the plane of the projection, it will be sufficient to remark, that these horizontal curves result from the different lengths of the perpendiculars that are supposed to be drawn in the plane of the projection, from the points where the lines that represent the ribbands intersect the frames, to the middle line. Hence, if the lengths of these perpendiculars are transferred to the lines corresponding to each frame in the horizontal plane, we shall have the points thro' which the curve that forms the ribband must pass.

But if these ribbands are to be represented upon a plane placed in an oblique position to the horizon, that is to say, a plane that has the same inclination to another plane erected perpendicularly upon the middle line of the keel, that the line that represents that ribband, has to the middle line in the plane of the projection; in that case, they will have a quite different form from what they have upon the plane of the horizon.

Now, to conceive the relation betwixt these and the lines that represent them upon the plane of the projection, it will be sufficient to remark, that if the several distances taken upon each diagonal intercepted betwixt the middle line and the points where these diagonals intersect the curves of the timbers in the plane of the projection; I say, if these be transferred to the lines that represent those timbers, we shall have the points thro' which the curves that form the ribbands must pass.

Again, if these ribbands are to be represented upon the plane of elevation, they will have a different form from any of the former; to find which we need only take the perpendicular distances from the points where the diagonals intersect the curves of the timbers in the plane
of

of the projection to the line that represents the upper side of the keel, and transfer them to the plane of elevation, setting them up from the upper side of the keel upon the line corresponding to the timber, from which they were taken upon the plane of the projection. This will give us the points thro' which their curves must pass.

Having thus given a general description of these curves, we shall now proceed to describe them upon the different planes.

To describe the Floor Ribband upon the Plane of Elevation.

1st. Take the perpendicular distance betwixt the point *a*, where the diagonal intersects frame 9, and the lower water line in the plane of the projection.

2d. Set up this distance from the point *S*, where the lower water line intersects frame 9 in the plane of elevation, and we shall have a point *G*, thro' which the curve must pass.

Now, it is plain, that we may, by repeating the same operations, have a point in each frame, thro' which the curve of the ribband must pass upon the plane of elevation. After the same manner are all the other ribbands formed.

To describe Ribbands upon the Horizontal Plane.

The breadth ribband is formed by transferring the lengths of all the perpendiculars that are supposed to be drawn from the points where the curve that represents this ribband intersects the timbers, to the middle line in the plane of the projection: This curve, in the plane of the projection, is drawn from the breadth in midships to the extremity of the wing transom.

1st. Lay off the length of the wing transom upon the perpendicular *uL*.

2d. Take the length of the perpendicular drawn, from the point where the curve that represents the breadth intersects frame 9, to the mid-line in the plane of the projection; lay off this from the line *AB* upon the perpendicular representing frame 9 in the horizontal plane, to the point *S*, which will be one of the points thro' which the curve of the ribband must pass. We may proceed in the same manner to find points upon all the perpendiculars, both afore and abaft, so shall the curve *L, S, Q, t*, be the form of the breadth ribband. But to compleat this ribband, the round aft of the wing transom must be set off.

To form the Oblique or Cant Ribbands.

We observed before, that these ribbands could not be formed either upon the horizontal plane or that of elevation, upon which account they were seldom drawn, because each must be drawn upon a separate plane. However, those who incline to draw them may use the following method:

Let it then be required to form the first ribband represented in the plane of the projection, by the diagonal H G.

1st. Produce line H G to the point *p* in the middle line upon the plane of the projection.

2d. Take the height of the point *p* above the line that represents the upper side of the keel in midships, in the plane of the projection; set up this from the same line in the plane of elevation, on a perpendicular, upon the post, from which point let fall a perpendicular to the point F in the line CD, and produce all the perpendiculars that represent the frames to the line CD; so F, Q will be the axis of the ribband from the post to the midships.

3d. Take upon the plane of the projection in the line H P, the distance *p* G, which set off upon the perpendicular from the point F to *f*,

4th. Take the distance on the diagonal from the point *p* in the middle line to its intersection with the frame 9. Set off this from the line CD upon the perpendicular corresponding to frame 9; this will give us a point thro' which the curve must pass. Do the same for all the other frames to the midship.

In like manner the curve for the fore part of the ribband is formed from the intersections of the diagonal 4, 5, with the curves of the frames in the plane of the projection; but it is evident, this is a different plane from that of the line H *p*; therefore we must have a different axis for the curve of the fore part of the ribband. In order to which, take from the plane of the projection the diagonal 4, 5; set off this from the point O to Z, and draw the line Z X parallel to CD. We must likewise take the height of the point 4 in the plane of the projection, and set it up on the stem; from which point letting fall a perpendicular to the line Z X, we shall limit the fore end of the ribband. The points thro' which the curve must pass will be found in the same manner as those for the after-body.

The builders make use of the cant ribbands to find the bevellings of the timbers; For we must represent each frame as one intire piece of circular timber, and being all fastened to the keel they form the side of the ship. They are square upon the upper side of the keel; but because
both

both the outside and inside of the ship's sides, length-ways, form curves, it is plain, that the sections of any of the frames, the midship only excepted, will produce a surface in the form of a lozenge or rhombus; the angles which are formed by these sections are what are called the bevellings of the timbers.

The ship-wrights take these angles mechanically by an instrument call'd a bevel; thus they draw, upon the plane of the ribband, a line parallel to that which represents the frame, and distant from it the whole breadth of the timber; and applying the stock of the bevel to the line that represents the frame, and the tongue to the ribband, they have the quantity of the angle which forms the bevelling of the timber at that place.

It is plain the angle b, a, c , which points to the midship frame, will be obtuse, whereas the angle b, a, d , which points to the post, will be acute.

Now, as every timber has two planes, that which points to the midships will have what they call a standing bevelling, and that which points either to the post or stem will be under bevelling.

We shall shew in another place how the modern builders, by putting the frames in an oblique position to the keel afore and abaft, lessen the bevellings.

CHAP. VI.

Another Method of laying down the Horizontal Plane, and the Plane of Projection.

THOSE who are well versed in the art of drawing have taken a method quite different from any of those we have described, which shall be the subject of this chapter.

After forming the plane of elevation, and drawing all the perpendiculars for the frames, as before, the following method must be observed:

I.

To lay down the Breadth Ribband on the Horizontal Plane.

The extremities of it on the stem and post, and the point thro' which it is to pass on the midship frame are found as directed in the preceeding chapter. It remains now to find the points in the balance frames, thro' which it is to pass.

To find the point in the fore balance frame take $\frac{11}{16}$ parts of half the main

main breadth, which set off on the line that represents that frame in the horizontal plane from K to L.

To find the point in the balance frame abaft, take $\frac{1}{10}$ parts of the half of the main breadth from M to N. It will be necessary to have another point in the fore-body, thro' which the curve must pass; for which purpose use the following method :

Divide the space contained betwixt the line that represents the balance frame afore and the rabbit of the stem, into two equal parts, and draw the line OP, on which set off the 160th part of the main breadth, which will give the point P, thro' which the curve is to pass. It must be observed, that the proportions for finding these points may be varied according to the form we propose to give to the ribband. After the points H, N, Q, L, P, I are thus set off, we may describe the curve either by moulds or penning battens.

II.

To lay down the Floor Ribband on the Horizontal Plane.

1st. The height of this ribband must be determined both upon the post and stem, from which points letting fall perpendiculars, we shall have the extremities of it on the horizontal plane, observing to allow for the rabbit.

2d. Take half the length of the midship floor timber, and set off on the line that represents the midship frame on the horizontal plane from *a* to S, which will be the point thro' which the curve must pass.

3d. Take $\frac{1}{10}$ of the line *a s*, the breadth at the midship frame, and set it off on the balance timber afore, from K to T, and set off $\frac{1}{10}$ of the same line, upon the balance timber abaft to V, and draw the curve thro' the points G, V, S, T, R.

III.

To lay down the after Balance Timber upon the Plane of Projection.

1st. Produce the line which represents it on the horizontal plane to the sheer rail, on the plane of elevation, and take the distance upon this line betwixt the upper side of the keel, and the lower edge of the second wale, which here represents the breadth ribband; set up this from A to C on the plane of the projection, from which point draw the line C D, perpendicular to the middle line. (*Plate II. Fig. 1, 2, 3.*)

2d. Take the line M N in the horizontal plane, and set off from D to E, which will give one point, through which the curve of the timber must pass.

3d. Take

3d, Take the height of the floor ribband, in the plane of elevation, and set it up on the plane of the projection to G; from the point H, at the end of the floor timber, draw the line H G which will represent the floor ribband on the plane of the projection. (*Plate II. fig. 1. 2. 3.*)

4th, Take the distance M V, in the horizontal plane, with a pair of compasses, and move the compasses with one foot, in the middle line, and the other in a line perpendicular to it, till it intersect the diagonal in the point L, thro' which the curve of the frame must pass. To those who are acquainted with drawing, the three points E, L, F, will be sufficient to form the timber; they who incline to have another point may divide the line A C into two equal parts by a perpendicular M K drawn to the middle line, from which setting off $\frac{1}{4}$ of the line M K we shall have another point thro' which the curve must pass.

IV.

To lay down the ninth Frame abaft on the Plane of the Projection.

Take the height of the breadth ribband at this frame, in the plane of elevation, and set it up on the plane of the projection from F to O, and draw the line O P perpendicular to the middle line. (*Plate II. fig. 1. 2. 3.*)

2d, Take the distance X S in the horizontal plane, and set off from O to P, which will be the point thro' which the curve must pass.

3d, Take the distance X Z in the horizontal plane, which set off from the middle line, to intersect the diagonal that represents the floor ribband, in the plane of projection in Q, observing to keep the compasses as before directed.

4th, Divide the line K O in two equal parts, and draw the line R S perpendicular to the middle line, on which set off $\frac{1}{4}$ of the line P O, from R to S, and draw the curve thro' the points P, S, Q, F, which will be the form of the ninth frame.

V.

To lay down the intermediate Ribbands abaft on the Plane of the Projection.

1st, Take the distance betwixt the upper side of the keel and the breadth, upon the line that represents the midship frame, in the plane of elevation, and set it up from A to T, and from B to T, in the plane of the projection, so shall the line T T give the height of the breadth ribband in midships.

2d, Divide the curve H M T into as many equal parts as there are to be intermediate ribbands; divide also the curve of the ninth frame Q S P into the same number, and, thro' these divisions, draw the diagonals which will represent the ribbands as in the plate.

VI.

To lay down the first intermediate Ribband upon the Horizontal Plane.

1st, Take the nearest distance of the point V (which is the extremity of the diagonal in the plane of the projection) to the middle line OF, set off this on the line which represents the midship frame in the horizontal plane, which will give the point thro' which the curve must pass at that place. After the same manner we may find the points in the lines that represent the balance and ninth frames in the horizontal plane.

2d, Take FZ, the height of the ribband upon the rabbet of the post, in the plane of the projection, and set it up on a perpendicular, from N to the point *k* on the line that represents the rabbet of the post in the plane of elevation; take the nearest distance of the point *k* to the perpendicular of the post, which set off from E to *e*, and this will be the end of the ribband: so a curve passing thro' the points *e*, *d*, *c*, *b*, will be the form of the ribband.

VII.

To lay down the Wing Transom upon the Plane of the Projection, and on the Horizontal Plane.

1st, Take the height of the upper side of the wing transom (including the round up) in the plane of elevation, and set it up in the plane of the projection to the point *e*.

2d, Take the height in the plane of elevation, without regarding the round up, and set off from F to *f*, and draw the line *fg* perpendicular to the middle line, on which set off the length of the transom from *f* to *g*, this is equal to the line GH in the horizontal plane. The curve *ge* represents the upper side of the wing transom.

The round aft of the transom is represented upon the horizontal plane by the curve *Lke*; HL is the square end of it.

VIII.

To lay down all the Frames in the after Body.

All these are laid down in the same manner as the ninth and balance frames before described, that is, by taking the half breadth of the ribbands at each frame in the horizontal plane, and setting them off from the middle line in the plane of the projection to intersect the diagonal corresponding to the ribband, as directed in forming the balance frame, by this means we shall divide each into as many points as there are frames: the curves drawn thro' these points will give the form of all the frames in the after body.

IX. To

IX.

To lay down the Position of the Fashion Piece on the Horizontal Plane.

Let fall the perpendicular GH, from the end of the wing transom, and draw the line H I, which will represent the plane of the fashion piece upon the horizontal plane, observing to make the angle G H I, about 25 degrees.

X.

To form the Fashion Piece in the same Manner it is to be, when put into its proper place in the Ship.

The fashion piece laid down in the plane of the projection, regards that frame as it would appear when viewed from abaft; but as the fashion pieces on each side are not in one plane, as all the rest of the frames are, we shall be much deceived, if we imagine that the fashion piece laid down in the plain of projection, will give the true form of that which is to be put in the ship. We must therefore lay it down upon another plane, and, to avoid confusion, we shall separate it from the plane of projection.

Note, The fashion piece, mention'd by our author, described in the plane of the projection, is that betwixt the ninth frame, and the curve f q o G, which represents the fashion piece of a square tuck; it is formed in the same manner as the rest of the frames, by transferring the lines n m, p o, &c. in the horizontal plane, to the plane of projection, to intersect the diagonals corresponding to these ribbands in the points i, l, &c:

1st, Draw the line f G, to represent the middle line of the plane of projection. (Fig. 4.)

2d, Draw the line f g perpendicular to G f, to represent the wing transom.

3d, From l, the point where the fashion piece intersects the floor ribband in the plane of the projection, take the nearest distance to the line f g, which represents the wing transom, and set off this distance in Fig. 4. from f to h, and draw the line b l parallel to f g.

4th, From the point l, where the fashion piece intersects the first intermediate diagonal, in the plane of projection take the nearest distance to the line f g, set it off from f to k, in Fig. 4. and draw the line k m, parallel to f g.

5th, In like manner, the points where the fashion piece intersects the second and third diagonals in the plane of projection, are to be transferr'd to the points q and n, Fig. 4. and the lines p q, n o drawn parallel to f g.

G 2

6th,

6th, To find the points through which the curve must pass: Take the line I/H , which represents the position of the fashion piece upon the horizontal plane; lay this off from f to g : Again, take the distance ly in the horizontal plane, which lay off from q to p , in like manner set off the distance lx , from n to o ; and the distance lp from k to m , and lastly, the distance ln , from b to l ; so a curve drawn through the points g, p, o, m, l , will give the true form of the fashion piece.

XI.

To lay down the Fashion Piece upon the Plane of Elevation.

1st, Take the several heights above the keel, of the points where the fashion piece intersects the diagonals in the plane of projection, and transferr them to the lines o, p, q, z, y , in the plane of elevation, drawn parallel to the keel, and the same height above it, that their corresponding points are in the plane of projection.

2d, Take the nearest distance of the point n , in the plane of elevation, to the line CF , the perpendicular from the head of the post, set off this from the same line in the plane of elevation upon the line p ; which will be the point through which the curve must pass.

3d, In like manner the points z, y , must be transferr'd from the horizontal plane, to the plane of elevation in the points z, x , a curve passing through these points will be the projection of the fashion piece on the plane of elevation.

We shall hear remark, that some builders to avoid giving a great bevelling to the timbers, and likewise that they may not require such compass timber, do change the direction of all the frames in the fore-body before that of the loof; that is, the lines that represent them in the horizontal plane make an acute angle with the line that represents the keel; these are called cant timbers, and may be formed in the same manner as the fashion piece, which we have now described. Tho' several builders form all the frames perpendicular to the keel, to have the floor timbers in one piece, which will be much stronger than when in two pieces, and this will inevitably be the case when the timbers are canted.

We might here shew how to lay down the top timbers, but as that part under water is the most material, we shall proceed to form the timbers afore.

XII.

To lay down the Frames for the Fore-body.

The balance and the eighth frame must first be formed in the same manner as the balance and ninth frame abaft: In order to which the curve that

that represents the breadth ribband must be laid down in the plane of the projection afore. The diagonal, which represents the floor ribband must likewise be laid down in the plane of the projection, for which purpose we must take the height of the ribband above the keel upon the rabbit of the stem, and set it upon the line that represents the rabbit of the post in the plane of the projection, to the point 4; from which draw a line to the floor head, so 4 5 will represent the floor ribband.

XII.

To space the Diagonals that represent the Ribbands afore, in the Plane of the Projection.

1st. As the points of their intersection at the midship frame are the same afore that they are abaft, we need only transfer them from abaft to the fore body.

2d. Take the height of the breadth ribband upon the stem in the plane of elevation, and set it up from F to 17 in the plane of projection.

3d. Divide the distance betwixt 4 and 17 into four equal parts, which will give the points in the plane of projection, where the intermediate diagonals end on the stem.

After the diagonals are drawn in the plane of the projection, the ribbands may be laid down in the horizontal plane, and from thence all the other frames may be laid down in the plane of projection, in the very same manner that the horizontal ribbands and the frames for the after-body were laid down.

C H A P. VII.

General Remarks on Ship Building.

ALL the rules we have hitherto laid down, collected from the principal dimensions of ships built by the most eminent masters, should only be so far regarded as they may assist the artist in forming the body in such a manner as to produce effects answerable to the service for which the vessel is designed.

In order to qualify a builder for such an undertaking, it is necessary he should understand the nature of fluids, and of such bodies as will float in the water; when he has made himself acquainted with these, I would recommend him to Mr *Bouguer's* treatise on ship-building.

The principal Qualities belonging to Ships.

1st. To be able to carry a good sail, not only because in forming the body, the water lines are all supposed to be described when a ship is upright in the water, but likewise for doubling a cape, or getting off a lee shore, which will be impossible to be done when a ship lies over in the water, this will likewise render her lower tier, if not all her guns useless.

2d. A ship should steer well, and feel the least motion of the helm.

3d. A ship should carry her lower tier of guns four feet and a half, or five feet out of the water, otherwise a great ship that cannot open her ports upon a wind, but in smooth water, may be taken by a small one, that can make use of her guns, or she must bare away before the wind, to have the use of her guns; on which account it will be proper to raise the ports higher before than in midships, because the fore part of the ship is often pressed into the water by carrying sail.

4th. A ship should be duly poised, so as not to dive or pitch hard, but go smooth and easy through the water, rising to the sea when it runs high, and the ship under her courses, or lying to under a mainsail, otherwise she will be in danger of carrying away her masts.

5th. A ship should sail well before the wind, large, but chiefly close hawled, keep a good wind, not fall off to the leeward.

Now the great difficulty consists in uniting so many different qualities in one ship, which seems indeed to be impossible; the whole art therefore consists in forming the body in such a manner, that none of these qualities shall be entirely destroyed, and in giving the preference to that which is most required in the particular service for which the vessel is built; in order to
which

which it will be necessary to know, at least nearly, what form will give a vessel one of these qualities, considered abstractly from the rest.

To make a Ship carry a good Sail.

A flat floor timber, and somewhat long, or the lower futtock pretty round, a streight upper futtock, the top timber to throw the breadth out aloft; at any rate to carry her main breadth as high as the lower deck: now, if the rigging be well adapted to such a body, and the upper works lightened as much as possible so that they all concur to lower the center of gravity, there will be no room to doubt of her carrying a good sail.

To make a Ship Steer well, and quickly Answer the Helm.

If the fashion pieces be well formed, and the tuck carried pretty high; the midship frame carried pretty forward; a considerable difference of the draught of water abaft more than afore, a great rake forward and none abaft, a snug quarer deck and forecastle, all these will make a ship steer well; but to make her feel the least motion of her helm, it will be necessary to regard her masts. There is one thing not to be forgot, that a ship which goes well will certainly steer well.

To make a Ship carry her Guns well out of the Water.

It is plain that a long floor timber, and not of a great rising, a very full midship frame, and low tuck with light upper works will make a ship carry her guns high.

To make a Ship go smoothly through the Water without pitching hard.

A long keel, a long floor not to rise to high afore and abaft, the area or space contained in the fore body, duly proportioned to that of the after body, according to the respective weights they are to carry; all these are necessary to make a ship go smoothly through the water.

To make a Ship keep a good Wind.

A good length by the keel, not too broad, but pretty deep in the hold, which will occasion her to have a short floor timber, and great rising.

As such a ship will meet with great resistance in the water going over the broad side, and little when going a head, she will not fall much to the leeward.

Now some builders imagine that it is not possible to make a ship carry her guns well; carry a good sail; and to be a prime sailer, because it
would

would require a very full bottom to gain the first two qualities, whereas a sharp ship will best answer for the latter; but when it is considered that a full ship will carry a great deal more sail than a sharp one, a good artist may so form the body as to have all these three good qualities, and likewise steer well, for which purpose I would recommend somewhat in length more than has been formerly practised.

After what has been said upon this head, I believe it will not be thought impossible to unite all these different qualities in one ship, so that all of them may be discerned in some degree of eminence, but when it happens otherwise, the fault must be owing to the builder, who has not applied himself to study the fundamental rules and principles of his art.

Excepting some antient builders, who were happily born with a natural genius, and our moderns, who being instructed in the principles of the mathematics, have truly laboured very hard to make a progress in the art of shipbuilding, one may, without violating the truth, affirm that the greatest part satisfy themselves with copying such ships as they esteem good sailers, and it is these servile mechanick methods, which to the great reproach of the art, are but too common, that have produced all these pretended rules of proportion, all these methods of describing the midship frame, and forming the rest of the timbers, which every builder endeavours if possible to conceal and keep wholly in his own family.

How low and mean is this? it is as if a great architect should endeavour to conceal the proportions of the different orders of architecture, whereas they are published every where, and so well known that many can raise a very beautiful porch or triumphal arch; but tho' the methods of describing the midship frame and forming the rest of the timbers be known to most apprentices, yet we have but few good master builders: This requires more than those mechanick rules, they should at least have such a knowledge of the mathematics, physics, mechanicks, of the nature of solids and fluids, as to be able to discover what figure would procure some good quality without hazarding or putting a bad one in its place.

Let us suppose one to have a collection of draughts of a vast number of ships, and whose good and bad qualities have been remarked with all possible exactness, such a valuable treasure would be of great service to a person who could calculate precisely by the draughts where the fault lay, and how it might be rectified. For instance, suppose a ship sails well, but carries her guns too low, a builder who is not acquainted with these principles would raise her deck, in consequence of which she would not sail well; whereas one that could exactly calculate how much the resistance of the fluid is diminished upon the prow, would take great care

to add no more to any of the other parts than he could find by an exact calculation might be done without augmenting the resistance in the fluids.

M. Bouguer has published several useful problems for making these calculations, to which we refer the reader, and only explain what regards the height of the gun deck, and the resistance of the fluid, in one example of a 70 gun ship.

C H A P. VIII.

To know by the Draught how high a Ship will carry her Guns out of the Water.

THIS is only to know if, when a ship is loaded with all her ammunition and provisions on board, and ready to sail, her seat in the water will then agree exactly with the load water line in the draught.

It may be demonstrated by several experiments, that any floating body of whatsoever figure will just sink so far in the water as to displace a bulk of water of equal weight with itself.

Hence it will be necessary, first to find a method of calculating the exact weight of a ship ready equipt for sea, and, secondly, to know the exact weight of the water the ship displaces, when loaded to the water line in the draught.

In order to the first, the exact weight of all the timber, iron, lead, masts, sails, rigging, and in short of all the materials, men, provisions, and every thing else on board the ship must be known.

It must be confessed that this is a very laborious task, yet the zeal of our modern builders has surmounted all these difficulties, and got the exact weight of a ship of each class with all its furniture, and six months provisions on board. It will be sufficient for our purpose to give the particulars, of the two followings, one of 30 and another of 50 guns, both ready equipt for sea, with six months provisions on board.

An Estimate of the Weight of the RENOMEE Frigate of Thirty Guns, with Six Months Provisions.

WEIGHT of the HULL.

	Cubic feet	Under water Pounds.	Above water Pounds.	Total in Tuns. Tons Pounds
Oak timber { under w. at 72 lb. per f.	5640	406080		
ber { above w. at 66 lb.	2920		192720	299 800
Fir at 50 lb. per foot { under water	600	30000		
{ above water	560		28000	29 000
Carved work			2200	1 200
Iron knees and standards		4200	7010	5 1210
Bolts, rudder irons, chain plates, nails		11650	6558	9 208
Lead for the haufe holes & scuppers		250	430	0 680
Locks			170	0 170
Oakum		1200	1830	1 1030
Pitch and Tar			650	0 650
Paint			440	0 440
In the Cook room			8000	4 000
Total		453380	248008	350 1388

WEIGHT of the FURNITURE.

	Pounds.	Pounds.	Tons	Pds
Mafts compleat set and spare	3000	37000	20	00
Blocks	1000	5444	2	444
Pumps	1734	670	1	404
Cables and Hawfers	24444		12	444
Sails and their Cafes	4222	3778	4	000
Anchors and their Stocks	2611	6944	4	1555
Cordage for the rigging		17282	8	1282
The master's Stores	3333		1	1333
Boats		6666	3	666
	40344	75784	58	128

WEIGHT of the PROVISIONS, &c.

	Under water Pounds.	Above water Pounds.	Total in Tuns. Tons Pounds
Provisions for 6 months for 200 men with all their equipage	245420		122 1420
Water for two months and a half	100000		50 000
Casks	32800		16 800
The Captain's table	15000	5000	10 000
Total	393220	5000	199 220

WEIGHT of the OFFICERS STORES.

	Under water Pounds.	Above water Pounds.	Total in Tons	
			Tons	Pds
The Carpenter's Stores	3000	1000	2	00
The Caulker's Stores	1000		0	1000
The Surgeon's Effects	2400		1	400
The Pilot's Effects	740	360	0	1100
The Chaplain's Effects		100	0	100
	7140	1460	4	600

WEIGHT of the GUNS and AMMUNITION.

	Under water Pounds.	Above water Pounds.	Total in Tons	
			Tons	Pds
Iron Guns		60300	30	300
Carriages fitted		14000	7	00
Balls round and crofs bar	11570	2430	7	00
Balls of one pound	600		0	600
Powder and Powder Barrels	7108	112	3	1220
Implements for the powder	1368	132	0	1500
Crows, Handspikes, Gunners Uten- fils, and Stores	3200	1500	2	700
Musquets, Cutlasses, and Pole Axes		900	0	900
	23846	79374	51	1220

WEIGHT of the MEN and their EQUIPAGE.

	Under water Pounds.	Above water Pounds.	Total in Tons.	
			Tons	Pounds
8 principal Officers and their Effects		4000	2	00
200 Men and their Effects		40000	20	00
Total		44000	22	00
BALLAST	200000		100	00

RECAP.

RECAPITULATION.

	Under water Pounds.	Above water Pounds.	Total in Tons.	
			Tons	Pounds
The Hull	453380	248008	350	1388
The Furniture	40344	75784	58	128
The Provisions	393220	5000	199	220
Officers Stores	7140	1460	4	600
Guns and Ammunition	23846	79374	51	1220
Weight of the Men		44000	22	00
Ballast	200000		100	00
Total	1117930	453626	785	1556

An Estimate of the Weight of a Frigate of Fifty Guns, with Six Months Provisions.

	Under water Pounds.	Above water Pounds.	Total in Tons.	
			Tons	Pounds
The Hull	774270	769134	771	1404
The Furniture	98237	163184	130	1421
Ballast	300000		150	000
Guns and Ammunition	67960	199320	133	1280
Provisions	659400	8000	383	1400
Stores	9800	2800	6	600
Men and their Equipage		77000	38	1000
	1909667	1219438	1564	1105

But as all ships of the same class are pretty near the same dimensions, and have the same number of guns, &c. we may have the exact weight of each only by examining the draught of water, and computing the weight of that column of water which is displaced by the ship.

Now if the *Intrepide* weighs 2718 tons, she must sink so far into the water till she has displaced a column of water containing 73459 $\frac{17}{47}$ cubick feet, for a cubick foot of salt water being supposed to weigh 74 lb. the 73459 $\frac{17}{47}$ will weigh 5436000 lb. or 2718 tons, or if she displaces 73459 $\frac{17}{47}$ cubick feet of salt water, we may thence conclude that she weighs 2718 tons.

In like manner, if the weight of the ship which is to be laid down in the

the draught be known; as, for instance, that of a ship of 78 guns, is 2350 tuns, we may with certainty know if the water line in the draught be properly placed, only by reducing the bottom into cubick feet.

The antient builders were unacquainted with the manner of performing this, but our moderns make an exact calculation of the contents of the bottom before they begin to build, whereby they will be sure to keep the lower tier of guns well out of the water.

If a ship's body were any regular figure, the solid contents of it could easily be found geometrically, but as the case is quite otherwise, we must be satisfied with dividing it into several parts, of which we may have a great number, and they will thereby become so small, that they may, without any sensible error, be esteemed as regular figures, limited by straight lines, tho' some of them are actually curves.

In the draught of the 70 gun ship which we have laid down, the bottom is divided on the plain of elevation into several parts, in a vertical way by the lines that represent the frames; and in an horizontal way by the water lines, so that the whole may be said to be divided into so many parallelopipedons, A, B, C, D , or a, b, c, d , contained betwixt the two frames 6 and 7, and limited on the side AB by a plain supposed to be erected vertically upon the keel, and on the other side by the round of the outside of the ship, at the height of the breadth water line, or ac . Now it is very plain that the area of the surface, which limits the lower part of this solid, is less than the area of the surface, which limits the upper part: But if we increase the water lines, and frames we may find the solid contents to a sufficient exactness for our purpose.

Now, in order to find the area of the upper surface $ABDC$, let AC be 16 feet 11 inches, and BD 13 feet 6 inches; add these two, the sum is 30 feet 5 inches, the half of which is 15 feet two inches and a half, and this sum multiplied by AB , which suppose 8 feet, the distance betwixt the frames, the product is 121 feet 8 inches, the area of the upper surface of the parallelopipedon.

The area of the lower surface of the parallelopipedon may be found after the same manner, which suppose 97 feet 4 inches. Now, if these two areas be added together their sum will be 219 feet, the half of which is 109 feet 6 inches for the mean area, and this multiplied by ab , the distance betwixt the water lines, which suppose 4 feet 4 inches, produces 474 feet 6 inches cubick.

By the same process we may find the solid contents of the other parallepipeds, and adding them together, and doubling that sum we shall have the

the solid content of the whole bottom of the ship in cubick feet to a sufficient degree of exactness.

I made use of this method before Mr *Bouguer's* treatise was published, where there is one which is more convenient and expeditious, for instead of finding the area of every single surface contained betwixt the frames upon the section of a water line, he finds by one operation the area of the whole surface formed by the horizontal section or water line, except that part intercepted betwixt the aftermost frame and the post, and the part contained betwixt the the foremost frame and the stem, which upon account of the rake must be measured separately, as also all that lies betwixt the upper side of the keel and the first water line. His method is as follows:

Take the lengths of all the lines that represent the frames on the horizontal plane, add all these together, excepting the foremost and aftermost, of which take only one half of each, so if it were required to find the area of the surface formed by a horizontal section in the plane of the load water line, it will be $\frac{1}{2} ZZ + BD + AC + IH + LK$, &c. $+ \frac{1}{2} X^S + AB$, supposing AB to be the distance betwixt the frames equally spaced betwixt ZZ and NO.

To demonstrate this, let it be considered by what operation the two trapezia ABDC and HIAC are measured. We observed in the preceeding article that this was performed by adding the length of the lines BD and AC together, and then taking half that sum; the length of the lines AC and HI, must likewise be added together, and the half of that sum taken; now it is evident that it will be same thing to take half the line BD, and half the line HI, and the whole line AC, and add all these three together, because the line AC, is common to both the trapezia.

After the areas of all the water lines are thus found, the solid content of the space contained betwixt the water lines may be had by multiplying the area by the distance between the water lines: But because the areas of the two surfaces which limit this part are unequal, a mean area must be found; this is half the sum of the two areas, so that all that is now to be done, is to add the areas of the water lines into one sum, excepting that of the uppermost and lowermost, of which only one half of each must be taken, and if this sum is multiplied by the distance betwixt the water lines, the product will give half the solid content of the bottom, observing that the water lines in the plain of elevation be equally distant from one another.

The application of this method in finding the cubick feet contained in a 70 gun ship laid down in the draught.

The

The forepart is divided into eight, and the after into nine equal parts, besides that betwixt the aftermoſt timber and the poſt, and that betwixt the foremoſt timber and the ſtem.

The bottom is likewiſe divided into four equal parts by water lines drawn parallel to the load water line, all which are formed upon the horizontal plane, for it will be very uſeful to know the ſolid content of each particular part contained betwixt the water lines, alſo to diſtinguiſh that of the fore body from the after body, whereby we may be enabled to know if the weight be duly poſed. We ſhall conſider all this in the following calculation.

Note, there muſt be four inches added to each line that represents the frames in the horizontal plane for the thickneſs of the plank, that being nearly a mean betwixt the thickneſs of the plank next the wale, and that next the keel.

The Area of the Upper Water Line abaft.

The breadth of the ſurface at the load water line, upon the midſhip frame a Q is 21 feet 2 inches,

		feet.	inch.
	one half is	10	7
Breadth at	1ſt Frame	21	2
	2d Frame	20	11
	3d Frame	20	9
	4th Frame	20	5
	5th Frame	19	11
	6th Frame	18	11
	7th Frame	17	4
	8th Frame	15	7
The 9th Frame X S is 12 feet 9 inches, one half of which is		6	4½
		Total	171 11½

which total doubled is 343 feet 11 inches, and multiplied by 8, the diſtance betwixt the frames, is the whole area of the water line from the midſhip to the after frame, in cubick feet

To this muſt be added the area of the trapezium X S L e

Now half of the lines X S and L e is 10 Feet 0 Inches

Diſtance betwixt them is

	9	9
Product is	97	6
which being doubled is		195 0
The whole area in cubick feet		2946 4

By uſing the ſame proceſs we may find the areas of all the other water lines, and adding all theſe areas together, excepting that of the firſt and fifth, of which taking only one half, multiply this ſum by 4 feet 5 inches, which

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 which is the distance betwixt them, we shall have the area in cubick feet
 of that part of the ship abaft the midship frame, contained betwixt the
 lower water line, and load water line.

	feet	inch.	l.	p.
Half the area of the load water line	1473	2	0	0
Whole area of the 4th water line	2516	1	4	0
Whole area of the 3d water line	2052	0	4	0
Whole area of the 2d water line	1452	10	7	6
Half the area of the 1st water line	144	3	2	0
Total	7638	5	5	6
Multiplied by the distance betwixt the water lines	4	5	0	0
Product in cubick feet betwixt the lower, and load water line.	33736	6	1	3 6
Betwixt the lower water line and keel	333	6	3	0 0
Keel and post	101	8	0	0 0
Cubick feet abaft the midship frame under water, when loaded	34171	8	4	3 6
Cubick ft. before the midship frame under water, when loaded	28928	6	1	0 0
Total cubick feet under water	63100	2	5	3 6
Multiply by the weight of a cubick foot of salt water		pounds	tuns.	pounds lb.
Weight of the whole ship with all her furniture provisions &c.	4661498	2334	1498	

We have omitted the operation for the fore part, because it is per-
 formed exactly by the same method with the after part.

It must be observed that in finding the cubick feet of that part contained
 betwixt the lower water line and upper side of the keel, we must take
 the heights of all the frames intercepted betwixt these two lines, and di-
 vide their sum by the number of frames abaft the midship, the quotient
 will be 1 foot 9 inches 9 lines.

	Feet	In.	Lines
The area of the lower water line is	288	6	4
The area of the upper side of the keel	79	6	0
Total	368	0	4
one half is	184	0	2
Area of that part contained betwixt the lower water line and keel	333	6	3

The use of the preceeding calculation is to know if the load water
 line upon the draught be properly placed.

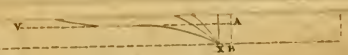
It has been found that a ship of 70 guns, with every thing on board,
 should weigh nearly 2350 tons, which is only 15 tons 1297 pound more
 than what is found by calculating from the load water line in the draught,
 this difference would occasion the ship not to draw above one inch more
 water

The first part of the paper is devoted to a general
 consideration of the problem. It is shown that the
 problem is equivalent to the problem of finding the
 minimum of a certain function. This function is
 defined by the following expression:

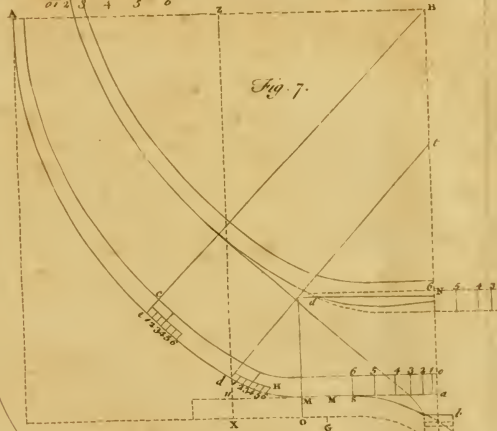
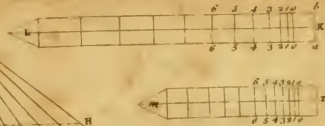
$$F(x) = \frac{1}{2} \int_0^x (f(t) - g(t))^2 dt$$
 where $f(t)$ and $g(t)$ are functions of t . The
 minimum of this function is found by the method
 of steepest descent. The results of the calculation
 are given in the following table:

x	$F(x)$
0	0.00
0.1	0.01
0.2	0.04
0.3	0.09
0.4	0.16
0.5	0.25
0.6	0.36
0.7	0.49
0.8	0.64
0.9	0.81
1.0	1.00

 The results show that the minimum of the function
 is reached at $x = 0$. This is the solution of the
 problem. The method of steepest descent is a
 very simple and effective method for finding the
 minimum of a function. It is applicable to a wide
 range of problems. The results of the calculation
 are in good agreement with the theoretical results.
 The method of steepest descent is a very useful
 tool for the solution of many problems in
 mathematics and physics. It is a simple and
 effective method for finding the minimum of a
 function. The results of the calculation are in
 good agreement with the theoretical results.









water it is not worth the regarding. But then by this calculation we discover that the ship is too lean before; for whereas the fore part should exceed the after part by 30 tuns, we find, by this calculation, that the after part exceeds the fore part by 193 tuns 1995 pounds.

Upon this account we must consider carefully if the midship frame is properly placed; it is here 5 feet before the middle. If the midship frame was exactly in the middle it would augment the weight of the fore part 102 tuns 307 pounds, and diminish that of the after part exactly the same quantity, by which means the fore part would be 1172 tuns 1014 pounds, and the after part 1162 tuns. Now we may fill out the fore part, so as to gain 15 tuns 636 pounds, which was deficient, to make the calculation taken from the draught agree with the real weight proposed for a ship of 70 guns fitted out for sea, with six months provision on board; and the fore-body will weigh 25 tuns 1255 pounds more than the after part, which may be judged sufficient.

C H A P. IX.

A method to calculate the Resistance of the Water upon the fore Part of the Ship.

DAILY Experience sufficiently proves that the fluids, by their motion, attack the solids that oppose them, as bridges, mills, &c. with such violence as to carry all before them; and this is agreeable to the very nature of fluids.

For all fluids are an assemblage of a prodigious number of small solid bodies of a globular form, each of which being easily put in motion will act upon any surface with the same force that any other solid body of the like mass would do. But as these particles have but a very small cohesion with each other, fluids cannot act with the same force as solids which have their parts united.

A mass of water of 20 cubical feet will not act with the same force upon the pier of a bridge which opposes it, as a mass of ice of the same dimensions; because the whole mass of ice having its parts so united together, that one cannot advance without the other, it gives the blow with the united force of all the parts at once, whereas the parts that compose the mass of water, being but slightly united, they cannot act jointly or in concert, and they exert their force one after another; they indeed succeed one another immediately, and are a little united by their reciprocal pres-

sure ; but as every part has its own peculiar velocity, so it makes its effort singly by itself, and, being easily put in motion, it will be as easily turned out of its direction, the parts being only retained together by the weight of those that come next them.

Fluids have a continual effort, because when a certain number have produced their effect they are succeeded by others as long as the current lasts.

Hence it will follow, that when a vessel is left to a current of the river, it can receive no more velocity than the current has, and its velocity will be accelerated till it is equal to that of the current.

If, on the contrary, any floating body receives a motion in a contrary direction to that of the current, it will be continually retarded, till it has none, and then it will change its direction to follow that of the current.

We shall here remark, that it is indifferent whether we ascribe the motion to the solid or to the fluid ; for the impression of the water upon the ship's stem is the same when under sail, as when at an anchor, provided the motion of the current be equal to that which the ship acquires by sailing.

The effort of fluids is as the square of the velocity of the current.

It is very plain that the more rapid the current is, the greater will the impression of the fluid be ; for the parts will then shock the solid with greater force than when it runs slowly ; so that the force is augmented in proportion to the velocity. Again, the number of the parts of the fluid that strike the solid in any space of time, is in proportion to the velocity of the current ; for the faster it runs the greater will be the number of the parts that strike the solid in a space of time ; so that not only the effort of the fluid, but likewise the number of parts that attack the solid, is augmented in proportion to the velocity of the current, and when these two are united, the effort of the fluid will be in a duplicate ratio of the velocity ; so that if the velocity be doubled, the shock will be quadrupled.

Hence, the faster a ship goes through the water, the greater will be the resistance she meets with, and this will be augmented in a duplicate ratio of the velocity with which she sails.

The impression of a fluid increases as the surfaces which oppose its current.

It is very plain, that if one surface is double another it will receive double the number of the parts of the fluid, and of consequence the impression will be double upon a surface, whose area is double the area of another surface. Hence those ships whose midship frames have the greatest capacity meet with most resistance.

The efforts of fluids will be less when the surfaces are in an oblique position to the current, than when in a perpendicular position.

Plate

Plate IV. Fig. 7.] Let E A represent the course of the fluid setting perpendicularly on any body A B ; it is plain, that it receives the impression of all the parts of the fluid contained between A and B ; whereas, if the point B be moved to D, the parts of the water contained betwixt B and G will have no impression upon A D. Hence the quantity of the fluid which attacks A B is to that which attacks A D as A B is to A G ; that is, as the radius is to the sine of the angle of incidence E A D. But if there were no other advantage gained by this oblique position, than being exposed to fewer parts of the fluid, it would be of very little service to a ship which must have a sufficient breadth, suppose A B ; it is plain, the number of the parts of the fluid which give the impression will be the same, when the fore part of the ship is in the form of A D B, as when it is flat in the form of A B ; but the fluid which exerts its force on the surface A D B does not produce the same impression as when it exerts its force on A B, because the direction of each particle of water, which strikes any surface obliquely, may be resolved into two directions, one perpendicular, and the other parallel to the plane.

In order to give us an idea of compound motions, and of the resolution of their forces, let us suppose two rulers A A and B B, (*Plate IV. Fig. 5.*) placed upon a plane at right angles to one another, and a small ball C placed at the angle of their meeting, it is plain, if we slide the ruler B B in a parallel position to itself, it will carry the ball C along the edge of the ruler A A ; but if both the rulers be made to slide together, so that they still preserve the same angle, in such a manner that when the ruler A A arrives at the line VII, VII, the ruler B B arrives only at the line 3, 3. It is plain, the ball will describe the diagonal of the parallelogram C, VII, D, 3. the sides of which will be proportional to the distance the rulers have moved, that is, D VII is to D 3 as 3 to 7 ; but if the rulers be supposed to be moved equally, so that when A A arrives at the line VII, VII, B B shall arrive at the line 7 7, the ball will describe the diagonal C F of the square C VII, F 7.

Now, if we substitute any other two agents in the place of the rulers, such as two hammers, and both be supposed to strike the ball with equal force at the same time, it is plain, the ball will go in the direction of the diagonal C F ; but if the force with which one hammer strikes the ball be to that by which the other hammer strikes the ball, as 7 to 3, then the ball will move in the direction of the line C D.

The principal Effects of Compound Motions. (Plate IV. Fig. 6.)

If two powers C and B act with equal force on the body A, but in the contrary directions of the lines CA and BA, the body A will remain at rest; but if one of the powers acts with greater force than the other, the body will follow the direction of that which predominates, diminished by the quantity of the smaller force.

2d, If two powers D and E act upon the body A in the same direction, viz. in the lines DA and EA, the body A will follow the direction of both, and pass through the point F, with this only difference, that it will go with greater velocity when impelled by both powers than with one.

3d. Let the two powers G and H strike the solid A, in the direction of the lines GA and HA, it will thereby receive a compound motion, the force and direction of which may be expressed by the diagonal of a parallelogram, as was before observed.

In order to construct this parallelogram, which is called the *resolution* of the forces, let the two powers G and H be supposed equal and expressed by the lines HA and GA; from the point G draw the line EG equal and parallel to HA, and the diagonal EA (the result of the two powers represented by the sides of the parallelogram HA and GA) shall express the velocity and direction of the compound motion; the effect of which will be, that the body A will be carried to the point F. But supposing the forces unequal, and let that of H, (Fig. 9) represented by the line HA, be double that of G, represented by the line RA; then from the point R draw the line RS equal and parallel to HA, which shall express the force and direction of the power H; and from the point H draw the line HS parallel to RA, which will express the force and direction of the power G; the diagonal SA expresses the velocity and direction of the body A, which will pass through the point T, whereas, if the powers were equal, it would pass through the point F.

It may be remarked, that two attractive powers placed at P and Q, would produce the same effect as two impulsive powers at G and H, and that the parallelogram may be constructed on the lines AQ and AP.

CONSEQUENCES.

1st. The acuter the angle of the direction of the power is, the nearer will they approach to one direction, and act with greater force; so the result of G and H is greater than that of K and I, supposing the powers to be equal.

2d,

2d. The greatest effect of two powers is, when they both act in the same direction, and the least when they act in contrary directions.

3d. When two equal powers act in such a direction that they form an angle of 120 degrees, as AK and AI ; in this and in no other case, the result will be equal to the single force of AI or AK ; it only changes the direction; for when the two powers act jointly, A will be carried to F , whereas if K only acted, it would be carried to T ; or if I only acted, A would be carried to V .

4th. If the direction of two powers make an angle less than 120 degrees, as GA and HA , they will assist one another; but if they form an angle greater than 120 degrees, as LA and AM , they will be reciprocally diminished.

The Results of a Motion impressed upon a Body A, in Relation to a Surface a b, which opposes its Motion. (Plate IV. Fig. 6.)

1st. When a body strikes a surface obliquely it will be with less force than when it strikes it perpendicularly; for it may strike it so obliquely as only to graze along it; between the perpendicular shock, which is the greatest, and the oblique, which approaches nearest to a parallel to the surface, there may be an infinite number of directions, less or more oblique, and the surface will be struck with more or less force.

2d. If the two powers are united in D , they will act, in the direction DF , with great force upon ab , because they not only act jointly, but likewise in a perpendicular direction upon the surface ab .

3d. If the two powers be equal in force, and act in the direction of the lines GA and HA , the body A will also fall perpendicularly on the surface ab , but with less force than in the first case, because of the obliquity of the directions.

4th. If the power H have double the force of the power G , then the direction will be changed into the line SA , (*Fig. 9.*) and the body will strike the surface obliquely in the direction of the line ST , but with less force than in the second case, not only on account of the diminution of the force of the power G , but also on account of the obliquity of the shock.

5th. It will be indifferent whether the body A receives its impulse from one single power, or from two, so that it strikes the surface ab in the same direction. Hence we shall have no occasion to consider the powers which give the motion, but only the velocity and the direction in which they strike the surface.

6th. It will produce the same effect, whether we change the line of direction

direction, in which the body *A* strikes the surface *ab*, or change the position of the surface *ab* in respect of the line of direction.

From what has been said, it will follow, that if the common effect of two powers acting upon the same body be known, and also the direction and force of one of them, then the direction and force of the other may be found; for let the body *C* (*Fig. 5.*) be carried to the point *D* by the action of two powers, and one represented by the line *C*, *VII*; draw the line *D 3* equal and parallel to *C*, *VII*, and compleat the parallelogram, so shall *C 3* express the force of the other power.

The Application of what has been said to the Shock of Fluids.

We have hitherto considered the shock of a solid body in different directions upon the surface of another solid, but we will readily grant that fluids do not act in the shock in the same manner that solids do. It is very probable, that when a fluid falls perpendicularly upon a surface, there is a mass of water that rests immoveable before the surface, which occupies the place of a solid body, and has nearly the same effect as if the surface was round, so that the fluid does not attack the body that opposes it in a direction perpendicular to its course; besides, the particles of water which attack a surface, whether obliquely or not, may rebound and change their direction, so that the laws of fluids are quite different from the laws of solids in the shock.

The oblique direction of a particle of water may be resolved into one that is perpendicular to the body which opposes its course, and one that is parallel to it.

In order to construct this resolution, (*Plate IV. Fig. II.*) upon the line *AC* inclined to the current, form the parallelogram *AHEF* (*AE* representing the velocity and direction of the current) making *EF* parallel to *CA* and *EH* perpendicular to *CA*. The diagonal *EA*, which represents a particle of water and its velocity, will be the result of a motion supposed to be produced by two powers, one parallel to *AC*, whose force and direction is represented by *EF*, the side of the parallelogram.

Hence it will follow, that when a surface is exposed to the shock of a current, in different oblique directions, the force of the direct shock is to that of the oblique, as the square of the radius is to the square of the sine of the oblique angle of incidence; for the effort of the particle *EA*, which strikes the body *AB*, in a perpendicular direction, is to the effort of the same particle of *EA*, which strikes the body *AC* in an oblique direction, as *EA* is to *EH*; but *EA* is to *EH* as *AB*, the sine of the right angle,

angle, is to AG , the sine of the oblique angle of incidence. But it was before observed, that the sum of the particles that strike AB is to the sum of the particles that strike AC as the radius is to the sine of the angle of incidence. Hence, by multiplying the effort of one particle, by the number of particles that strike AB ; (that is, the effort of the whole water upon AB) and multiplying the effort of one particle, by the number of particles that fall upon AC , (that is, the effort of the whole fluid upon AC) we shall have the following proportion: The effort of the whole fluid upon AB is to its effort upon AC as the square of the radius is to the square of the angle of incidence.

When the surfaces AB and AD , which oppose the current AE , are unequal (*Plate IV. Fig. 10.*) the quantities of water which strike these surfaces are as the product of the surfaces by the sines of the angles of incidence; from whence we shall have the following proportion: The effort of the fluid upon AD is to that upon AB as the square of AG , the sine of the angle of incidence multiplied by the surface AD , is to the square of AB the radius, multiplied by the surface AB .

CONSEQUENCES.

1st, If two equal surfaces, exposed to the same current, receive its shock in different obliquities, the impressions will be to one another as the squares of the sines of the angle of incidence.

2d. A surface parallel to the current can receive no shock, because there is no angle of incidence.

3d, If two unequal surfaces are exposed to the same current, the impressions they receive by the shock in different obliquities, are to one another as the products of the squares of the sines of the angles of incidence, and of the surfaces that receive the shock.

4th. If two equal surfaces receive the shock of two unequal currents, the impressions will be to one another as the products of the squares of the velocities, and of the squares of the angles of incidence.

5th. If two unequal surfaces are exposed to two unequal currents, which strike them with different obliquities, the impressions will be to one another as the products of the squares of their velocities; of the squares of the sines; of the angles of incidence; and of the surfaces.

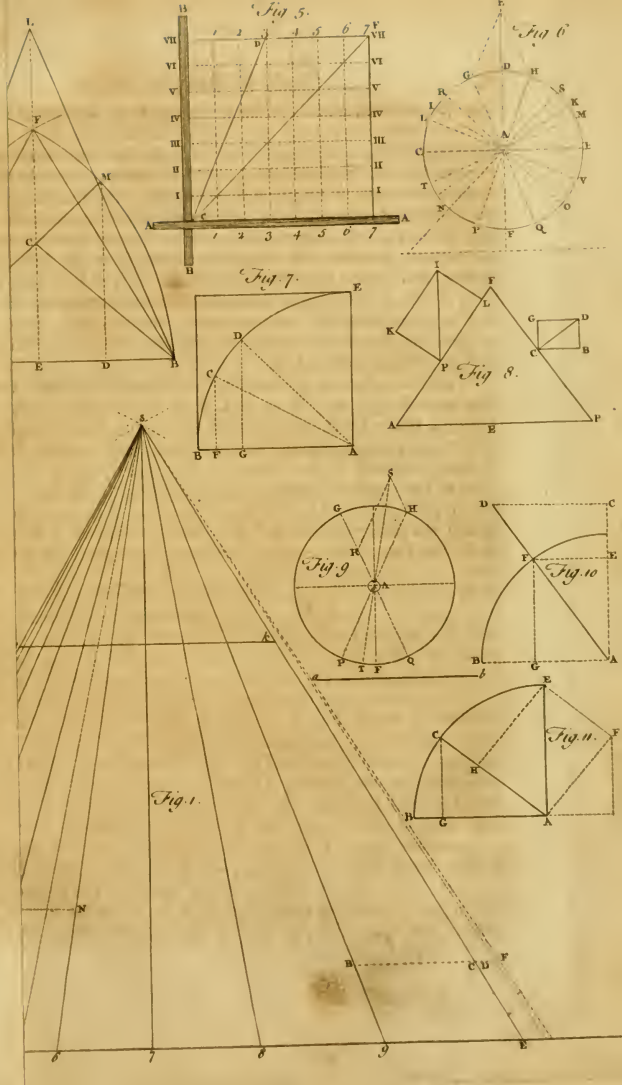
All these consequences may be deduced from the preceding principles; it only remains to apply them.

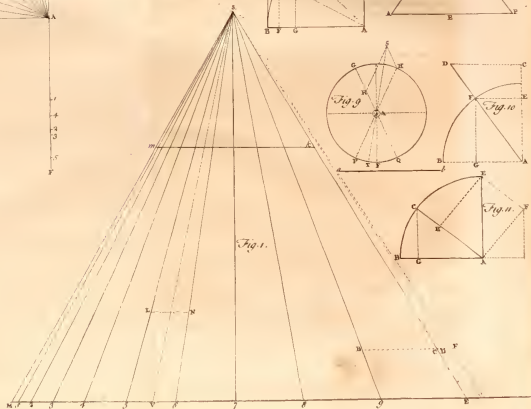
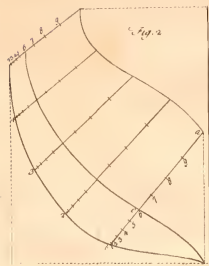
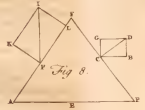
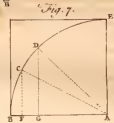
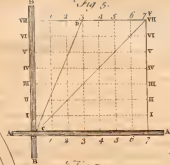
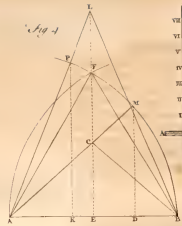
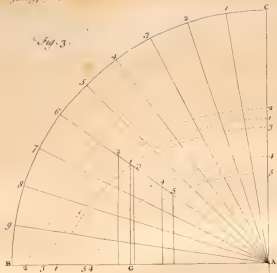
Let AB (*Plate V. Fig. 4.*) represent the extreme breadth of a vessel, and let the fore part be formed according to the angles ACB , or AFB , or ALB . In order to find the efforts of the fluid, supposing

ing the velocity and direction to be the same, and parallel to the keel, in the three cases; upon the middle of the line AB , erect the perpendicular EL , which will pass through the tops of all the triangles; then to find the sines of the angles of incidence, with the radius AB describe two arches AF and BF to intersect one another in the vertex of the equilateral triangle; the arches will intersect the sides of the angle, that is less than 60 degrees; but not the sides of that which is more than 60 degrees, produce one of the sides from C to M (*Plate IV. Fig. 4.*) Lastly, let fall the perpendiculars MD , FE , PK , upon the line AB , from all the points where the arches intersect, either the sides of the triangles, or the sides that are produced. So shall AE , AD , and AK , represent the sines of the angles of incidence upon the different triangles AFB , ACB , and ALB .

It will be easier to observe, that the effort of the fluid upon the intire fore parts ACB , AFB , or ALB , is to the effort upon the extreme breadth, as the effort upon AC , AE , or AL , is to the effort upon AE ; but it was proved before, that the impressions received by two unequal surfaces, opposed to the direction of a current, are as the squares of the sines of the angles of incidence multiplied by the surfaces, so, in this case, the impression on AC will be to that on AE , or (which is the same thing) the impression on ACB will be to that on AB , as the square of AD , the sine of the angle of incidence multiplied by ACB , is to the square of the radius multiplied by AB .

We have also the effort on AFB to the effort on AB ; as AFB , multiplied by the square of AE , the sine of the angle of incidence, is to AB , multiplied by the square of AB the radius. This proportion would shew the effort of the fluid upon the prow AFB , in a perpendicular direction to the sides AF and BF , which would be very useful, if it were required to determine the dimensions of the timber, that is, to resist that pressure of water; but in the present case, where only the relative effort upon the prow is considered in the direction of the keel, we must form another resolution. Let then CD (*Fig. 8*) represent the effort upon FB , perpendicular to that surface, if from the point F we let fall the perpendicular DH , and compleat the parallelogram $GCDH$, CG shall represent the relative effort upon the prow in the direction of the keel, so the whole effort upon FB , may be represented by FB multiplied by the square of the angle of incidence, which is to the relative effort as FB is to EB : The relative effort then is equal to the square of the sine of the angle of incidence multiplied by EB , or by the sum of the particles which fall upon FB ; so to find the relative effort on FB , we must multiply





multiply the square of the angle of incidence by the projection of the plane FB upon the beam EB .

Tho' this method cannot be truly applied but to rectilineal triangles, yet, by dividing curves into a number of small parts, each may be considered, without any sensible error, as a strait line. *M. Bouguer* makes use of this method of approximation to a sufficient degree of exactness. What we have already said upon that head, it is to be hoped, will facilitate this description to such as have only a slight knowledge of the mathematics; so that all that remains now is to apply this to the draught of a ship of 70 guns, which has been already laid down.

A Calculation of the Resistance of a Fluid upon the Prow of the ship of 70 Guns, which we have laid down in a draught compared with the Effort of the same Fluid upon the Area of the Midship Frame.

As the operations are to be performed upon the plane of projection before laid down, all the frames in the fore part must be exactly formed as before in *Plate II*, in order to which it will be necessary to make use of a larger scale, as in *Plate V*:

It will be very convenient to draw the water lines $I, II, III, \&c.$ and the frames $1, 2, 3, \&c.$ to the midship frame at equal distances from one another.

It is plain, that the water lines and frames divide the prow into trapezia, such as $ra, sb, 7c, \&c.$ corresponding to the trapezium ab , and parallelogram ad in the plane of elevation *Plate II*.

It will be necessary to observe, that there must be so many water lines and frames that the lines $sa, 7b, 6c, \&c.$ which are curves, may be esteemed strait lines.

We must draw the diagonals $ra, sb, 7c, \&c.$ through the trapezia; but we may take two trapezia at once near to the midships, because the ships sides are there nearly parallel to the current.

It will likewise be proper to observe that these diagonals are the projections of the diagonals of the parallelograms represented upon the plane of elevation, at least, on the surface of the ship; as for instance, the diagonal sb , on the plane of the projection, is the projection of the diagonal sd drawn on the plane of elevation.

These diagonals divide the prow into the triangles $1, 2, 3, 4, \&c.$ which strike the fluid with different degrees of obliquity.

We have not the entire areas of these triangles, by reason of the curving of the prow, but only their projection on the midship frame; but this is all we want, for the sum of all the particles of water that strike the tri-

angles are proportioned to the triangle projected on the midship frame, since the water that ranges along each triangle may be considered as a triangular prism, the base of which is equal to the triangle $ra8$, *Fig. 1, Plate V.* and this was the thing in question. Mr. *Bouguer* proposes to calculate the effort of the fluid on each triangle, their sum will give the shock of the water upon the whole prow, and compare this to the shock of the fluid upon a surface parallel to the area of the midship frame.

To attain this Mr. *Bouguer* lets fall perpendiculars to every frame, from the angles formed at the intersection of the water lines and the diagonals that were drawn to form the triangle; for instance, upon the frame 8, the perpendiculars lb and rp ; upon the frame 7 8, the perpendicular $8m$ on one side, and the perpendicular nc on the other side; &c.

This method requires that there be as many right angled triangles formed as there are triangles on the prow, and as there must be a great number of them, it will be necessary to find some method of forming them. The following seems to me to be the most expeditious.

Draw two parallel lines BD and CR ; let the distance betwixt them be equal to that betwixt the frames on the plane of elevation, and by this one operation we have the height of all the triangles that are contained betwixt the parallels.

As the base of all the rectangles should be equal to the perpendicular of the corresponding triangle of the prow, we may set off the length of each perpendicular upon the parallel CR ; so shall CH *Fig. 2*, be equal to rp , *Fig. 1*; HL , *Fig. 2*, equal to sa , *Fig. 1*; LE equal to $8M$, &c. to compleat the triangles, draw the perpendiculars HN , LM , and the hypotenuses DH , NL , &c.

If one of these rectangles be considered singly, DH may represent the radius, and CH will be the sine of the angle of incidence.

All these triangles being described, we may begin to find their areas on the plane of the projection, because it is upon this that the relative impulse in the direction of the keel depends, which is the thing now required, as was before observed.

The surface of a triangle is found by multiplying half the base by the perpendicular, so the surface of the triangle $r, a, 8$, will be the product of the perpendicular rp , (equal to CH) multiplied by half the base $a8$, and this will be the sum of all the particles which strike the triangle $r, a, 8$, which is an element of the prow of the ship.

In order to find the relative force of the fluid in the direction of the keel on that part of the bottom corresponding to the triangle $r, a, 8$, it is only

only multiplying the surface of the triangle by the square of the sine of the incidence; in place of multiplying it by the square of the radius, which would give the impulse the triangle would receive from the water in a perpendicular direction; now if we divide this impulse by the oblique one, the quotient will give the quantity that the impulse is diminished by the obliquity of the prow; but there will be no occasion for this last step, for as the sum of all the products of the triangles multiplied by the square of the radius, gives the effort of the fluid upon the midship frame; the direct effort may be found by multiplying the area of the midship frame by the square of the radius, and if this be divided by the sum of the products of all the triangles multiplied by the squares of the sines of the angles of incidence on each triangle, we shall know the diminution of the resistance which the prow meets with in proportion to that of the midship frame.

It would be almost impracticable to multiply the surface of each triangle by the square of the sine of the angle of incidence, upon which account Mr *Bouguer* substituted proportional lines in place of the squares of the sines, which we shall now explain.

It was before observed, that if DH be the radius, CH will be the sine of the angle of incidence.

If we let fall the perpendicular CO upon the line DH , we shall have the triangle DCH similar to DOC ; so taking the equal lines CD , and NH for the radius, CO will be the sine of the angle of incidence.

If we draw OP perpendicular to DC , the triangles DOC and COP will be similar, therefore the triangles DCH and COP will be similar, and DH is to CH as OC to CP ; but DH is to CH as DC to CO , and by multiplying these two proportions, the square of DH will be to the square CH as DC is to PC ; that is, if DC represent the square of the radius, PC will be the square of the sine of the angle of incidence CDH . So the lines CP , HQ , LG , &c. give the squares of the sines of the angles of incidence in the triangles 1, 2, 3, &c. which are to be multiplied by the surfaces of the triangles; and the parallels DC , NH , &c. always represent the radius.

A Calculation of the Resistance of the Fluid upon the Prow of a Ship of 70 Guns, compared with that upon the Midship Frame.

Triangles.	Perpendicular.			Multiplied by half the base.			Product.				Multiplied by the square of the sine of the angle of incidence.			Product gives the effort of the water upon the triangle.			
ft.	f.	in.	lin.	f.	in.	lin.	f.	in.	lin.	poi.	f.	in.	lin.	f.	in.	lin.	poi.
1	6	6	0	1	9	6	11	7	9	0	3	2	0	36	10	6	6
2	5	7	6	1	6	3	8	6	7	10	2	8	0	22	9	9	0
3	4	6	6	1	9	6	8	1	7	9	1	11	6	15	11	2	8
4	4	6	6	1	9	6	8	1	7	9	1	11	6	15	11	2	8
5	2	9	6	1	8	6	4	9	2	9	0	11	0	4	4	5	6
6	2	8	0	1	9	6	4	9	4	0	0	9	0	3	7	0	0
7	1	11	6	1	7	6	3	2	2	3	0	5	0	1	3	10	11
8	2	2	0	1	9	0	3	9	6	0	0	6	6	2	0	7	9
9	1	4	0	1	7	6	2	2	0	0	0	2	6	0	5	5	0
10	1	4	0	1	7	6	2	2	0	0	0	2	6	0	5	5	0
11	0	5	0	1	6	6	0	7	8	6	0	1	0	0	0	7	8
12	0	10	0	1	7	6	1	4	3	0	0	2	0	0	2	8	6
13	0	2	6	1	6	6	0	3	10	3	0	0	8	0	0	2	6
14	0	4	0	1	6	6	0	6	2	0	0	1	2	0	0	7	2
15	0	1	0	1	6	6	0	1	6	6	0	0	3	0	0	0	4
16	0	5	0	1	6	6	0	7	8	6	0	1	6	0	0	11	6

The total effort of the first piece *r* V

Of the second

Of the third

Of the fourth

Of the fifth

Of the sixth

104	2	8	8
57	10	6	2
43	8	6	5
28	0	8	10
12	9	8	7
3	2	8	11

Total 249 10 11 7

We must in the next place find the direct effort of the water upon the area of the midship frame, by multiplying the area by the square of the radius.

O P E R A T I O N.

Half the 6th water line r VI	10	2	0	
The whole 5th water line V	20	0	0	
The 4th water line	10	5	0	
The 3d water line	18	1	0	
The 2d water line	15	11	6	
The 1st water line	12	0	0	
The breadth of the keel	0	0	0	
Total	95	9	6	
Multiplied by the distance betwixt the water lines, which is	}	3	2	0
Product is the area of the midship frame		303	4	1
Multiplied by the distance betwixt the frames		8	0	0
Product		2426	8	8

This being divided by 249 : 10 : 11, the sum of the efforts of the fluid upon the triangles of the prow; the quotient is $9\frac{7}{11}$, which shews that the effort of the fluid upon the prow is to that upon the midship frame as 1 is to $9\frac{7}{11}$, which is a sufficient diminution of the resistance for a ship of this force. Hence we may conclude that the water lines in the fore-body are well formed, but a frigate will require more diminution, as will appear by the following examples.

The *Brillant*, as $3\frac{1}{2}$ to 1, a very bad sailer.

The *Tigre*, as 5 to 1, a company keeper.

A ship of 50 guns, designed by M. *Boyer*, but not built, as 8 to 1.

The *Monarque*, of 74 guns, built by M. *Ollivier* in 1745, as $9\frac{1}{2}$ to 1.

The *Palme*, of 12 guns, 4 lb. shot, built by M. *Ollivier* in 1744, as $13\frac{1}{2}$ to 1.

The *Alcid*, of 64 guns, by Mr *Ollivier*, at *Brest* 1741, as $6\frac{1}{2}$ to 1.

The *Renommee*, built at *Brest* by Mr *Desalieux* 1744, as 10 to 1.—this ship, by the account of the captians, was a very fine sailer.

The *Badine*, 6 guns of 3 lb. shot, as $7\frac{1}{2}$ to 1.

The *Pantbere*, of 20 guns, 6 lb. shot, as $10\frac{1}{2}$ to 1.

The *Amazon*, of 44 guns, built by M. *Blaise Penguist*, as $8\frac{1}{3}$ to 1.

The *Superbe*, built by M. *Helie*, as $5\frac{1}{2}$ to 1.

The *Mutine*, of 24 guns, built by M. *Geffroi*, senior, as $10\frac{3}{4}$ to 1.

We have compared the efforts of the fluid upon the prow of each ship, with that upon a plane, equal to the area of the midship frame.

It will be proper also to examine if the resistance in those be less than in ships which are known to be good sailers; but it may happen that a ship,

ship, whose midship frame has a small area, may meet with little resistance, tho' her prow be not diminished in proportion to that of her midship frame; so it will not be sufficient to know this proportion only, to be assured whether or not the ship will be a fine sailer. We must also compare the areas of the midship frames, and not rest satisfied with comparing the efforts of the fluid; upon the prow of the ship we have laid down, with that upon the prow of a ship of the same rate, which has gained a good character.

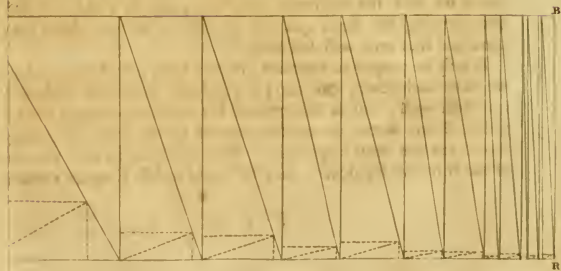
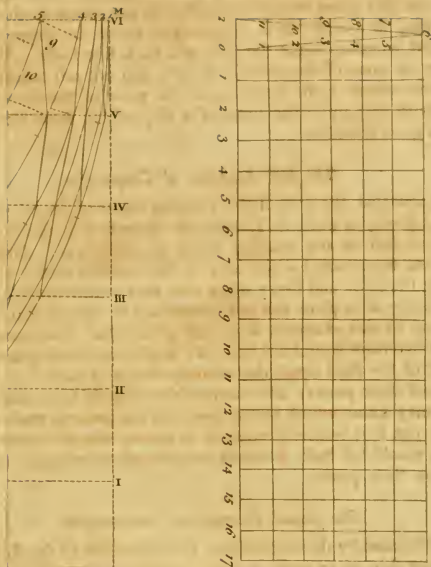
The first Example of Comparison.

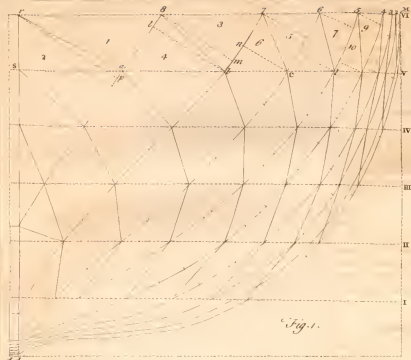
We know that the area of the midship frame of the 70 gun ship, we have laid down, is 606 : 8 : 2, and that the effort of the fluid upon the prow is to that on the midship frame as 1 is to $9\frac{2}{5}$. Now if another ship of the same rate has the area of her midship frame 7 or 800 feet, supposing the sails, and every thing that may contribute to sailing, to be alike in both; it is plain this last cannot sail so well as that we have laid down, and by this example, it is very plain, that if we would calculate which of two ships would sail best, we must, after finding how much the resistance of the fluid upon the midship frame of each is diminished by the form of their prows; also compare the areas of their midship frames, that we may know which of the two has the greatest mass of water to displace; but if it was only required to know, which of two ships of the same rate would sail best, it would be sufficient to compare the efforts of fluids upon their prows.

The second Example of Comparison.

We have found by the calculation, that the effort of the fluid upon the prow of our ship of 70 guns, is 249 : 10 : 11 : 7; but if by a like calculation we find the resistance upon the prow of a ship of the same force, and carrying the same quantity of sail, to be 300 feet, we may thence conclude that ours will sail best.

It will be proper to examine, by the same calculation, whether the ship we have laid down, can carry a good sail, drive but little to the leeward, and steer well; but as this treatise has already exceeded the bounds I proposed, I am forced to confine myself to the two preceding conditions, which are the most important. The methods to find the other qualities of the ships we lay down, may be found in Mr *Bouguer's* treatise.





*An EXPLICATION of the SIGNS or CHARACTERS made use of in this
TREATISE.*

SIGNS.	NAMES.	SIGNIFICATIONS.
+	Plus, or more.	The sign of addition, as $3 + 5$, is 3 more 5, that is, the character + placed between any two or more figures, signifies that they are to be added into one sum.
—	Minus, or less	The sign of subtraction, as $5 - 3$, is 5 less 3, and signifies that 3 is to be taken from 5.
×	Multiplied by	The sign of multiplication, as 5×3 , signifies 5 multiplied by 3.
÷	Divided by	The sign of division, as $8 \div 4$, signifies 8 divided by 4.
=	Equal to	The sign of equality, that is when this is placed betwixt numbers or quantities, it signifies that they are equal. as $5 + 3 = 8$, or $10 - 2 = 5 + 3$, that is, 5 more 3, is equal to 8, and 10 less 2, is equal to 5 more 3.
::	So is	The sign similarity of ratio. It is always placed betwixt the two middle terms or numbers in proportion, thus $3 : 9 :: 8 : 24$, that is, as 3 is to 9, so is 8 to 24.

R. Radius.

S. Sine.

T. Tangent,

Sec. Secant.

S. c. Sine complement.

T. c. tangent complement.

Sec. c. Secant complement.

H. Hypothenufe.

B. Base.

P. Perpendicular.

E R R A T A.

Page	Line	for	read
14	19	a in	in a
25	15	of division	in the divisions
26	31	<i>f</i> B	<i>f</i> b
30	Margin	Fig. 25	Fig. 25 at Prop. VIII.
30	30	<i>C f</i> e	<i>C f</i> E
35	13	from the point N, &c.	cancel
38	Margin	Fig. 5.	cancel
40	Margin	Plate II.	Plate I. Fig. 37.
40	Margin	Fig. 1.	Plate II. Fig. 1.
41	Margin	Fig. 6, 7, 8, 9, 10.	cancel
41	30	Example	Examples
43	Margin	Fig. 30.	cancel
49	4	40	50
49	10	50	40
49	13	base at A . . . <i>also</i> for 50	base ; at A <i>read</i> 40
53	15	$T x = x E$	$T x = x F$
53	18	$m, s, n, t,$	$m s, n t,$
53	34	F D	F d
50	17	the	then
56	27	$r t \frac{1}{2} r s$	$r t \times \frac{1}{2} r s$
58	12	1.14592	3.14592
58	31	785	.785
59	5	whose diame-	whose diameter is 1.
59	30	line, as	line
64	11	<i>C m</i>	<i>C s</i>
64	12	$m r \times C +$	$m s \times C t$
		<i>C s</i>	<i>C s</i>
64	21	including the areas	including half the areas
68	Margin		before Case 2d. prefix Fig. 53
82	Margin	Plate I.	Plate II.
82	17	5.9990	5.6990
102	Column 8	Length	Depth
103	22	as in the columns	cancel
104	7	6.25	.625
105	2	height	length
105	16	double line ; on the rule	double line on the rule ; look
106	21	11,	1, 1,
106	22	11,	1, 1,
106	last line	A C	A D
110	31	<i>f</i> o B	<i>t</i> o B
111	4	Fig. 2.	Fig. 9.
111	5	S S	S S Fig. 2.
114	5	remarked	marked
121	36	1 s, 2 e, 3 b,	1 r, 2 d, 3 a.
122	34	G A E	C A E
		provided the dimensions and inclination of these planes to one another be given.	cancel
126	32	<i>s q c b</i>	<i>s q 7</i>

E R R A T A.

Page	Line	for	read
130	18	the body	the body plane
131	16	ribband lines	ribbands
			See Plate VII. where the curves
			here defined are distinguished
133	after line 22	add	by their initial letters.
136	36	M to y	M to S
137	37	C	c
138	19	BEND	Bend Mould
138	26	H lw	Hollow mould
144	20	floor	floor timbers
152	10	so	fo
162	29	b l	b l
168	40	when this is	cancel
169	1	hewed of	cancel
169	1	of the outside	(in that direction)
199	8	+ .697997	+ .697997
201	17	56° 15'	56° 15' to 33° 45'
204	4	stance	distance
214	11	E o: E n	E n: E o
217	1	50	15
218	16	C x	C X
225	5	Long line	Log line
225	17	A B	A C
225	18	B E	A E
228	31	difference of R	difference of latitude; R.
231	7	by 3 15	cancel
237	4	north	south
244	24	65338	65338
	25	18668	18688
249	6	27°. 2' R: difference of latitude	27° 2': R:: difference of lati- tude 1063:
249	7	distance: 1191	distance 1191
249	14	2.738920	2.739030
249	28	75.7	75.5
249	29	18.4	21.1
249	30	546.9	549.6
250	35	side	fine
253	10	30	60
253	11	60	30
253	33	25	15
254	1	25	15
254	2	15	25
254	6	15	25
256	4	P p	P P
256	6	P p	P P
258	12	transcribe	transcribes
258	34	make	makes

ERRATA to the APPENDIX.

Page	Line	for	read
18	22	of D	D
22	33	<i>d</i> and K	<i>d</i> and S
22	35	to <i>g</i>	to <i>l</i>
22	36	<i>s</i> and I	<i>s</i> and <i>l</i>
23	5	<i>d</i> also for <i>d x</i>	<i>b</i> read <i>b x</i>
27	7	point X	point π
27	14	R S	R
27	17	point X	point π
35	26	lines	line
37	26	<i>n</i> L	N L,
38	17	HP	H ρ
40	23	S	<i>s</i>
42	10	from N	strike it out
43	27	3d from <i>l</i>	3d from <i>i</i>
44	20	πx	πy
54	17	$\frac{1}{2} \text{Sc.} + \frac{1}{2} \text{XS} + \text{AB}$	$\frac{1}{2} \text{Sc.} + \text{XS} \times \text{AB}$
62	25	Plate IV. Fig II.	Plate IV. Fig. 11.
62	line 32	add	and the other perpendicular to it whose force and direction is represented by the line EH.

Where the References to the Plates are omitted in Part II. See Plate VII.
and in the Appendix, See Plate 2.

TABLES of the SUN'S DECLINATION: *Adapted to the New Style.*

First after Leap-Year. Sun's Declination 1753, 1757, 1761, 1765, 1769.

Lat.	Janu. South	Feb. South	March South	April North	May North	June North	July North	Aug. North	Sept. North	Oct. South	Nov. South	Dec. South	Day
1	23 00	16 58	07 24	04 41	15 13	22 08	23 08	17 59	08 11	03 11	14 16	21 55	1
2	22 44	16 41	07 01	05 00	15 31	22 16	23 04	17 44	07 49	03 44	14 55	22 04	2
3	22 28	16 23	06 38	05 19	15 48	22 23	22 59	17 28	07 27	04 07	15 14	22 13	3
4	22 12	16 05	06 15	05 05	16 05	22 30	22 54	17 11	07 05	04 30	15 33	22 19	4
5	22 35	15 47	05 52	04 53	16 23	22 37	22 48	16 56	06 42	04 54	15 51	22 29	5
6	22 10	15 29	05 29	04 37	16 40	22 43	22 42	16 39	06 20	05 17	16 09	22 36	6
7	22 23	15 10	05 06	04 00	16 58	22 49	22 30	16 21	05 57	05 40	16 39	22 43	7
8	22 12	14 51	04 42	03 38	17 13	22 55	22 19	16 05	05 35	05 12	16 44	22 49	8
9	22 04	14 31	04 19	03 45	17 29	23 00	22 12	15 48	05 14	05 26	17 01	22 55	9
10	21 55	14 12	03 55	03 07	17 44	23 05	22 15	15 31	04 40	06 48	17 18	23 00	10
11	21 45	13 52	03 32	02 08	18 00	23 09	22 07	15 15	04 26	07 11	17 35	23 05	11
12	21 35	13 32	03 08	01 18	18 15	23 11	21 58	14 55	04 03	07 34	17 51	23 10	12
13	21 25	13 12	02 44	00 13	18 30	23 16	21 50	14 36	03 40	07 56	18 07	23 14	13
14	21 14	12 51	02 21	00 34	18 44	23 19	22 41	14 18	03 17	08 19	18 21	23 17	14
15	21 03	12 31	01 57	00 55	18 59	23 22	21 31	13 50	02 54	08 41	18 38	23 20	15
16	20 52	12 10	01 33	10 17	19 13	23 24	21 22	13 40	02 31	09 03	18 54	23 23	16
17	20 40	11 49	01 10	10 38	19 29	23 26	21 11	13 21	02 08	09 25	19 08	23 25	17
18	20 28	11 28	00 46	10 59	19 39	23 27	21 01	13 02	01 44	09 47	19 23	23 27	18
19	20 15	11 06	00 23	11 20	19 52	23 28	20 50	12 42	01 21	10 09	19 37	23 28	19
20	20 02	10 45	00 00	11 40	20 04	23 29	20 38	12 22	00 57	10 31	19 50	23 29	20
21	19 49	10 23	00 25	12 01	20 17	23 29	20 27	12 02	00 34	10 52	20 04	23 29	21
22	19 35	10 01	00 49	12 21	20 29	23 29	20 15	11 42	00 11	11 11	20 17	23 29	22
23	19 21	09 39	01 12	12 41	20 41	23 28	20 03	11 22	00 11	11 34	20 29	23 28	23
24	19 06	09 17	01 36	13 01	20 52	23 27	19 51	11 01	00 36	11 55	20 41	23 27	24
25	18 51	08 55	02 00	13 20	21 03	23 26	19 38	10 40	00 11	12 16	20 53	23 25	25
26	18 36	08 32	02 23	13 39	21 13	23 24	19 25	10 20	00 21	12 37	21 04	23 23	26
27	18 21	08 10	02 47	13 59	21 23	23 21	19 12	09 58	01 47	12 57	21 15	23 21	27
28	18 05	07 47	03 10	14 18	21 31	23 19	18 57	09 37	02 10	13 17	21 26	23 18	28
29	17 49		03 33	14 36	21 42	23 15	18 43	09 16	02 34	13 37	21 36	23 14	29
30	17 32		03 57	14 55	21 51	23 12	18 29	08 54	02 57	13 57	21 46	23 10	30
31	17 15		04 23		22 08		18 14	08 33		14 17		23 06	31

Second after Leap-Year. Sun's Declination, 1754, 1758, 1762, 1766, 1770.

Lat.	Janu. South	Febru. South	March South	April North	May North	June North	July North	Aug. North	Sept. North	Oct. South	Nov. South	Dec. South	Day
1	23 01	17 03	07 30	04 37	15 08	22 06	23 08	18 01	08 15	03 14	14 31	21 54	1
2	22 55	16 45	07 07	05 00	15 26	22 14	23 04	17 47	07 54	03 36	14 50	22 03	2
3	22 50	16 27	06 44	05 21	15 43	22 22	22 59	17 34	07 37	04 12	15 09	22 12	3
4	22 43	16 10	06 21	05 46	16 01	22 29	22 54	17 15	07 10	04 25	15 28	22 20	4
5	22 37	15 52	05 58	06 09	16 18	22 36	22 49	16 59	06 47	04 48	15 47	22 27	5
6	22 30	15 33	05 35	06 32	16 35	22 43	22 43	16 43	06 25	05 11	16 05	22 34	6
7	22 23	15 14	05 12	06 54	16 52	22 49	23 37	16 27	06 02	05 34	16 23	22 41	7
8	22 15	14 55	04 48	07 17	17 09	22 54	22 30	16 10	05 39	05 57	16 41	22 48	8
9	22 06	14 36	04 25	07 39	17 25	22 59	22 23	15 53	05 17	06 20	16 58	22 54	9
10	21 57	14 17	04 02	08 01	17 41	23 04	22 16	15 35	04 54	06 43	17 15	23 00	10
11	21 48	13 57	03 38	08 23	17 56	23 08	22 08	15 17	04 32	07 07	17 32	23 05	11
12	21 38	13 37	03 14	08 45	18 11	23 11	22 00	14 59	04 09	07 28	17 48	23 10	12
13	21 28	13 17	02 51	09 07	18 26	23 15	21 52	14 41	03 45	07 50	18 04	23 14	13
14	21 17	12 57	02 26	09 29	18 41	23 18	21 43	14 23	03 23	08 13	18 20	23 17	14
15	21 06	12 36	02 03	09 50	18 56	23 21	21 24	14 04	03 00	08 35	18 35	23 20	15
16	20 55	12 15	01 40	10 11	19 10	23 23	21 14	13 45	02 37	08 57	18 50	23 21	16
17	20 43	11 54	01 16	10 32	19 24	23 25	21 14	13 26	02 14	09 19	19 05	23 25	17
18	20 31	11 33	00 52	10 53	19 37	23 27	21 04	13 07	01 50	09 41	19 19	23 27	18
19	20 18	11 11	00 29	11 14	19 50	23 28	20 53	12 47	01 27	10 03	19 33	23 28	19
20	20 05	10 50	00 05	11 35	20 02	23 29	20 42	12 27	01 03	10 25	19 47	23 29	20
21	19 52	10 28	00 19	11 55	20 14	23 29	20 30	12 07	00 39	10 47	20 00	23 29	21
22	19 38	10 07	00 43	12 15	20 26	23 29	20 18	11 47	00 16	11 08	20 13	23 29	22
23	19 24	09 45	01 08	12 35	20 38	23 28	20 06	11 27	00 11	11 29	20 26	23 28	23
24	19 10	09 23	01 30	12 55	20 50	23 27	19 54	11 06	00 31	11 50	20 38	23 27	24
25	18 55	09 00	01 53	13 15	21 01	23 26	19 41	10 45	00 54	12 11	20 51	23 26	25
26	18 40	08 38	02 17	13 34	21 11	23 24	19 29	10 34	01 17	12 32	21 02	23 24	26
27	18 25	08 15	02 40	13 53	21 21	23 21	19 15	10 21	01 40	12 51	21 11	23 21	27
28	18 09	07 53	03 04	14 12	21 31	23 18	19 01	09 42	02 04	13 11	21 24	23 18	28
29	17 53		03 27	14 31	21 40	23 15	18 47	09 23	02 28	13 31	21 34	23 15	29
30	17 37		03 45	14 50	21 49	23 12	18 31	08 58	02 51	13 41	21 41	23 11	30
31	17 20		04 14		21 58		18 18	08 36		14 11		23 07	31

TABLES of the SUN'S DECLINATION. Adapted to the New Stile.

Third after Leap-Year. Sun's Declination 1755, 1759, 1763, 1767, 1771.

Days	Janu. South	Feb South	Marco South	April South	May North	June North	July North	August North	Septem. North	Oct. South	Nov. South	Decem. South	Days
1	23 02	17 06	07 36	04 31	15 04	22 04	23 09	23 06	08 22	03 08	14 26	21 52	1
2	22 57	16 49	07 33	04 54	15 22	22 12	23 05	23 01	08 00	03 32	14 46	22 01	2
3	22 45	16 34	08 10	05 17	15 40	22 18	23 01	23 01	07 38	03 55	15 05	22 09	3
4	22 30	15 56	08 09	05 40	15 58	22 27	22 56	22 56	07 16	04 18	15 24	22 17	4
5	22 13	15 06	08 01	06 03	16 15	22 34	22 50	22 50	06 32	04 41	15 42	22 25	5
6	22 02	14 38	07 55	06 26	16 32	22 40	22 44	22 44	06 05	05 04	16 00	22 32	6
7	22 24	15 19	07 57	06 48	16 49	22 46	22 38	22 38	05 46	05 27	16 18	22 45	7
8	22 16	15 00	08 04	07 10	17 05	22 52	22 32	22 32	05 16	05 50	16 36	22 53	8
9	22 08	14 41	08 10	07 31	17 22	22 57	22 25	22 25	04 57	06 36	17 11	22 57	9
10	21 59	14 22	08 07	07 56	17 37	23 02	22 18	22 18	04 38	06 59	17 28	23 02	10
11	21 50	14 02	08 03	08 18	17 52	23 07	22 10	22 10	04 18	07 22	17 44	23 07	11
12	21 40	13 42	08 03	08 40	18 07	23 11	22 02	22 02	04 04	07 22	17 44	23 07	12
13	21 30	13 22	08 06	09 02	18 22	23 14	21 54	21 54	03 52	07 44	18 00	23 11	13
14	21 20	13 02	08 12	09 23	18 37	23 18	21 45	21 45	03 29	08 07	18 16	23 15	14
15	21 09	12 42	08 18	09 45	18 52	23 21	21 35	21 35	03 06	08 30	18 32	23 18	15
16	20 58	12 21	08 21	10 07	19 06	23 25	21 26	21 26	02 43	08 52	18 47	23 21	16
17	20 46	12 00	08 21	10 28	19 20	23 28	21 16	21 16	02 19	09 14	19 02	23 24	17
18	20 34	11 39	08 20	10 49	19 34	23 31	21 05	21 05	01 51	09 36	19 16	23 27	18
19	20 21	11 18	08 19	11 10	19 47	23 33	20 54	20 54	01 31	09 58	19 30	23 27	19
20	20 09	10 56	08 17	11 30	20 00	23 36	20 41	20 41	01 09	10 20	19 44	23 28	20
21	19 56	10 34	08 14	11 50	20 12	23 39	20 31	20 31	12 32	10 42	19 57	23 29	21
22	19 42	10 12	08 10	12 10	20 24	23 40	20 23	20 23	11 52	11 04	20 10	23 29	22
23	19 28	09 50	08 05	12 30	20 36	23 42	20 11	20 11	11 32	11 25	20 23	23 29	23
24	19 14	09 38	08 01	12 50	20 47	23 43	19 58	19 58	11 12	11 45	20 35	23 28	24
25	18 59	09 06	08 01	13 10	20 58	23 47	19 45	19 45	10 51	12 05	20 47	23 26	25
26	18 44	08 44	08 01	13 30	21 09	23 50	19 31	19 31	10 30	12 12	20 59	23 24	26
27	18 29	08 21	08 01	13 48	21 19	23 53	19 18	19 18	10 09	12 17	21 10	23 22	27
28	18 13	07 58	08 01	14 07	21 30	23 56	19 04	19 04	09 43	12 17	21 31	23 19	28
29	17 56		08 01	14 26	21 38	23 57	18 50	18 50	09 27	12 22	21 31	23 16	29
30	17 40		08 01	14 45	21 47	23 58	18 36	18 36	09 06	12 27	21 42	23 13	30
31	17 23		08 01		21 56		18 21	18 21	08 44	12 31	21 47	23 09	31

Leap-Year. Sun's Declination, 1756, 1760, 1764, 1768, 1772.

Days	Janu. South	Febru. South	Mar. South	April North	May North	June North	July North	August North	Septem. North	Oct. South	Nov. South	Decem. South	Days
1	23 01	17 11	07 28	04 49	15 17	22 10	23 07	23 05	08 05	03 27	14 41	21 58	1
2	22 58	16 54	08 05	05 12	15 34	22 18	23 03	23 03	07 43	03 50	15 00	22 07	2
3	22 52	16 36	08 32	05 35	15 52	22 25	22 58	22 58	07 21	04 13	15 19	22 15	3
4	22 46	16 18	08 09	05 57	16 10	22 32	22 53	22 53	06 59	04 36	15 37	22 23	4
5	22 40	16 00	08 06	06 20	16 27	22 39	22 47	22 47	06 37	04 59	15 55	22 30	5
6	22 33	15 42	08 03	06 43	16 44	22 45	22 41	22 41	06 14	05 22	16 13	22 37	6
7	22 26	15 24	08 04	07 05	17 00	22 51	22 34	22 34	05 52	05 45	16 31	22 44	7
8	22 18	15 06	08 04	07 27	17 17	23 06	22 27	22 27	05 29	06 08	16 40	22 50	8
9	22 10	14 46	08 04	07 50	17 33	23 01	22 20	22 20	05 06	06 31	17 06	22 56	9
10	22 01	14 27	08 04	08 12	17 48	23 06	22 13	22 13	04 44	06 54	17 23	23 01	10
11	21 52	14 07	08 02	08 34	18 04	23 10	22 05	22 05	04 21	07 17	17 39	23 06	11
12	21 42	13 47	08 02	08 56	18 19	23 14	21 56	21 56	04 03	07 39	17 55	23 11	12
13	21 31	13 27	08 03	09 18	18 34	23 17	21 48	21 48	03 34	08 08	18 11	23 16	13
14	21 23	13 07	08 02	09 39	18 48	23 20	21 38	21 38	03 11	08 24	18 27	23 18	14
15	21 12	12 46	08 01	10 00	19 02	23 22	21 29	21 29	02 54	08 47	18 42	23 21	15
16	21 01	12 26	08 01	10 21	19 16	23 25	21 19	21 19	02 36	09 18	18 57	23 23	16
17	20 49	12 05	08 01	10 42	19 29	23 28	21 09	21 09	02 16	09 31	19 12	23 25	17
18	20 37	11 43	08 00	11 03	19 42	23 32	20 58	20 58	01 58	09 53	19 26	23 27	18
19	20 25	11 22	08 00	11 24	19 55	23 35	20 47	20 47	01 35	10 14	19 40	23 28	19
20	20 12	11 01	08 00	11 45	20 08	23 39	20 36	20 36	01 17	10 36	19 54	23 29	20
21	19 59	10 40	08 00	12 05	20 20	23 42	20 24	20 24	00 57	10 57	20 07	23 29	21
22	19 45	10 18	08 00	12 26	20 33	23 45	20 13	20 13	00 37	11 19	20 20	23 30	22
23	19 32	09 56	08 01	12 45	20 46	23 48	20 00	20 00	00 17	11 40	20 32	23 31	23
24	19 17	09 34	08 01	13 05	20 59	23 51	19 48	19 48	00 02	12 01	20 44	23 32	24
25	19 03	09 12	08 02	13 25	21 12	23 54	19 35	19 35	00 06	12 21	20 56	23 33	25
26	18 48	08 40	08 02	13 44	21 31	23 57	19 21	19 21	00 01	12 42	21 07	23 34	26
27	18 32	08 27	08 02	14 03	21 50	24 00	19 08	19 08	00 01	13 02	21 18	23 35	27
28	18 17	08 04	08 03	14 22	22 10	24 03	18 54	18 54	00 02	13 22	21 29	23 36	28
29	18 01	07 41	08 03	14 41	22 30	24 06	18 40	18 40	00 03	13 42	21 40	23 37	29
30	17 45		08 03	15 00	22 50	24 09	18 25	18 25	00 03	14 02	21 48	23 38	30
31	17 28		08 03		23 02		18 10	18 10	00 03	14 22		23 39	31

A Large and very Useful

TABLE OF DIFFERENCE

OF

LATITUDE and DEPARTURE

IN

MINUTES and TENTH PARTS

TO

Every DEGREE and QUARTER-POINT

OF THE

C O M P A S S,

For the exact Working of a

T R A V E R S E.

A TABLE of DIFFERENCE

Diff.	1 Deg.		2 Deg.		$\frac{1}{2}$ Point.		3 Deg.		4 Deg.		5 Deg.		$\frac{1}{2}$ Point.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	01.0	00.0	01.0	00.0	01.0	00.0	01.0	00.1	01.0	00.1	01.0	00.1	01.0	00.1	1
2	02.0	00.0	02.0	00.1	02.0	00.1	02.0	00.1	02.0	00.1	02.0	00.2	02.0	00.2	2
3	03.0	00.1	03.0	00.1	03.0	00.1	03.0	00.2	03.0	00.2	03.0	00.3	03.0	00.3	3
4	04.0	00.1	04.0	00.1	04.0	00.2	04.0	00.2	04.0	00.3	04.0	00.3	04.0	00.4	4
5	05.0	00.1	05.0	00.2	05.0	00.2	05.0	00.3	05.0	00.3	05.0	00.4	05.0	00.5	5
6	06.0	00.1	06.0	00.2	06.0	00.3	06.0	00.3	06.0	00.4	06.0	00.5	06.0	00.6	6
7	07.0	00.1	07.0	00.2	07.0	00.3	07.0	00.4	07.0	00.5	07.0	00.6	07.0	00.7	7
8	08.0	00.1	08.0	00.3	08.0	00.4	08.0	00.4	08.0	00.6	08.0	00.7	08.0	00.8	8
9	09.0	00.2	09.0	00.3	09.0	00.4	09.0	00.5	09.0	00.6	09.0	00.8	09.0	00.9	9
10	10.0	00.2	10.0	00.4	10.0	00.5	10.0	00.5	10.0	00.7	10.0	00.9	10.0	01.0	10
11	11.0	00.2	11.0	00.4	11.0	00.5	11.0	00.6	11.0	00.8	11.0	01.0	10.9	01.1	11
12	12.0	00.2	12.0	00.4	12.0	00.6	12.0	00.6	12.0	00.6	12.0	01.0	11.9	01.2	12
13	13.0	00.2	13.0	00.5	13.0	00.6	13.0	00.7	13.0	00.9	12.9	01.1	12.9	01.3	13
14	14.0	00.2	14.0	00.5	14.0	00.7	14.0	00.7	14.0	01.0	13.9	01.2	13.9	01.4	14
15	15.0	00.3	15.0	00.5	15.0	00.7	15.0	00.8	15.0	01.0	14.9	01.3	14.9	01.5	15
16	16.0	00.3	16.0	00.6	16.0	00.8	16.0	00.8	16.0	01.1	15.9	01.4	15.9	01.6	16
17	17.0	00.3	17.0	00.6	17.0	00.8	17.0	00.9	17.0	01.2	16.9	01.5	16.9	01.7	17
18	18.0	00.3	18.0	00.6	18.0	00.9	18.0	00.9	18.0	01.3	17.9	01.6	17.9	01.8	18
19	19.0	00.3	19.0	00.7	19.0	00.9	19.0	01.0	19.0	01.3	18.9	01.7	18.9	01.9	19
20	20.0	00.4	20.0	00.7	20.0	01.0	20.0	01.0	20.0	01.4	19.9	01.7	19.9	02.0	20
21	21.0	00.4	21.0	00.7	21.0	01.0	21.0	01.1	20.9	01.5	20.9	01.8	20.9	02.1	21
22	22.0	00.4	22.0	00.8	22.0	01.1	22.0	01.1	21.9	01.5	21.9	01.9	21.9	02.2	22
23	23.0	00.4	23.0	00.8	23.0	01.1	23.0	01.2	22.9	01.6	22.9	02.0	22.9	02.3	23
24	24.0	00.4	24.0	00.8	24.0	01.2	24.0	01.3	23.9	01.7	23.9	02.1	23.9	02.4	24
25	25.0	00.4	25.0	00.9	25.0	01.2	25.0	01.3	24.9	01.7	24.9	02.2	24.9	02.4	25
26	26.0	00.5	26.0	00.9	26.0	01.3	26.0	01.4	25.9	01.8	25.9	02.3	25.9	02.5	26
27	27.0	00.5	27.0	00.9	27.0	01.3	27.0	01.4	26.9	01.9	26.9	02.4	26.9	02.6	27
28	28.0	00.5	28.0	01.0	28.0	01.4	28.0	01.5	27.9	02.0	27.9	02.4	27.9	02.7	28
29	29.0	00.5	29.0	01.0	29.0	01.4	29.0	01.5	28.9	02.0	28.9	02.5	28.9	02.8	29
30	30.0	00.5	30.0	01.1	30.0	01.5	30.0	01.6	29.9	02.1	29.9	02.6	29.9	02.9	30
31	31.0	00.5	31.0	01.1	31.0	01.5	31.0	01.6	30.9	02.2	30.9	02.7	30.8	03.0	31
32	32.0	00.6	32.0	01.1	32.0	01.6	32.0	01.7	31.9	02.2	31.9	02.8	31.8	03.1	32
33	33.0	00.6	33.0	01.2	33.0	01.6	33.0	01.7	32.9	02.3	32.9	02.9	32.8	03.2	33
34	34.0	00.6	34.0	01.2	34.0	01.7	34.0	01.8	33.9	02.4	33.9	03.0	33.8	03.3	34
35	35.0	00.6	35.0	01.2	35.0	01.7	35.0	01.8	34.9	02.4	34.9	03.1	34.8	03.4	35
36	36.0	00.6	36.0	01.3	36.0	01.8	35.9	01.9	35.9	02.5	35.9	03.1	35.8	03.5	36
37	37.0	00.6	37.0	01.3	37.0	01.8	36.9	01.9	36.9	02.6	36.9	03.2	36.8	03.6	37
38	38.0	00.7	38.0	01.3	38.0	01.9	37.9	02.0	37.9	02.7	37.9	03.3	37.8	03.7	38
39	39.0	00.7	39.0	01.4	39.0	01.9	38.9	02.0	38.9	02.7	38.9	03.4	38.8	03.8	39
40	40.0	00.7	40.0	01.4	40.0	02.0	40.0	02.1	39.9	02.8	39.8	03.5	39.8	03.9	40
41	41.0	00.7	41.0	01.4	41.0	02.0	40.9	02.1	40.9	02.9	40.8	03.6	40.8	04.0	41
42	42.0	00.7	42.0	01.5	42.0	02.1	41.9	02.2	41.9	02.9	41.8	03.7	41.8	04.1	42
43	43.0	00.8	43.0	01.5	42.9	02.1	42.9	02.2	42.9	03.0	42.8	03.8	42.8	04.2	43
44	44.0	00.8	44.0	01.5	43.9	02.2	43.9	02.3	43.9	03.1	43.8	03.8	43.8	04.3	44
45	45.0	00.8	45.0	01.6	44.9	02.2	44.9	02.4	44.9	03.1	44.8	03.9	44.8	04.4	45
46	46.0	00.8	46.0	01.6	45.9	02.3	45.9	02.4	45.9	03.2	45.8	04.0	45.8	04.5	46
47	47.0	00.8	47.0	01.6	46.9	02.3	46.9	02.5	46.9	03.3	46.8	04.1	46.8	04.6	47
48	48.0	00.8	48.0	01.7	47.9	02.4	47.9	02.5	47.9	03.4	47.8	04.2	47.8	04.7	48
49	49.0	00.9	49.0	01.7	48.9	02.4	48.9	02.6	48.9	03.4	48.8	04.3	48.8	04.8	49
50	50.0	00.9	50.0	01.7	49.9	02.5	49.9	02.6	49.9	03.5	49.8	04.4	49.8	04.9	50
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	89 Deg.		88 Deg.		$7\frac{1}{2}$ Point.		87 Deg.		86 Deg.		85 Deg.		$7\frac{1}{2}$ Point.		

Of LATITUDE and DEPARTURE.

5

Dist.	1 Deg.		2 Deg.		½ Point.		3 Deg.		4 Deg.		5 Deg.		½ Point.		Dist.	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.		
51	51.0	00.9	51.0	01.8	50.9	2.5	50.9	02.7	50.9	03.6	50.8	04.4	50.8	05.0	51	
52	52.0	00.9	52.0	01.8	51.9	02.6	51.9	02.7	51.9	03.6	51.8	4.5	51.7	05.1	52	
53	53.0	00.9	53.0	01.8	52.9	02.6	52.9	02.8	52.9	03.7	52.8	04.6	52.7	05.2	53	
54	54.0	00.9	54.0	01.9	53.9	02.7	53.9	02.8	53.9	03.8	53.8	04.7	53.7	05.3	54	
55	55.0	01.0	55.0	01.9	54.9	2.7	54.9	02.8	54.9	03.8	54.8	4.8	54.7	05.4	55	
56	56.0	01.0	56.0	02.0	55.9	02.7	55.9	02.9	55.9	03.9	55.8	04.9	55.7	05.5	56	
57	57.0	01.0	57.0	02.0	56.9	02.8	56.9	03.0	56.9	04.0	56.8	05.0	56.7	05.6	57	
58	58.0	01.0	58.0	02.0	57.9	02.8	57.9	03.0	57.9	04.0	57.8	05.1	57.7	05.7	58	
59	59.0	01.0	59.0	02.1	58.9	02.9	58.9	03.1	58.9	04.1	58.8	05.1	58.7	05.8	59	
60	60.0	01.1	60.0	02.1	59.9	02.9	59.9	03.1	59.9	04.2	59.8	05.2	59.7	05.9	60	
61	61.0	01.1	61.0	02.1	60.9	03.0	60.9	03.2	60.9	04.3	60.8	05.3	60.7	06.0	61	
62	62.0	01.1	62.0	02.2	61.9	03.0	61.9	03.2	61.9	04.3	61.8	05.4	61.7	06.1	62	
63	63.0	01.1	63.0	02.2	62.9	03.1	62.9	03.3	62.8	04.4	62.8	05.5	62.7	06.2	63	
64	64.0	01.1	64.0	02.3	63.9	03.1	63.9	03.3	63.8	04.5	63.8	05.6	63.7	06.3	64	
65	65.0	01.1	65.0	02.3	64.9	03.2	64.9	03.4	64.8	04.5	64.8	05.7	64.7	06.4	65	
66	66.0	01.2	66.0	02.3	65.9	03.2	65.9	03.5	65.8	04.6	65.7	05.8	65.7	06.5	66	
67	67.0	01.2	67.0	02.3	66.9	03.3	66.9	03.5	66.8	04.7	66.7	05.8	66.7	06.6	67	
68	68.0	01.2	68.0	02.4	67.9	03.3	67.9	03.6	67.8	04.7	67.7	05.9	67.7	06.7	68	
69	69.0	01.2	69.0	02.4	68.9	03.4	68.9	03.6	68.8	04.8	68.7	06.0	68.7	06.8	69	
70	70.0	01.2	70.0	02.4	69.9	03.4	69.9	03.7	69.8	04.9	69.7	06.1	69.7	06.9	70	
71	71.0	01.2	71.0	02.5	70.9	03.5	70.9	03.7	70.8	05.0	70.7	06.2	70.7	07.0	71	
72	72.0	01.3	72.0	02.5	71.9	03.5	71.9	03.8	71.8	05.0	71.7	06.3	71.7	07.1	72	
73	73.0	01.3	73.0	02.5	72.9	03.6	72.9	03.8	72.8	05.1	72.7	06.4	72.6	07.2	73	
74	74.0	01.3	74.0	02.6	73.9	03.6	73.9	03.9	73.8	05.2	73.7	06.5	73.6	07.3	74	
75	75.0	01.3	75.0	02.6	74.9	03.7	74.9	03.9	74.8	05.2	74.7	06.5	74.6	07.3	75	
76	76.0	01.3	76.0	02.7	75.9	03.7	75.9	04.0	75.8	05.3	75.7	06.6	75.6	07.4	76	
77	77.0	01.3	77.0	02.7	76.9	03.8	76.9	04.0	76.8	05.4	76.7	06.7	76.6	07.5	77	
78	78.0	01.4	78.0	02.7	77.9	03.8	77.9	04.1	77.8	05.4	77.7	06.8	77.6	07.6	78	
79	79.0	01.4	79.0	02.8	78.9	03.9	78.9	04.1	78.8	05.5	78.7	06.9	78.6	07.7	79	
80	80.0	01.4	80.0	02.8	79.9	03.9	79.9	04.2	79.8	05.6	79.7	07.0	79.6	07.8	80	
81	81.0	01.4	81.0	02.8	80.9	04.0	80.9	04.2	80.8	05.7	80.7	07.1	80.6	07.9	81	
82	82.0	01.4	81.9	02.9	81.9	04.0	81.9	04.3	81.8	05.7	81.7	07.2	81.6	08.0	82	
83	83.0	01.5	82.9	02.9	82.9	04.1	82.9	04.3	82.8	05.8	82.7	07.2	82.6	08.1	83	
84	84.0	01.5	83.9	02.9	83.9	04.1	83.9	04.4	83.8	05.9	83.7	07.3	83.6	08.2	84	
85	85.0	01.5	84.9	03.0	84.9	04.2	84.9	04.4	84.8	05.9	84.7	07.4	84.6	08.3	85	
86	86.0	01.5	85.9	03.0	85.9	04.2	85.9	04.5	85.8	06.0	85.7	07.5	85.6	08.4	86	
87	87.0	01.5	86.9	03.0	86.9	04.3	86.9	04.6	86.8	06.1	86.7	07.6	86.6	08.5	87	
88	88.0	01.5	87.9	03.1	87.9	04.3	87.9	04.6	87.8	06.1	87.7	07.7	87.6	08.6	88	
89	89.0	01.6	88.9	03.1	88.9	04.4	88.9	04.7	88.8	06.2	88.7	07.8	88.6	08.7	89	
90	90.0	01.6	89.9	03.1	89.9	04.4	89.9	04.7	89.8	06.3	89.7	07.8	89.6	08.8	90	
91	91.0	01.6	90.9	03.2	90.9	04.5	90.9	04.8	90.8	6.4	90.7	07.9	90.6	08.9	91	
92	92.0	01.6	91.9	03.2	91.9	04.5	91.9	04.8	91.8	06.4	91.6	08.0	91.6	09.0	92	
93	93.0	01.6	92.9	03.2	92.9	04.6	92.9	04.9	92.8	06.5	92.6	08.1	92.6	09.1	93	
94	94.0	01.6	93.9	03.3	93.9	04.6	93.9	04.9	93.8	06.6	93.6	08.2	93.5	09.2	94	
95	95.0	01.7	94.9	03.3	94.9	04.7	94.9	05.0	94.8	06.6	94.6	08.3	94.5	09.3	95	
96	96.0	01.7	95.9	03.4	95.9	04.7	95.9	05.0	95.8	06.7	95.6	08.4	95.5	09.4	96	
97	97.0	01.7	96.9	03.4	96.9	04.8	96.9	05.1	96.8	06.8	96.6	08.5	96.5	09.5	97	
98	98.0	01.7	97.9	03.4	97.9	04.8	97.9	05.1	97.8	06.8	97.6	08.5	97.5	09.6	98	
99	99.0	01.7	98.9	03.5	98.9	04.9	98.9	05.2	98.8	06.9	98.6	08.6	98.5	09.7	99	
100	100.0	01.7	99.9	03.5	99.9	04.9	99.9	05.2	99.8	07.0	99.6	08.7	99.5	09.8	100	
Dist.	Dep.	Lat.	Dep.	Lat.	Dist.	Dep.	Lat.	Dep.	Lat.	Dist.	Dep.	Lat.	Dist.	Dep.	Lat.	Dist.
89 Deg.			88 Deg.		77 Point.			87 Deg.		86 Deg.		85 Deg.			77 Point.	

Diff.	6 Deg.		7 Deg.		8 Deg.		$\frac{1}{2}$ Point.		9 Deg.		10 Deg.		11 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	01.0	00.1	01.0	00.1	01.0	00.1	01.0	00.1	01.0	00.2	01.0	00.2	01.0	00.2	1
2	02.0	00.2	02.0	00.2	02.0	00.3	02.0	00.3	02.0	00.3	02.0	00.3	02.0	00.4	2
3	03.0	00.3	03.0	00.4	03.0	00.4	03.0	00.4	03.0	00.5	03.0	00.5	02.9	00.6	3
4	04.0	00.4	04.0	00.5	04.0	00.6	04.0	00.6	03.9	00.6	03.9	00.7	03.9	00.8	4
5	05.0	00.5	05.0	00.6	05.0	00.7	04.9	00.7	04.9	00.8	04.9	00.9	04.9	01.0	5
6	06.0	00.6	06.0	00.7	05.9	00.8	05.9	00.9	05.9	00.9	05.9	01.0	05.9	01.1	6
7	07.0	00.7	06.9	00.9	06.9	01.0	06.9	01.0	06.9	01.1	06.9	01.2	06.9	01.3	7
8	08.0	00.8	07.9	01.0	07.9	01.1	07.9	01.2	07.9	01.3	07.9	01.4	07.9	01.5	8
9	08.9	00.9	08.9	01.1	08.9	01.2	08.9	01.3	08.9	01.4	08.9	01.6	08.8	01.7	9
10	09.9	01.0	09.9	01.2	09.9	01.4	09.9	01.5	09.9	01.6	09.8	01.7	09.8	01.9	10
11	10.9	01.1	10.9	01.3	10.9	01.5	10.9	01.6	10.9	01.7	10.8	01.9	10.8	02.1	11
12	11.9	01.3	11.9	01.5	11.9	01.7	11.9	01.8	11.9	01.9	11.8	02.1	11.8	02.3	12
13	12.9	01.4	12.9	01.6	12.9	01.8	12.9	01.9	12.8	02.0	12.8	02.3	12.8	02.5	13
14	13.9	01.5	13.9	01.7	13.9	01.9	13.8	02.1	13.8	02.2	13.8	02.4	13.7	02.7	14
15	14.9	01.6	14.9	01.8	14.9	02.1	14.8	02.2	14.8	02.3	14.8	02.6	14.7	02.9	15
16	15.9	01.7	15.9	01.9	15.8	02.2	15.8	02.3	15.8	02.5	15.8	02.8	15.7	03.1	16
17	16.9	01.8	16.9	02.1	16.8	02.4	16.8	02.5	16.8	02.7	16.7	03.0	16.7	03.2	17
18	17.9	01.9	17.9	02.2	17.8	02.5	17.8	02.6	17.8	02.8	17.7	03.1	17.7	03.4	18
19	18.9	02.0	18.9	02.3	18.8	02.6	18.8	02.8	18.8	03.0	18.7	03.3	18.6	03.6	19
20	19.9	02.1	19.8	02.4	19.8	02.8	19.8	02.9	19.8	03.1	19.7	03.5	19.6	03.8	20
21	20.9	02.2	20.8	02.6	20.8	02.9	20.8	03.1	20.7	03.3	20.7	03.6	20.6	04.0	21
22	21.9	02.3	21.8	02.7	21.8	03.1	21.8	03.2	21.7	03.4	21.7	03.8	21.6	04.2	22
23	22.9	02.4	22.8	02.8	22.8	03.2	22.8	03.4	22.7	03.6	22.6	04.0	22.6	04.4	23
24	23.9	02.5	23.8	02.9	23.8	03.3	23.7	03.5	23.7	03.8	23.6	04.2	23.6	04.6	24
25	24.9	02.6	24.8	03.0	24.8	03.5	24.7	03.7	24.7	03.9	24.6	04.3	24.5	04.8	25
26	25.9	02.7	25.8	03.2	25.7	03.6	25.7	03.8	25.7	04.1	25.6	04.5	25.5	05.0	26
27	26.9	02.8	26.8	03.3	26.7	03.8	26.7	04.0	26.7	04.2	26.6	04.7	26.5	05.2	27
28	27.8	02.9	27.8	03.4	27.7	03.9	27.7	04.1	27.7	04.4	27.6	04.9	27.5	05.3	28
29	28.8	03.0	28.8	03.5	28.7	04.0	28.7	04.3	28.6	04.5	28.6	05.0	28.5	05.5	29
30	29.8	03.1	29.8	03.7	29.7	04.2	29.7	04.4	29.6	04.7	29.5	05.2	29.4	05.7	30
31	30.8	03.2	30.8	03.8	30.7	04.3	30.7	04.5	30.6	04.9	30.5	05.4	30.4	05.9	31
32	31.8	03.3	31.8	03.9	31.7	04.5	31.7	04.7	31.6	05.1	31.5	05.6	31.4	06.1	32
33	32.8	03.4	32.8	04.0	32.7	04.6	32.6	04.8	32.6	05.2	32.5	05.7	32.4	06.3	33
34	33.8	03.6	33.7	04.1	33.7	04.7	33.6	05.0	33.6	05.4	33.5	05.9	33.4	06.5	34
35	34.8	03.7	34.7	04.3	34.7	04.9	34.6	05.1	34.6	05.5	34.5	06.1	34.4	06.7	35
36	35.8	03.8	35.7	04.4	35.6	05.0	35.6	05.3	35.6	05.6	35.4	06.2	35.3	06.9	36
37	36.8	03.9	36.7	04.5	36.6	05.1	36.6	05.4	36.5	05.8	36.4	06.4	36.3	07.1	37
38	37.8	04.0	37.7	04.6	37.6	05.3	37.6	05.6	37.5	05.9	37.4	06.6	37.3	07.3	38
39	38.8	04.1	38.7	04.8	38.6	05.4	38.6	05.7	38.5	06.1	38.4	06.8	38.3	07.4	39
40	39.8	04.2	39.7	04.9	39.6	05.6	39.6	05.9	39.5	06.3	39.4	06.9	39.3	07.6	40
41	40.8	04.3	40.7	05.0	40.6	05.7	40.6	06.0	40.5	06.4	40.4	07.1	40.2	07.8	41
42	41.8	04.4	41.7	05.1	41.6	05.8	41.5	06.2	41.5	06.6	41.4	07.3	41.2	08.0	42
43	42.8	04.5	42.7	05.2	42.6	06.0	42.5	06.3	42.5	06.7	42.3	07.5	42.2	08.2	43
44	43.8	04.6	43.7	05.4	43.6	06.1	43.5	06.5	43.5	06.9	43.3	07.6	43.2	08.4	44
45	44.8	04.7	44.7	05.5	44.6	06.3	44.5	06.6	44.4	07.0	44.3	07.8	44.2	08.6	45
46	45.7	04.8	45.7	05.6	45.6	06.4	45.5	06.7	45.4	07.2	45.3	08.0	45.2	08.8	46
47	46.7	04.9	46.6	05.7	46.5	06.6	46.5	06.9	46.4	07.3	46.3	08.2	46.1	09.0	47
48	47.7	05.0	47.6	05.9	47.5	06.7	47.5	07.0	47.4	07.5	47.3	08.3	47.1	09.2	48
49	48.7	05.1	48.6	06.0	48.5	06.8	48.5	07.2	48.4	07.7	48.3	08.5	48.1	09.3	49
50	49.7	05.2	49.6	06.1	49.5	07.0	49.5	07.3	49.4	07.8	49.2	08.7	49.1	09.5	50
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
Diff.	84 Deg.		83 Deg.		82 Deg.		74 Point.		81 Deg.		80 Deg.		79 Deg.		Diff.

Of LATITUDE and DEPARTURE.

7

Diff.	6 Deg.		7 Deg.		8 Deg.		$\frac{1}{2}$ Point.		9 Deg.		10 Deg.		11 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	50.7	05.3	50.6	06.2	50.5	07.1	50.4	07.5	50.4	08.0	50.2	08.9	50.1	09.7	51
52	51.7	05.4	51.6	06.3	51.5	07.2	51.4	07.6	51.4	08.1	51.2	09.0	51.0	09.9	52
53	52.7	05.5	52.6	06.5	52.5	07.4	52.4	07.8	52.3	08.3	52.2	09.2	52.0	10.1	53
54	53.7	05.6	53.6	06.6	53.5	07.5	53.4	07.9	53.3	08.4	53.2	09.4	53.0	10.3	54
55	54.7	05.7	54.6	06.7	54.5	07.7	54.4	08.1	54.3	08.6	54.2	09.5	54.0	10.5	55
56	55.7	05.9	55.6	06.8	55.5	07.8	55.4	08.2	55.3	08.8	55.1	09.7	55.0	10.7	56
57	56.7	06.0	56.6	06.9	56.4	07.9	56.4	08.4	56.3	08.9	56.1	09.9	56.0	10.9	57
58	57.7	06.1	57.6	07.1	57.4	08.1	57.4	08.5	57.3	09.1	57.1	10.1	56.9	11.1	58
59	58.7	06.2	58.6	07.2	58.4	08.2	58.4	08.7	58.3	09.2	58.1	10.2	57.9	11.3	59
60	59.7	06.3	59.5	07.3	59.4	08.4	59.4	08.8	59.3	09.4	59.1	10.4	58.9	11.4	60
61	60.7	06.4	60.5	07.4	60.4	08.5	60.3	08.9	60.2	09.5	60.1	10.6	59.9	11.6	61
62	61.7	06.5	61.5	07.6	61.4	08.6	61.3	09.1	61.2	09.7	61.1	10.8	60.9	11.8	62
63	62.7	06.6	62.5	07.7	62.4	08.8	62.3	09.2	62.2	09.9	62.0	10.9	61.8	12.0	63
64	63.6	06.7	63.5	07.8	63.4	08.9	63.3	09.4	63.2	10.0	63.0	11.1	62.8	12.2	64
65	64.6	06.8	64.5	07.9	64.4	09.0	64.3	09.5	64.2	10.2	64.0	11.3	63.8	12.4	65
66	65.6	06.9	65.5	08.0	65.4	09.2	65.3	09.7	65.2	10.3	65.0	11.5	64.8	12.6	66
67	66.6	07.0	66.5	08.2	66.3	09.3	66.3	09.8	66.2	10.5	66.0	11.6	65.8	12.8	67
68	67.6	07.1	67.5	08.3	67.3	09.5	67.3	10.0	67.2	10.6	67.0	11.8	66.7	13.0	68
69	68.6	07.2	68.5	08.4	68.3	09.6	68.3	10.1	68.2	10.8	68.0	12.0	67.7	13.2	69
70	69.6	07.3	69.5	08.5	69.3	09.7	69.2	10.3	69.1	10.9	68.9	12.2	68.7	13.4	70
71	70.6	07.4	70.5	08.7	70.3	09.9	70.2	10.4	70.1	11.1	69.9	12.3	69.7	13.5	71
72	71.6	07.5	71.5	08.8	71.3	10.0	71.2	10.6	71.1	11.3	70.9	12.5	70.7	13.7	72
73	72.6	07.6	72.5	08.9	72.3	10.2	72.2	10.7	72.1	11.4	71.9	12.7	71.7	13.9	73
74	73.6	07.7	73.4	09.0	73.3	10.3	73.2	10.9	73.1	11.6	72.9	12.8	72.6	14.1	74
75	74.6	07.8	74.4	09.1	74.3	10.4	74.2	11.0	74.1	11.7	73.9	13.0	73.6	14.3	75
76	75.6	07.9	75.4	09.3	75.3	10.6	75.2	11.1	75.1	11.9	74.8	13.2	74.6	14.5	76
77	76.6	08.0	76.4	09.4	76.3	10.7	76.2	11.3	76.1	12.0	75.8	13.4	75.6	14.7	77
78	77.6	08.1	77.4	09.5	77.2	10.9	77.2	11.4	77.0	12.2	76.8	13.5	76.6	14.9	78
79	78.6	08.3	78.4	09.6	78.2	11.0	78.1	11.6	78.0	12.4	77.8	13.7	77.5	15.1	79
80	79.6	08.4	79.4	09.8	79.2	11.1	79.1	11.7	79.0	12.5	78.8	13.9	78.5	15.3	80
81	80.6	08.5	80.4	09.9	80.2	11.3	80.1	11.9	80.0	12.7	79.8	14.1	79.5	15.5	81
82	81.5	08.6	81.4	10.0	81.2	11.4	81.1	12.0	81.0	12.8	80.8	14.2	80.5	15.6	82
83	82.5	08.7	82.4	10.1	82.2	11.6	82.1	12.2	82.0	13.0	81.7	14.4	81.5	15.8	83
84	83.5	08.8	83.4	10.2	83.2	11.7	83.1	12.3	83.0	13.1	82.7	14.6	82.5	16.0	84
85	84.5	08.9	84.4	10.4	84.2	11.8	84.1	12.5	84.0	13.3	83.7	14.8	83.4	16.2	85
86	85.5	09.0	85.4	10.5	85.2	12.0	85.1	12.6	84.9	13.4	84.7	14.9	84.4	16.4	86
87	86.5	09.1	86.3	10.6	86.2	12.1	86.0	12.8	85.9	13.6	85.7	15.1	85.4	16.6	87
88	87.5	09.2	87.3	10.7	87.1	12.2	87.0	12.9	86.9	13.8	86.7	15.3	86.4	16.8	88
89	88.5	09.3	88.3	10.8	88.1	12.4	88.0	13.1	87.9	13.9	87.6	15.4	87.4	17.0	89
90	89.5	09.4	89.3	11.0	89.1	12.5	89.0	13.2	88.9	14.1	88.6	15.6	88.3	17.2	90
91	90.5	09.5	90.3	11.1	90.1	12.7	90.0	13.3	89.9	14.2	89.6	15.8	89.3	17.4	91
92	91.5	09.6	91.3	11.2	91.1	12.8	91.0	13.5	90.9	14.4	90.6	16.0	90.3	17.6	92
93	92.5	09.7	92.3	11.3	92.1	12.9	92.0	13.6	91.9	14.5	91.6	16.1	91.3	17.7	93
94	93.5	09.8	93.3	11.5	93.1	13.1	93.0	13.8	92.8	14.7	92.6	16.3	92.3	17.9	94
95	94.5	09.9	94.3	11.6	94.1	13.2	94.0	13.9	93.8	14.9	93.6	16.5	93.3	18.1	95
96	95.5	10.0	95.3	11.7	95.1	13.4	95.0	14.1	94.8	15.0	94.5	16.7	94.2	18.3	96
97	96.5	10.1	96.3	11.8	96.1	13.5	96.0	14.2	95.8	15.2	95.5	16.8	95.2	18.5	97
98	97.5	10.2	97.3	11.9	97.0	13.6	96.9	14.4	96.8	15.3	96.5	17.0	96.2	18.7	98
99	98.5	10.3	98.3	12.1	98.0	13.8	97.9	14.5	97.8	15.5	97.5	17.2	97.2	18.9	99
100	99.4	10.4	99.2	12.2	99.0	13.9	98.9	14.7	98.8	15.6	98.5	17.4	98.2	19.1	100
Diff.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Diff.
	84 Deg.		83 Deg.		82 Deg.		$7\frac{1}{2}$ Point.		81 Deg.		80 Deg.		79 Deg.		

Diff.	1 Point.		12 Deg.		13 Deg.		14 Deg.		15 Point.		15 Deg.		16 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.2	01.0	00.3	01.0	00.3	1
2	02.0	00.4	02.0	00.4	01.9	00.4	01.9	00.5	01.9	00.5	01.9	00.5	01.9	00.6	2
3	02.9	00.6	02.9	00.6	02.9	00.7	02.9	00.7	02.9	00.7	02.9	00.8	02.9	00.8	3
4	03.9	00.8	03.9	00.8	03.9	00.9	03.9	01.0	03.9	01.0	03.9	01.0	03.8	01.1	4
5	04.9	01.0	04.9	01.0	04.9	01.1	04.9	01.2	04.8	01.2	04.8	01.3	04.8	01.4	5
6	05.9	01.2	05.9	01.2	05.8	01.3	05.8	01.4	05.8	01.5	05.8	01.5	05.8	01.7	6
7	06.9	01.4	06.8	01.5	06.8	01.6	06.8	01.7	06.8	01.7	06.8	01.8	06.7	01.9	7
8	07.8	01.6	07.8	01.7	07.8	01.8	07.8	01.9	07.8	01.9	07.7	02.1	07.7	02.2	8
9	08.8	01.8	08.8	01.9	08.8	02.0	08.7	02.2	08.7	02.2	08.7	02.3	08.7	02.5	9
10	09.8	02.0	09.8	02.1	09.7	02.2	09.7	02.4	09.7	02.4	09.7	02.6	09.6	02.8	10
11	10.8	02.1	10.8	02.3	10.7	02.5	10.7	02.7	10.7	02.7	10.6	02.8	10.6	03.0	11
12	11.8	02.3	11.7	02.5	11.7	02.7	11.6	02.9	11.6	02.9	11.6	03.1	11.5	03.3	12
13	12.7	02.5	12.7	02.7	12.7	02.9	12.6	03.1	12.6	03.2	12.6	03.4	12.5	03.6	13
14	13.7	02.7	13.7	02.9	13.6	03.1	13.6	03.4	13.6	03.4	13.5	03.6	13.5	03.9	14
15	14.7	02.9	14.7	03.1	14.6	03.4	14.6	03.6	14.5	03.6	14.5	03.9	14.4	04.1	15
16	15.7	03.1	15.6	03.3	15.6	03.6	15.5	03.9	15.5	03.9	15.5	04.1	15.4	04.4	16
17	16.7	03.3	16.6	03.5	16.6	03.8	16.5	04.1	16.5	04.1	16.4	04.4	16.3	04.7	17
18	17.7	03.5	17.6	03.7	17.5	04.0	17.5	04.4	17.5	04.4	17.4	04.7	17.3	05.0	18
19	18.6	03.7	18.6	03.9	18.5	04.3	18.4	04.6	18.4	04.6	18.4	04.9	18.3	05.2	19
20	19.6	03.9	19.6	04.2	19.5	04.5	19.4	04.8	19.4	04.9	19.3	05.2	19.2	05.5	20
21	20.6	04.1	20.5	04.4	20.5	04.7	20.4	05.1	20.4	05.1	20.3	05.4	20.2	05.8	21
22	21.6	04.3	21.5	04.6	21.4	04.9	21.3	05.3	21.3	05.3	21.2	05.7	21.1	06.1	22
23	22.6	04.5	22.5	04.8	22.4	05.2	22.3	05.6	22.3	05.6	22.2	06.0	22.1	06.3	23
24	23.5	04.7	23.5	05.0	23.4	05.4	23.3	05.8	23.3	05.8	23.2	06.2	23.1	06.6	24
25	24.5	04.9	24.5	05.2	24.4	05.6	24.3	06.0	24.2	06.1	24.1	06.5	24.0	06.9	25
26	25.5	05.1	25.4	05.4	25.3	05.8	25.2	06.3	25.2	06.3	25.1	06.7	25.0	07.2	26
27	26.5	05.3	26.4	05.6	26.3	06.1	26.2	06.5	26.2	06.6	26.1	07.0	26.0	07.4	27
28	27.5	05.5	27.4	05.8	27.3	06.3	27.2	06.8	27.2	06.8	27.0	07.2	26.9	07.7	28
29	28.4	05.7	28.4	06.0	28.3	06.5	28.1	07.0	28.1	07.0	28.0	07.5	27.9	08.0	29
30	29.4	05.9	29.3	06.2	29.2	06.7	29.1	07.3	29.1	07.3	29.0	07.8	28.8	08.3	30
31	30.4	06.0	30.3	06.4	30.2	07.0	30.1	07.5	30.1	07.5	29.9	08.0	29.8	08.5	31
32	31.4	06.2	31.3	06.7	31.2	07.2	31.0	07.7	31.0	07.8	30.9	08.3	30.8	08.8	32
33	32.4	06.4	32.3	06.9	32.2	07.4	32.0	08.0	32.0	08.0	31.9	08.5	31.7	09.1	33
34	33.3	06.6	33.3	07.1	33.1	07.6	33.0	08.2	33.0	08.3	32.8	08.8	32.7	09.4	34
35	34.3	06.8	34.2	07.3	34.1	07.9	34.0	08.5	33.9	08.5	33.8	09.1	33.6	09.6	35
36	35.3	07.0	35.2	07.5	35.1	08.1	34.9	08.7	34.9	08.7	34.8	09.3	34.6	09.9	36
37	36.3	07.2	36.2	07.7	36.1	08.3	35.9	09.0	35.9	09.0	35.7	09.6	35.6	10.2	37
38	37.3	07.4	37.2	07.9	37.0	08.5	36.9	09.2	36.9	09.2	36.7	09.8	36.6	10.5	38
39	38.3	07.6	38.1	08.1	38.0	08.8	37.8	09.4	37.8	09.5	37.7	10.1	37.5	10.7	39
40	39.2	07.8	39.1	08.3	39.0	09.0	38.8	09.7	38.8	09.7	38.6	10.4	38.5	11.0	40
41	40.2	08.0	40.1	08.5	39.9	09.2	39.8	09.9	39.8	10.0	39.6	10.6	39.4	11.3	41
42	41.2	08.2	41.1	08.7	40.9	09.4	40.8	10.2	40.7	10.2	40.6	10.9	40.4	11.6	42
43	42.2	08.4	42.1	08.9	41.9	09.7	41.7	10.4	41.7	10.4	41.5	11.1	41.3	11.8	43
44	43.2	08.6	43.0	09.1	42.9	09.9	42.7	10.6	42.7	10.7	42.5	11.4	42.3	12.1	44
45	44.1	08.8	44.0	09.4	43.8	10.1	43.7	10.9	43.6	10.9	43.5	11.6	43.3	12.4	45
46	45.1	09.0	45.0	09.6	44.8	10.3	44.6	11.1	44.6	11.2	44.4	11.9	44.2	12.7	46
47	46.1	09.2	46.0	09.8	45.8	10.6	45.6	11.4	45.6	11.4	45.4	12.2	45.2	13.0	47
48	47.1	09.4	47.0	10.0	46.8	10.8	46.6	11.6	46.6	11.7	46.4	12.4	46.1	13.2	48
49	48.1	09.6	47.9	10.2	47.7	11.0	47.5	11.9	47.5	11.9	47.3	12.7	47.1	13.5	49
50	49.0	09.8	48.9	10.4	48.7	11.2	48.5	12.1	48.5	12.1	48.3	12.9	48.1	13.8	50
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	7 Point.		78 Deg.		77 Deg.		76 Deg.		63 Point.		75 Deg.		74 Deg.		

Of LATITUDE and DEPARTURE.

9

Diff.	1 Point.		12 Deg.		13 Deg.		14 Deg.		15 Point.		15 Deg.		16 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	50.0	10.0	49.9	10.6	49.7	11.5	49.5	12.3	49.5	12.4	49.3	13.2	49.0	14.1	51
52	51.0	10.1	50.9	10.8	50.7	11.7	50.5	12.6	50.4	12.6	50.2	13.5	50.0	14.3	52
53	52.0	10.3	51.8	11.0	51.6	11.9	51.4	12.8	51.4	12.9	51.2	13.7	50.9	14.6	53
54	53.0	10.5	52.8	11.2	52.6	12.1	52.4	13.1	52.4	13.1	52.2	14.0	51.9	14.9	54
55	53.9	10.7	53.8	11.4	53.6	12.4	53.4	13.3	53.3	13.4	53.1	14.2	52.9	15.2	55
56	54.9	10.9	54.8	11.6	54.6	12.6	54.3	13.5	54.3	13.6	54.1	14.5	53.8	15.4	56
57	55.9	11.1	55.8	11.8	55.5	12.8	55.3	13.8	55.3	13.9	55.1	14.8	54.8	15.7	57
58	56.9	11.3	56.7	12.1	56.5	13.0	56.3	14.0	56.3	14.1	56.0	15.0	55.8	16.0	58
59	57.9	11.5	57.7	12.3	57.5	13.3	57.2	14.3	57.2	14.3	57.0	15.3	56.7	16.3	59
60	58.8	11.7	58.7	12.5	58.5	13.5	58.2	14.5	58.2	14.6	58.0	15.5	57.7	16.5	60
61	59.8	11.9	59.7	12.7	59.4	13.7	59.2	14.8	59.2	14.8	58.9	15.8	58.6	16.8	61
62	60.8	12.1	60.6	12.9	60.4	13.9	60.2	15.1	60.1	15.1	59.9	16.0	59.6	17.1	62
63	61.8	12.3	61.6	13.1	61.4	14.2	61.1	15.2	61.1	15.3	60.9	16.3	60.6	17.4	63
64	62.8	12.5	62.6	13.3	62.4	14.4	62.1	15.5	62.1	15.6	61.8	16.6	61.5	17.6	64
65	63.8	12.7	63.6	13.5	63.3	14.6	63.1	15.7	63.0	15.8	62.8	16.8	62.5	17.9	65
66	64.7	12.9	64.6	13.7	64.3	14.8	64.0	16.0	64.0	16.0	63.7	17.1	63.4	18.2	66
67	65.7	13.1	65.5	13.9	65.3	15.1	65.0	16.2	65.0	16.3	64.7	17.3	64.4	18.5	67
68	66.7	13.3	66.5	14.1	66.3	15.3	66.0	16.4	66.0	16.5	65.7	17.6	65.4	18.7	68
69	67.7	13.5	67.5	14.3	67.2	15.5	66.9	16.7	66.9	16.8	66.6	17.9	66.3	19.0	69
70	68.7	13.7	68.5	14.6	68.2	15.7	67.9	16.9	67.9	17.0	67.6	18.1	67.3	19.3	70
71	69.6	13.9	69.4	14.8	69.2	16.0	68.9	17.2	68.9	17.3	68.6	18.4	68.2	19.6	71
72	70.6	14.0	70.4	15.0	70.2	16.2	69.9	17.4	69.8	17.5	69.5	18.6	69.2	19.8	72
73	71.6	14.2	71.4	15.2	71.1	16.4	70.8	17.7	70.8	17.7	70.5	18.9	70.2	20.1	73
74	72.6	14.4	72.4	15.4	72.1	16.6	71.8	17.9	71.8	18.0	71.5	19.2	71.1	20.4	74
75	73.6	14.6	73.4	15.6	73.1	16.9	72.8	18.1	72.7	18.2	72.4	19.4	72.1	20.7	75
76	74.5	14.8	74.3	15.8	74.1	17.1	73.7	18.4	73.7	18.5	73.4	19.7	73.1	20.9	76
77	75.5	15.0	75.3	16.0	75.0	17.3	74.7	18.6	74.7	18.7	74.4	19.9	74.0	21.2	77
78	76.5	15.2	76.3	16.2	76.0	17.5	75.7	18.9	75.7	18.9	75.3	20.2	75.0	21.5	78
79	77.5	15.4	77.3	16.4	77.0	17.8	76.7	19.1	76.6	19.2	76.3	20.4	75.9	21.8	79
80	78.5	15.6	78.2	16.6	78.0	18.0	77.6	19.4	77.6	19.4	77.3	20.7	76.9	22.0	80
81	79.4	15.8	79.2	16.8	78.9	18.2	78.6	19.6	78.6	19.7	78.2	21.0	77.9	22.3	81
82	80.4	16.0	80.2	17.0	79.9	18.4	79.6	19.8	79.5	19.9	79.2	21.2	78.8	22.6	82
83	81.4	16.2	81.2	17.3	80.9	18.7	80.5	20.1	80.5	20.2	80.2	21.5	79.8	22.9	83
84	82.4	16.4	82.2	17.5	81.8	18.9	81.5	20.3	81.5	20.4	81.1	21.7	80.7	23.1	84
85	83.4	16.6	83.1	17.7	82.8	19.1	82.5	20.6	82.4	20.7	82.1	22.0	81.7	23.4	85
86	84.3	16.8	84.1	17.9	83.8	19.3	83.4	20.8	83.4	20.9	83.1	22.3	82.7	23.7	86
87	85.3	17.0	85.1	18.1	84.8	19.6	84.4	21.0	84.4	21.1	84.0	22.5	83.6	24.0	87
88	86.3	17.2	86.1	18.3	85.7	19.8	85.4	21.3	85.4	21.4	85.0	22.8	84.6	24.3	88
89	87.3	17.4	87.1	18.5	86.7	20.0	86.4	21.5	86.3	21.6	86.0	23.0	85.6	24.5	89
90	88.3	17.6	88.0	18.7	87.7	20.2	87.3	21.8	87.3	21.9	86.9	23.3	86.5	24.8	90
91	89.3	17.8	89.0	18.9	88.7	20.5	88.3	22.0	88.3	22.1	87.9	23.5	87.5	25.1	91
92	90.2	17.9	90.0	19.1	89.6	20.7	89.3	22.3	89.2	22.4	88.9	23.8	88.4	25.4	92
93	91.2	18.1	91.0	19.3	90.6	20.9	90.2	22.5	90.2	22.6	89.8	24.1	89.4	25.6	93
94	92.2	18.3	91.9	19.5	91.6	21.1	91.2	22.7	91.2	22.8	90.8	24.3	90.4	25.9	94
95	93.2	18.5	92.9	19.7	92.6	21.4	92.2	23.0	92.1	23.1	91.8	24.6	91.3	26.2	95
96	94.2	18.7	93.9	20.0	93.5	21.6	93.1	23.2	93.1	23.3	92.7	24.8	92.3	26.5	96
97	95.1	18.9	94.9	20.2	94.5	21.8	94.1	23.5	94.1	23.6	93.7	25.1	93.2	26.7	97
98	96.1	19.1	95.9	20.4	95.5	22.0	95.1	23.7	95.1	23.8	94.7	25.4	94.2	27.0	98
99	97.1	19.3	96.8	20.6	96.5	22.3	96.1	23.9	96.0	24.1	95.6	25.6	95.2	27.3	99
100	98.1	19.5	97.8	20.8	97.4	22.5	97.0	24.2	97.0	24.3	96.6	25.9	96.1	27.6	100
Diff.	7 Point.		78 Deg.		77 Deg.		76 Deg.		61 Point.		75 Deg.		74 Deg.		Diff.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	

Diff.	1 $\frac{1}{2}$ Point.		17 Deg.		18 Deg.		19 Deg.		1 $\frac{1}{2}$ Point.		20 Deg.		21 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	01.0	00.3	01.0	00.3	01.0	00.3	00.9	00.3	00.9	00.3	00.9	00.3	00.9	00.4	1
2	01.9	00.6	01.0	00.6	01.0	00.6	01.0	00.7	01.0	00.7	01.0	00.7	01.0	00.7	2
3	02.9	00.9	02.9	00.9	02.9	00.9	02.8	01.0	02.8	01.0	02.8	01.0	02.8	01.1	3
4	03.8	01.2	03.8	01.2	03.8	01.2	03.8	01.3	03.8	01.3	03.8	01.4	03.7	01.4	4
5	04.8	01.5	04.8	01.5	04.8	01.5	04.7	01.6	04.7	01.7	04.7	01.7	04.7	01.8	5
6	05.7	01.7	05.7	01.8	05.7	01.9	05.7	02.0	05.6	02.0	05.6	02.1	05.6	02.1	6
7	06.7	02.0	06.7	02.0	06.7	02.2	06.6	02.3	06.6	02.4	06.6	02.4	06.5	02.5	7
8	07.7	02.3	07.6	02.3	07.6	02.5	07.6	02.6	07.5	02.7	07.5	02.7	07.5	02.9	8
9	08.6	02.6	08.6	02.6	08.6	02.8	08.5	02.9	08.5	03.0	08.5	03.1	08.4	03.2	9
10	09.6	02.9	09.6	02.9	09.5	03.1	09.5	03.3	09.4	03.4	09.4	03.4	09.3	03.6	10
11	10.5	03.2	10.5	03.2	10.5	03.4	10.4	03.6	10.4	03.7	10.3	03.8	10.3	03.9	11
12	11.5	03.5	11.5	03.5	11.4	03.7	11.3	03.9	11.3	04.0	11.3	04.1	11.2	04.3	12
13	12.4	03.8	12.4	03.8	12.4	04.0	12.3	04.2	12.2	04.4	12.2	04.4	12.1	04.7	13
14	13.4	04.1	13.4	04.1	13.3	04.3	13.2	04.6	13.2	04.7	13.2	04.8	13.1	05.0	14
15	14.4	04.4	14.3	04.4	14.3	04.6	14.2	04.9	14.1	05.1	14.1	05.1	14.0	05.4	15
16	15.3	04.6	15.3	04.7	15.2	04.9	15.1	05.2	15.1	05.4	15.0	05.5	14.9	05.7	16
17	16.3	04.9	16.3	05.0	16.2	05.3	16.1	05.5	16.0	05.7	16.0	05.8	15.9	06.1	17
18	17.2	05.2	17.2	05.3	17.1	05.6	17.0	05.9	16.9	06.1	16.9	06.2	16.8	06.4	18
19	18.2	05.5	18.2	05.6	18.1	05.9	18.0	06.2	17.9	06.4	17.9	06.5	17.7	06.8	19
20	19.1	05.8	19.1	05.8	19.0	06.2	18.9	06.5	18.8	06.7	18.8	06.8	18.7	07.2	20
21	20.1	06.1	20.1	06.1	20.0	06.5	19.9	06.8	19.8	07.1	19.7	07.2	19.6	07.5	21
22	21.1	06.4	21.0	06.4	20.9	06.8	20.8	07.2	20.7	07.4	20.7	07.5	20.5	07.9	22
23	22.0	06.7	22.0	06.7	21.9	07.1	21.7	07.5	21.7	07.7	21.6	07.9	21.5	08.2	23
24	23.0	07.0	22.9	07.0	22.8	07.4	22.7	07.8	22.6	08.1	22.6	08.2	22.4	08.6	24
25	23.9	07.3	23.9	07.3	23.8	07.7	23.6	08.1	23.5	08.4	23.5	08.5	23.3	09.0	25
26	24.9	07.5	24.9	07.6	24.7	08.0	24.6	08.5	24.5	08.8	24.4	08.9	24.3	09.3	26
27	25.8	07.8	25.8	07.9	25.7	08.3	25.5	08.8	25.4	09.1	25.4	09.2	25.2	09.7	27
28	26.8	08.1	26.8	08.2	26.6	08.7	26.5	09.1	26.4	09.4	26.3	09.6	26.1	10.0	28
29	27.8	08.4	27.7	08.5	27.6	09.0	27.4	09.4	27.3	09.8	27.3	09.9	27.1	10.4	29
30	28.7	08.7	28.7	08.8	28.5	09.3	28.4	09.8	28.2	10.1	28.2	10.3	28.0	10.8	30
31	29.7	09.0	29.6	09.1	29.5	09.6	29.3	10.1	29.2	10.4	29.1	10.6	28.9	11.1	31
32	30.6	09.3	30.6	09.4	30.4	10.0	30.3	10.4	30.1	10.8	30.1	10.9	29.9	11.5	32
33	31.6	09.6	31.6	09.6	31.4	10.2	31.2	10.7	31.1	11.1	31.0	11.3	30.8	11.8	33
34	32.5	09.9	32.5	09.9	32.3	10.5	32.1	11.1	32.0	11.5	31.9	11.6	31.7	12.2	34
35	33.5	10.2	33.5	10.2	33.3	10.8	33.1	11.4	33.0	11.8	32.9	12.0	32.7	12.5	35
36	34.4	10.4	34.4	10.5	34.2	11.1	34.0	11.7	33.9	12.1	33.8	12.3	33.6	12.9	36
37	35.4	10.7	35.4	10.8	35.2	11.2	35.0	12.0	34.8	12.5	34.8	12.7	34.5	13.3	37
38	36.4	11.0	36.3	11.1	36.1	11.7	35.9	12.4	35.8	12.8	35.7	13.0	35.5	13.6	38
39	37.3	11.3	37.3	11.4	37.1	12.0	36.9	12.7	36.7	13.1	36.6	13.3	36.4	14.0	39
40	38.3	11.6	38.3	11.7	38.0	12.4	37.8	13.0	37.7	13.5	37.6	13.7	37.3	14.3	40
41	39.2	11.9	39.2	12.0	39.0	12.7	38.8	13.3	38.6	13.8	38.5	14.0	38.3	14.7	41
42	40.2	12.2	40.2	12.3	39.9	13.0	39.7	13.7	39.5	14.1	39.5	14.4	39.2	15.1	42
43	41.1	12.5	41.1	12.6	40.9	13.3	40.7	14.0	40.5	14.5	40.4	14.7	40.1	15.4	43
44	42.1	12.8	42.1	12.9	41.8	13.6	41.6	14.3	41.4	14.8	41.3	15.0	41.1	15.8	44
45	43.1	13.1	43.0	13.1	42.8	13.9	42.5	14.7	42.4	15.2	42.3	15.4	42.0	16.1	45
46	44.0	13.4	44.0	13.4	43.7	14.2	43.5	15.0	43.3	15.5	43.2	15.7	42.9	16.5	46
47	45.0	13.6	44.9	13.7	44.7	14.5	44.4	15.3	44.2	15.8	44.2	16.1	43.9	16.8	47
48	45.9	13.9	45.9	14.0	45.7	14.8	45.4	15.6	45.2	16.2	45.1	16.4	44.8	17.2	48
49	46.9	14.2	46.9	14.3	46.6	15.1	46.3	16.0	46.1	16.5	46.0	16.8	45.7	17.6	49
50	47.8	14.5	47.8	14.6	47.6	15.4	47.3	16.3	47.1	16.8	47.0	17.1	46.7	17.9	50
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	6 $\frac{1}{2}$ Point.		73 Deg.		72 Deg.		71 Deg.		6 $\frac{1}{2}$ Point.		70 Deg.		69 Deg.		

Of LATITUDE *and* DEPARTURE.

11

Lat.	1 st Point.		17 Deg.		18 Deg.		19 Deg.		1 st Point.		20 Deg.		21 Deg.		Lat.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	48.8	14.8	48.8	14.9	48.5	15.8	48.2	16.6	48.0	17.2	47.9	17.4	47.6	18.3	51
52	49.7	15.1	49.7	15.2	49.4	16.1	49.2	16.9	49.0	17.5	48.9	17.8	48.5	18.6	52
53	50.7	15.3	50.7	15.5	50.4	16.4	50.1	17.3	49.9	17.9	49.8	18.1	49.5	19.0	53
54	51.7	15.7	51.6	15.8	51.3	16.7	51.0	17.6	50.8	18.2	50.7	18.5	50.4	19.4	54
55	52.6	16.0	52.6	16.1	52.3	17.0	52.0	17.9	51.8	18.5	51.7	18.8	51.3	19.7	55
56	53.6	16.2	53.5	16.4	53.3	17.3	52.9	18.2	52.7	18.9	52.6	19.2	52.3	20.1	56
57	54.5	16.5	54.5	16.7	54.2	17.6	53.8	18.6	53.7	19.2	53.6	19.5	53.2	20.4	57
58	55.5	16.8	55.5	17.0	55.2	17.9	54.9	18.9	54.6	19.5	54.5	19.8	54.1	20.8	58
59	56.5	17.1	56.4	17.3	56.1	18.2	55.8	19.2	55.5	19.9	55.4	20.2	55.1	21.1	59
60	57.4	17.4	57.4	17.5	57.1	18.5	56.7	19.5	56.5	20.2	56.4	20.5	56.0	21.5	60
61	58.4	17.7	58.3	17.8	58.0	18.8	57.7	19.9	57.4	20.6	57.3	20.9	56.9	21.9	61
62	59.3	18.0	59.3	18.1	59.0	19.2	58.6	20.2	58.4	20.9	58.3	21.2	57.9	22.2	62
63	60.3	18.3	60.2	18.4	59.9	19.5	59.6	20.5	59.3	21.2	59.2	21.5	58.8	22.6	63
64	61.2	18.6	61.2	18.7	60.9	19.8	60.5	20.8	60.3	21.6	60.1	21.9	59.7	22.9	64
65	62.2	18.9	62.2	19.0	61.8	20.1	61.5	21.2	61.2	21.9	61.1	22.2	60.7	23.3	65
66	63.2	19.2	63.1	19.3	62.8	20.4	62.4	21.5	62.1	22.2	62.0	22.6	61.6	23.7	66
67	64.1	19.4	64.1	19.6	63.7	20.7	63.3	21.8	63.1	22.6	63.0	22.9	62.6	24.0	67
68	65.1	19.7	65.0	19.9	64.7	21.0	64.3	22.1	64.0	22.9	63.9	23.3	63.5	24.4	68
69	66.0	20.0	66.0	20.2	65.6	21.3	65.2	22.5	65.0	23.2	64.8	23.6	64.4	24.7	69
70	67.0	20.3	66.9	20.5	66.6	21.6	66.2	22.8	65.9	23.6	65.8	23.9	65.4	25.1	70
71	67.9	20.6	67.9	20.8	67.5	21.9	67.1	23.1	66.8	23.9	66.7	24.3	66.3	25.4	71
72	68.9	20.9	68.8	21.1	68.5	22.2	68.1	23.4	67.8	24.3	67.7	24.6	67.2	25.8	72
73	69.9	21.2	69.8	21.3	69.4	22.6	69.0	23.8	68.7	24.6	68.6	25.0	68.2	26.2	73
74	70.8	21.5	70.8	21.6	70.4	22.9	70.0	24.1	69.7	24.9	69.5	25.3	69.1	26.5	74
75	71.8	21.8	71.7	21.9	71.3	23.2	70.9	24.4	70.6	25.3	70.5	25.6	70.0	26.9	75
76	72.7	22.1	72.7	22.2	72.3	23.5	71.9	24.7	71.6	25.6	71.4	26.0	71.0	27.2	76
77	73.7	22.4	73.6	22.5	73.2	23.8	72.8	25.1	72.5	25.9	72.4	26.3	71.9	27.6	77
78	74.6	22.6	74.6	22.8	74.2	24.1	73.7	25.4	73.4	26.3	73.3	26.7	72.8	28.0	78
79	75.6	22.9	75.5	23.1	75.1	24.4	74.7	25.7	74.4	26.6	74.2	27.0	73.8	28.3	79
80	76.6	23.2	76.5	23.4	76.1	24.7	75.6	26.0	75.3	27.0	75.2	27.4	74.7	28.7	80
81	77.5	23.5	77.5	23.7	77.0	25.0	76.6	26.4	76.3	27.3	76.1	27.7	75.6	29.0	81
82	78.5	23.8	78.4	24.0	78.0	25.3	77.5	26.7	77.2	27.6	77.1	28.0	76.6	29.4	82
83	79.4	24.1	79.4	24.3	78.9	25.6	78.5	27.0	78.1	28.0	78.0	28.4	77.5	29.7	83
84	80.4	24.4	80.3	24.5	79.9	26.0	79.4	27.3	79.1	28.3	78.9	28.7	78.4	30.1	84
85	81.3	24.7	81.3	24.8	80.8	26.3	80.4	27.7	80.0	28.6	79.9	29.1	79.4	30.5	85
86	82.3	25.0	82.2	25.1	81.8	26.6	81.3	28.0	81.0	29.0	80.8	29.4	80.3	30.8	86
87	83.3	25.3	83.2	25.4	82.7	26.9	82.3	28.3	81.9	29.3	81.8	29.8	81.2	31.2	87
88	84.2	25.5	84.2	25.7	83.7	27.2	83.2	28.7	82.9	29.6	82.7	30.1	82.2	31.5	88
89	85.2	25.8	85.1	26.0	84.6	27.5	84.1	29.0	83.8	30.0	83.6	30.4	83.1	31.9	89
90	86.1	26.1	86.1	26.3	85.6	27.8	85.1	29.3	84.7	30.3	84.6	30.8	84.0	32.3	90
91	87.1	26.4	87.0	26.6	86.5	28.1	86.0	29.6	85.7	30.7	85.5	31.1	85.0	32.6	91
92	88.0	26.7	88.0	26.9	87.5	28.4	87.0	30.0	86.6	31.0	86.5	31.5	85.9	33.0	92
93	89.0	27.0	88.9	27.2	88.4	28.7	87.9	30.3	87.6	31.3	87.4	31.8	86.8	33.3	93
94	90.0	27.3	89.9	27.5	89.4	29.0	88.9	30.6	88.5	31.7	88.3	32.1	87.8	33.7	94
95	90.9	27.6	90.8	27.8	90.4	29.4	89.8	30.9	89.4	32.0	89.3	32.5	88.7	34.0	95
96	91.9	27.9	91.8	28.1	91.3	29.7	90.8	31.3	90.4	32.3	90.2	32.8	89.6	34.4	96
97	92.8	28.2	92.8	28.4	92.3	30.0	91.7	31.6	91.3	32.7	91.1	33.2	90.6	34.8	97
98	93.8	28.4	93.7	28.7	93.2	30.3	92.7	31.9	92.3	33.0	92.1	33.5	91.5	35.1	98
99	94.7	28.7	94.7	28.9	94.2	30.6	93.6	32.2	93.2	33.4	93.0	33.9	92.4	35.5	99
100	95.7	29.0	95.6	29.2	95.1	30.9	94.5	32.6	94.2	33.7	94.0	34.2	93.4	35.8	100
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	6 th Point.		73 Deg.		72 Deg.		71 Deg.		6 th Point.		70 Deg.		69 Deg.		

A TABLE of DIFFERENCE

Diff.	22 Deg.		2 Points		23 Deg.		24 Deg.		25 Deg.		2½ Point.		26 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	00.9	00.4	1
2	01.9	00.7	01.8	00.8	01.8	00.8	01.8	00.8	01.8	00.8	01.8	00.9	01.8	00.9	2
3	02.8	01.1	02.8	01.1	02.8	01.2	02.7	01.2	02.7	01.3	02.7	01.3	02.7	01.3	3
4	03.7	01.5	03.7	01.5	03.7	01.6	03.6	01.6	03.6	01.7	03.6	01.7	03.6	01.8	4
5	04.6	01.9	04.6	01.9	04.6	02.0	04.6	02.0	04.5	02.1	04.5	02.1	04.5	02.2	5
6	05.6	02.2	05.5	02.3	05.5	02.3	05.5	02.4	05.4	02.5	05.4	02.6	05.4	02.6	6
7	06.5	02.6	06.5	02.7	06.4	02.7	06.4	02.8	06.3	03.0	06.3	03.0	06.3	03.1	7
8	07.4	03.0	07.4	03.1	07.4	03.1	07.3	03.2	07.2	03.4	07.2	03.4	07.2	03.5	8
9	08.3	03.4	08.3	03.4	08.3	03.5	08.2	03.7	08.2	03.8	08.1	03.8	08.1	03.9	9
10	09.3	03.7	09.2	03.8	09.2	03.9	09.1	04.1	09.1	04.2	09.0	04.3	09.0	04.4	10
11	10.2	04.1	10.2	04.2	10.1	04.3	10.0	04.5	10.0	04.6	09.9	04.7	09.9	04.8	11
12	11.1	04.5	11.1	04.6	11.0	04.7	11.0	04.9	10.9	05.1	10.8	05.1	10.8	05.3	12
13	12.1	04.9	12.0	05.0	12.0	05.1	11.9	05.3	11.8	05.5	11.7	05.6	11.7	05.7	13
14	13.0	05.2	12.9	05.4	12.9	05.5	12.8	05.7	12.7	05.9	12.7	06.0	12.6	06.1	14
15	13.9	05.6	13.9	05.7	13.8	05.9	13.7	06.1	13.6	06.3	13.6	06.4	13.5	06.6	15
16	14.8	06.0	14.8	06.1	14.7	06.2	14.6	06.5	14.5	06.8	14.5	06.8	14.4	06.9	16
17	15.8	06.4	15.7	06.5	15.6	06.6	15.5	06.9	15.4	07.2	15.4	07.3	15.3	07.5	17
18	16.7	06.7	16.6	06.9	16.6	07.0	16.4	07.3	16.3	07.6	16.3	07.7	16.2	07.9	18
19	17.6	07.1	17.6	07.3	17.5	07.4	17.4	07.7	17.2	08.0	17.2	08.1	17.1	08.3	19
20	18.5	07.5	18.5	07.7	18.4	07.8	18.3	08.1	18.1	08.5	18.1	08.6	18.0	08.8	20
21	19.5	07.9	19.4	08.0	19.3	08.2	19.2	08.5	19.0	08.9	19.0	09.0	18.9	09.2	21
22	20.4	08.2	20.3	08.4	20.3	08.6	20.1	08.9	19.9	09.3	19.9	09.4	19.8	09.6	22
23	21.3	08.6	21.2	08.8	21.2	09.0	21.0	09.4	20.8	09.7	20.8	09.8	20.7	10.1	23
24	22.3	09.0	22.2	09.2	22.1	09.4	21.9	09.8	21.8	10.1	21.7	10.3	21.6	10.5	24
25	23.2	09.4	23.1	09.6	23.0	09.8	22.8	10.2	22.7	10.6	22.6	10.7	22.5	11.0	25
26	24.1	09.7	24.0	09.9	23.9	10.2	23.8	10.6	23.6	11.0	23.5	11.1	23.4	11.4	26
27	25.0	10.1	24.9	10.2	24.9	10.5	24.7	11.0	24.5	11.4	24.4	11.5	24.3	11.8	27
28	26.0	10.5	25.9	10.7	25.8	10.9	25.6	11.4	25.4	11.8	25.3	12.0	25.2	12.3	28
29	26.9	10.9	26.8	11.1	26.7	11.3	26.5	11.8	26.3	12.3	26.2	12.4	26.1	12.7	29
30	27.8	11.2	27.7	11.5	27.6	11.7	27.4	12.2	27.2	12.7	27.1	12.8	27.0	13.2	30
31	28.7	11.6	28.6	11.9	28.5	12.1	28.3	12.6	28.1	13.1	28.0	13.3	27.9	13.6	31
32	29.7	12.0	29.6	12.3	29.5	12.5	29.2	13.0	29.0	13.5	28.9	13.7	28.8	14.0	32
33	30.6	12.4	30.5	12.6	30.4	12.9	30.1	13.4	29.9	13.9	29.8	14.1	29.7	14.5	33
34	31.5	12.7	31.4	13.0	31.3	13.3	31.1	13.8	30.8	14.4	30.7	14.5	30.6	14.9	34
35	32.5	13.1	32.3	13.4	32.2	13.7	32.0	14.2	31.7	14.8	31.6	15.1	31.5	15.3	35
36	33.4	13.5	33.3	13.8	33.1	14.1	32.9	14.6	32.6	15.2	32.5	15.4	32.4	15.8	36
37	34.3	13.9	34.2	14.2	34.1	14.4	33.8	15.0	33.5	15.6	33.4	15.8	33.3	16.2	37
38	35.2	14.2	35.1	14.5	35.0	14.8	34.7	15.5	34.4	16.1	34.3	16.2	34.2	16.7	38
39	36.2	14.6	36.0	14.9	35.9	15.2	35.6	15.9	35.3	16.5	35.3	16.7	35.1	17.1	39
40	37.1	15.0	37.0	15.3	36.8	15.6	36.5	16.3	36.3	16.9	36.2	17.1	36.0	17.5	40
41	38.0	15.4	37.9	15.7	37.7	16.0	37.5	16.7	37.2	17.3	37.1	17.5	36.8	18.0	41
42	38.9	15.7	38.8	16.1	38.7	16.4	38.4	17.1	38.1	17.7	38.0	18.0	37.7	18.4	42
43	39.9	16.1	39.7	16.5	39.6	16.8	39.3	17.5	39.0	18.2	38.9	18.4	38.6	18.9	43
44	40.8	16.5	40.7	16.8	40.5	17.2	40.2	17.9	39.9	18.6	39.8	18.8	39.5	19.3	44
45	41.7	16.9	41.6	17.2	41.4	17.6	41.1	18.3	40.8	19.0	40.7	19.2	40.4	19.7	45
46	42.7	17.2	42.5	17.6	42.3	18.0	42.0	18.7	41.7	19.4	41.6	19.7	41.3	20.2	46
47	43.6	17.6	43.4	18.0	43.3	18.4	42.9	19.1	42.6	19.9	42.5	20.1	42.2	20.6	47
48	44.5	18.0	44.3	18.4	44.2	18.8	43.8	19.5	43.5	20.3	43.4	20.5	43.1	21.0	48
49	45.4	18.4	45.3	18.8	45.1	19.2	44.8	19.9	44.4	20.7	44.3	20.9	44.0	21.5	49
50	46.4	18.7	46.2	19.1	46.0	19.5	45.7	20.3	45.3	21.1	45.2	21.4	44.9	21.9	50
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	68 Deg.	6 Points			67 Deg.	66 Deg.	65 Deg.	65 Deg.	5½ Point.	64 Deg.					

Diff.	22 Deg.		2 Points		23 Deg.		24 Deg.		25 Deg.		24 Point.		26 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	47.3	19.1	47.1	19.5	46.9	19.9	46.6	20.7	46.2	21.6	46.1	21.8	45.8	22.4	51
52	48.2	19.5	48.0	19.9	47.9	20.3	47.5	21.1	47.1	22.0	47.0	22.2	46.7	22.8	52
53	49.1	19.9	49.0	20.3	48.8	20.7	48.4	21.6	48.0	22.4	47.9	22.7	47.6	23.2	53
54	50.1	20.2	49.9	20.7	49.7	21.1	49.3	22.0	48.9	22.8	48.8	23.1	48.5	23.7	54
55	51.0	20.6	50.8	21.0	50.6	21.5	50.2	22.4	49.8	23.2	49.7	23.5	49.4	24.1	55
56	51.9	21.0	51.7	21.4	51.5	21.9	51.2	22.8	50.8	23.7	50.6	23.9	50.3	24.5	56
57	52.8	21.4	52.7	21.8	52.5	22.3	52.1	23.2	51.7	24.1	51.5	24.4	51.2	25.0	57
58	53.8	21.7	53.6	22.2	53.4	22.7	53.0	23.6	52.6	24.5	52.4	24.8	52.1	25.4	58
59	54.7	22.1	54.5	22.6	54.3	23.1	53.9	24.0	53.5	24.9	53.3	25.2	53.0	25.9	59
60	55.6	22.5	55.4	23.0	55.2	23.4	54.8	24.4	54.4	25.4	54.2	25.7	53.9	26.3	60
61	56.5	22.8	56.4	23.3	56.1	23.8	55.7	24.8	55.3	25.8	55.1	26.1	54.8	26.7	61
62	57.5	23.2	57.3	23.7	57.1	24.2	56.6	25.2	56.2	26.2	56.0	26.5	55.7	27.2	62
63	58.4	23.6	58.2	24.1	58.0	24.6	57.5	25.6	57.1	26.6	57.0	26.9	56.6	27.6	63
64	59.3	24.0	59.1	24.5	58.9	25.0	58.5	26.0	58.0	27.0	57.9	27.4	57.5	28.0	64
65	60.3	24.3	60.1	24.9	59.8	25.4	59.4	26.4	58.9	27.5	58.8	27.8	58.4	28.5	65
66	61.2	24.7	61.0	25.3	60.8	25.8	60.3	26.8	59.8	27.9	59.7	28.2	59.3	28.9	66
67	62.1	25.1	61.9	25.6	61.7	26.2	61.2	27.2	60.7	28.3	60.6	28.6	60.2	29.4	67
68	63.0	25.5	62.8	26.0	62.6	26.6	62.1	27.7	61.6	28.7	61.5	29.1	61.1	29.8	68
69	64.0	25.8	63.7	26.4	63.5	27.0	63.0	28.1	62.5	29.2	62.4	29.5	62.0	30.2	69
70	64.9	26.2	64.7	26.8	64.4	27.3	63.9	28.5	63.4	29.6	63.3	29.9	62.9	30.7	70
71	65.8	26.6	65.6	27.2	65.4	27.7	64.9	28.9	64.3	30.0	64.2	30.4	63.8	31.1	71
72	66.8	27.0	66.5	27.6	66.3	28.1	65.8	29.3	65.2	30.4	65.1	30.8	64.7	31.6	72
73	67.7	27.3	67.4	27.9	67.2	28.5	66.7	29.7	66.2	30.8	66.0	31.2	65.6	32.0	73
74	68.6	27.7	68.4	28.3	68.1	28.9	67.6	30.1	67.1	31.3	66.9	31.6	66.5	32.4	74
75	69.5	28.1	69.3	28.7	69.0	29.3	68.5	30.5	68.0	31.7	67.8	32.1	67.4	32.9	75
76	70.5	28.5	70.2	29.1	70.0	29.7	69.4	30.9	68.9	32.1	68.7	32.5	68.3	33.3	76
77	71.4	28.8	71.1	29.5	70.9	30.1	70.3	31.3	69.8	32.5	69.6	32.9	69.2	33.8	77
78	72.3	29.2	72.1	29.8	71.8	30.5	71.3	31.7	70.7	33.0	70.5	33.3	70.1	34.2	78
79	73.2	29.6	73.0	30.2	72.7	30.9	72.2	32.1	71.6	33.4	71.4	33.8	71.0	34.6	79
80	74.2	30.0	73.9	30.6	73.6	31.3	73.1	32.5	72.5	33.8	72.3	34.2	71.9	35.1	80
81	75.1	30.3	74.8	31.0	74.6	31.6	74.0	32.9	73.4	34.2	73.2	34.6	72.8	35.5	81
82	76.0	30.7	75.8	31.4	75.5	32.0	74.9	33.3	74.3	34.7	74.1	35.1	73.7	35.9	82
83	77.0	31.1	76.7	31.8	76.4	32.4	75.8	33.7	75.2	35.1	75.0	35.5	74.6	36.4	83
84	77.9	31.5	77.6	32.1	77.3	32.8	76.7	34.1	76.1	35.5	75.9	35.9	75.5	36.8	84
85	78.8	31.8	78.5	32.5	78.2	33.2	77.6	34.6	77.0	35.9	76.8	36.3	76.4	37.3	85
86	79.7	32.2	79.5	32.9	79.2	33.6	78.6	35.0	77.9	36.3	77.7	36.8	77.3	37.7	86
87	80.7	32.6	80.4	33.3	80.1	34.0	79.5	35.4	78.8	36.8	78.6	37.2	78.2	38.1	87
88	81.6	33.0	81.3	33.7	81.0	34.4	80.4	35.8	79.8	37.2	79.6	37.6	79.1	38.6	88
89	82.5	33.3	82.2	34.1	81.9	34.8	81.3	36.2	80.7	37.6	80.5	38.1	80.0	39.0	89
90	83.4	33.7	83.2	34.4	82.8	35.2	82.2	36.6	81.6	38.0	81.4	38.5	80.9	39.5	90
91	84.4	34.1	84.1	34.8	83.8	35.6	83.1	37.0	82.5	38.5	82.3	38.9	81.8	39.9	91
92	85.3	34.5	85.0	35.2	84.7	35.9	84.0	37.4	83.4	38.9	83.2	39.3	82.7	40.3	92
93	86.2	34.8	85.9	35.6	85.6	36.3	85.0	37.8	84.3	39.3	84.1	39.8	83.6	40.8	93
94	87.2	35.2	86.8	36.0	86.5	36.7	85.9	38.2	85.2	39.7	85.0	40.2	84.5	41.2	94
95	88.1	35.6	87.8	36.4	87.4	37.1	86.8	38.6	86.1	40.1	85.9	40.6	85.4	41.6	95
96	89.0	36.0	88.7	36.7	88.3	37.5	87.7	39.0	87.0	40.6	86.8	41.0	86.3	42.1	96
97	89.9	36.3	89.6	37.1	89.3	37.9	88.6	39.4	87.9	41.0	87.7	41.5	87.2	42.5	97
98	90.8	36.7	90.5	37.5	90.2	38.4	89.5	39.9	88.8	41.4	88.6	41.9	88.1	43.0	98
99	91.7	37.1	91.5	37.9	91.1	38.7	90.4	40.3	89.7	41.8	89.5	42.3	89.0	43.4	99
100	92.6	37.5	92.4	38.3	92.0	39.1	91.4	40.7	90.6	42.3	90.4	42.8	89.9	43.8	100
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	68 Deg.		6 Points		67 Deg.		66 Deg.		65 Deg.		54 Point.		64 Deg.		

Diff.	27 Deg.		28 Deg.		29 Point.		29 Deg.		30 Deg.		31 Point.		31 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.9	00.5	00.9	00.9	00.9	00.5	00.9	00.5	00.9	00.5	00.9	00.5	00.9	00.5	1
2	01.8	00.9	01.8	00.9	01.8	00.9	01.7	01.0	01.7	01.0	01.7	01.0	01.7	01.0	2
3	02.7	01.4	02.6	01.4	02.6	01.4	02.6	01.5	02.6	01.5	02.6	01.5	02.6	01.5	3
4	03.6	01.8	03.5	01.9	03.5	01.9	03.5	01.9	03.5	02.0	03.4	02.1	03.4	02.1	4
5	04.5	02.3	04.4	02.3	04.4	02.4	04.4	02.4	04.3	02.5	04.3	02.6	04.3	02.6	5
6	05.3	02.7	05.3	02.8	05.3	02.8	05.2	02.9	05.2	03.0	05.1	03.1	05.1	03.1	6
7	06.2	03.2	06.2	03.3	06.2	03.3	06.1	03.4	06.1	03.5	06.0	03.6	06.0	03.6	7
8	07.1	03.6	07.1	03.8	07.1	03.8	07.0	03.9	06.9	04.0	06.9	04.1	06.9	04.1	8
9	08.0	04.1	07.9	04.2	07.9	04.2	07.9	04.4	07.8	04.5	07.7	04.6	07.7	04.6	9
10	08.9	04.5	08.8	04.7	08.8	04.7	08.7	04.8	08.7	05.0	08.6	05.1	08.6	05.1	10
11	09.8	05.0	09.7	05.2	09.7	05.2	09.6	05.3	09.5	05.5	09.4	05.7	09.4	05.7	11
12	10.7	05.4	10.6	05.6	10.6	05.7	10.5	05.8	10.4	06.0	10.3	06.2	10.3	06.2	12
13	11.6	05.9	11.5	06.1	11.5	06.1	11.4	06.3	11.3	06.5	11.1	06.7	11.1	06.7	13
14	12.5	06.4	12.4	06.6	12.3	06.6	12.2	06.8	12.1	07.0	12.0	07.2	12.0	07.2	14
15	13.4	06.8	13.2	07.0	13.2	07.1	13.1	07.3	13.0	07.5	12.9	07.7	12.9	07.7	15
16	14.3	07.3	14.1	07.5	14.1	07.5	14.0	07.8	13.9	08.0	13.7	08.2	13.7	08.2	16
17	15.1	07.7	15.0	08.0	15.0	08.0	14.9	08.2	14.7	08.5	14.6	08.7	14.6	08.8	17
18	16.0	08.2	15.9	08.5	15.9	08.5	15.7	08.7	15.6	09.0	15.4	09.3	15.4	09.3	18
19	16.9	08.6	16.8	08.9	16.8	09.0	16.6	09.2	16.5	09.5	16.3	09.8	16.3	09.8	19
20	17.8	09.1	17.7	09.4	17.6	09.4	17.5	09.7	17.3	10.0	17.2	10.3	17.1	10.3	20
21	18.7	09.5	18.5	09.9	18.5	09.9	18.4	10.2	18.2	10.5	18.0	10.8	18.0	10.8	21
22	19.6	10.0	19.4	10.3	19.4	10.4	19.2	10.7	19.1	11.0	18.9	11.3	18.9	11.3	22
23	20.5	10.4	20.3	10.8	20.3	10.8	20.1	11.1	19.9	11.5	19.7	11.8	19.7	11.8	23
24	21.4	10.9	21.2	11.3	21.2	11.3	21.0	11.6	20.8	12.0	20.6	12.3	20.6	12.3	24
25	22.3	11.3	22.1	11.7	22.1	11.8	21.9	12.1	21.6	12.5	21.4	12.9	21.4	12.9	25
26	23.2	11.8	23.0	12.2	22.9	12.3	22.7	12.6	22.5	13.0	22.3	13.4	22.3	13.4	26
27	24.1	12.3	23.8	12.7	23.8	12.7	23.6	13.1	23.4	13.5	23.2	13.9	23.1	13.9	27
28	24.9	12.7	24.7	13.1	24.7	13.2	24.5	13.6	24.2	14.0	24.0	14.4	24.0	14.4	28
29	25.8	13.2	25.6	13.6	25.6	13.7	25.4	14.1	25.1	14.5	24.9	14.9	24.9	14.9	29
30	26.7	13.6	26.5	14.1	26.5	14.1	26.2	14.5	26.0	15.0	25.7	15.4	25.7	15.4	30
31	27.6	14.1	27.4	14.6	27.3	14.6	27.1	15.0	26.8	15.5	26.6	15.9	26.6	16.0	31
32	28.5	14.5	28.3	15.0	28.2	15.1	28.0	15.5	27.7	16.0	27.4	16.4	27.4	16.5	32
33	29.4	15.0	29.1	15.5	29.1	15.6	28.9	16.0	28.6	16.5	28.3	17.0	28.3	17.0	33
34	30.3	15.4	30.0	16.0	30.0	16.0	29.7	16.5	29.4	17.0	29.2	17.5	29.1	17.5	34
35	31.2	15.9	30.9	16.4	30.9	16.5	30.6	17.0	30.3	17.5	30.0	18.0	30.0	18.0	35
36	32.1	16.3	31.8	16.9	31.7	17.0	31.5	17.5	31.2	18.0	30.9	18.5	30.9	18.5	36
37	33.0	16.8	32.7	17.4	32.6	17.4	32.4	17.9	32.0	18.5	31.7	19.0	31.7	19.1	37
38	33.9	17.3	33.5	17.8	33.5	17.9	33.2	18.4	32.9	19.0	32.6	19.5	32.6	19.6	38
39	34.7	17.7	34.4	18.3	34.4	18.4	34.1	18.9	33.8	19.5	33.4	20.0	33.4	20.1	39
40	35.6	18.2	35.3	18.8	35.3	18.9	35.0	19.4	34.6	20.0	34.0	20.6	34.3	20.6	40
41	36.5	18.6	36.2	19.2	36.2	19.3	35.9	19.9	35.5	20.5	35.2	21.1	35.1	21.1	41
42	37.4	19.1	37.1	19.7	37.0	19.8	36.7	20.4	36.4	21.0	36.0	21.6	36.0	21.6	42
43	38.3	19.5	38.0	20.2	37.9	20.3	37.6	20.8	37.2	21.5	36.9	22.1	36.9	22.1	43
44	39.2	20.0	38.8	20.7	38.8	20.7	38.5	21.3	38.1	22.0	37.7	22.6	37.7	22.7	44
45	40.1	20.4	39.7	21.1	39.7	21.2	39.4	21.8	39.0	22.5	38.6	23.1	38.6	23.2	45
46	41.0	20.9	40.6	21.6	40.6	21.7	40.2	22.3	39.8	23.0	39.5	23.6	39.4	23.7	46
47	41.9	21.3	41.5	22.1	41.4	22.2	41.1	22.8	40.7	23.5	40.3	24.2	40.3	24.2	47
48	42.8	21.8	42.4	22.5	42.3	22.6	42.0	23.3	41.6	24.0	41.2	24.7	41.1	24.7	48
49	43.7	22.2	43.3	23.0	43.2	23.1	42.9	23.8	42.4	24.5	42.0	25.2	42.0	25.2	49
50	44.5	22.7	44.1	23.5	44.1	23.6	43.7	24.2	43.3	25.0	42.9	25.7	42.9	25.7	50
Diff.	Dep.		Dep.		Dep.		Dep.		Dep.		Dep.		Dep.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
63	63	Deg.	62	Deg.	51	Point.	61	Deg.	60	Deg.	51	Point.	59	Deg.	

Of LATITUDE and DEPARTURE.

15

Diff.	27 Deg.		28 Deg.		29 Point.		29 Deg.		30 Deg.		31 Point.		31 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	45.4	23.2	45.0	23.9	45.0	24.0	44.6	24.7	44.2	25.5	43.7	26.2	43.7	26.3	51
52	40.3	23.6	45.9	24.4	45.9	24.5	45.5	25.2	45.0	26.0	44.6	26.7	44.6	26.8	52
53	47.2	24.1	46.8	24.9	46.7	25.0	46.4	25.7	45.9	26.5	45.5	27.2	45.4	27.3	53
54	48.1	24.5	47.7	25.4	47.6	25.5	47.2	26.2	46.8	27.0	46.3	27.8	46.3	27.8	54
55	49.0	25.0	48.0	25.8	48.5	25.9	48.1	26.7	47.6	27.5	47.2	28.3	47.1	28.3	55
56	49.9	25.4	49.4	26.3	49.4	26.4	49.0	27.1	48.5	28.0	48.0	28.8	48.0	28.8	56
57	50.8	25.9	50.3	26.8	50.3	26.9	49.9	27.6	49.4	28.5	48.9	29.4	48.9	29.4	57
58	51.7	26.3	51.2	27.2	51.2	27.3	50.7	28.1	50.2	29.0	49.7	29.8	49.7	29.9	58
59	52.6	26.8	52.1	27.7	52.0	27.8	51.6	28.6	51.1	29.5	50.6	30.3	50.6	30.4	59
60	53.5	27.2	53.0	28.2	52.9	28.3	52.5	29.1	52.0	30.0	51.5	30.8	51.4	30.9	60
61	54.4	27.7	53.9	28.6	53.8	28.8	53.3	29.6	52.8	30.5	52.3	31.4	52.3	31.4	61
62	55.2	28.1	54.7	29.1	54.7	29.2	54.2	30.1	53.7	31.0	53.2	31.9	53.1	31.9	62
63	56.1	28.6	55.6	29.6	55.6	29.7	55.1	30.5	54.6	31.5	54.0	32.4	54.0	32.4	63
64	57.0	29.1	56.5	30.0	56.4	30.2	56.0	31.0	55.4	32.0	54.9	32.9	54.9	33.0	64
65	57.9	29.5	57.4	30.5	57.3	30.6	56.8	31.5	56.3	32.5	55.7	33.4	55.7	33.5	65
66	58.8	30.0	58.3	31.0	58.2	31.1	57.7	32.0	57.2	33.0	56.6	33.9	56.6	34.0	66
67	59.7	30.4	59.2	31.5	59.1	31.6	58.6	32.5	58.0	33.5	57.5	34.4	57.4	34.5	67
68	60.6	30.9	60.0	32.0	60.0	32.1	59.5	33.0	58.9	34.0	58.3	35.0	58.3	35.0	68
69	61.5	31.3	60.9	32.4	60.9	32.5	60.3	33.5	59.8	34.5	59.2	35.5	59.1	35.5	69
70	62.4	31.8	61.8	32.9	61.7	33.0	61.2	33.9	60.6	35.0	60.0	36.0	60.0	36.0	70
71	63.3	32.2	62.7	33.3	62.6	33.5	62.1	34.4	61.5	35.5	60.9	36.5	60.9	36.6	71
72	64.2	32.7	63.6	33.8	63.5	33.9	63.0	34.9	62.4	36.0	61.8	37.0	61.7	37.2	72
73	65.0	33.1	64.5	34.3	64.4	34.4	63.8	35.4	63.2	36.5	62.6	37.5	62.6	37.6	73
74	65.9	33.6	65.3	34.7	65.3	34.9	64.7	35.9	64.1	37.0	63.5	38.0	63.4	38.1	74
75	66.8	34.1	66.2	35.2	66.1	35.4	65.6	36.4	64.9	37.5	64.3	38.6	64.3	38.6	75
76	67.7	34.5	67.1	35.7	67.0	35.8	66.5	36.8	65.8	38.0	65.2	39.1	65.1	39.1	76
77	68.6	35.0	68.0	36.2	67.9	36.3	67.3	37.3	66.7	38.5	66.0	39.6	66.0	39.7	77
78	69.5	35.4	68.9	36.6	68.8	36.8	68.2	37.8	67.5	39.0	66.9	40.1	66.9	40.2	78
79	70.4	35.9	69.7	37.1	69.7	37.2	69.1	38.3	68.4	39.5	67.8	40.6	67.7	40.7	79
80	71.3	36.3	70.6	37.6	70.6	37.7	70.0	38.8	69.3	40.0	68.6	41.1	68.6	41.2	80
81	72.2	36.8	71.5	38.0	71.4	38.2	70.8	39.3	70.1	40.5	69.5	41.6	69.4	41.7	81
82	73.1	37.2	72.4	38.5	72.3	38.7	71.7	39.8	71.0	41.0	70.3	42.2	70.3	42.2	82
83	74.0	37.7	73.3	39.0	73.2	39.1	72.6	40.2	71.9	41.5	71.2	42.7	71.1	42.7	83
84	74.8	38.1	74.2	39.4	74.1	39.6	73.5	40.7	72.7	42.0	72.0	43.2	72.0	43.3	84
85	75.7	38.6	75.0	39.9	75.0	40.1	74.3	41.2	73.6	42.5	72.9	43.7	72.9	43.8	85
86	76.6	39.0	75.9	40.4	75.8	40.5	75.2	41.7	74.5	43.0	73.8	44.2	73.7	44.3	86
87	77.5	39.5	76.8	40.8	76.7	41.0	76.1	42.2	75.3	43.5	74.6	44.7	74.6	44.8	87
88	78.4	40.0	77.7	41.3	77.6	41.5	77.0	42.7	76.2	44.0	75.5	45.2	75.4	45.3	88
89	79.3	40.4	78.6	41.8	78.5	42.0	77.8	43.1	77.1	44.5	76.3	45.8	76.3	45.8	89
90	80.2	40.9	79.5	42.3	79.4	42.4	78.7	43.6	77.9	45.0	77.2	46.3	77.1	46.3	90
91	81.1	41.3	80.3	42.7	80.3	42.9	79.6	44.1	78.8	45.5	78.1	46.8	78.0	46.9	91
92	82.0	41.8	81.2	43.2	81.1	43.4	80.5	44.6	79.7	46.0	78.9	47.3	78.9	47.4	92
93	82.9	42.2	82.1	43.7	82.0	43.8	81.3	45.1	80.5	46.5	79.8	47.8	79.7	47.9	93
94	83.8	42.7	83.0	44.1	82.9	44.3	82.2	45.6	81.4	47.0	80.6	48.3	80.6	48.4	94
95	84.6	43.1	83.9	44.6	83.8	44.8	83.1	46.1	82.3	47.5	81.5	48.8	81.4	48.9	95
96	85.5	43.6	84.8	45.1	84.7	45.3	84.0	46.5	83.1	48.0	82.3	49.4	82.3	49.4	96
97	86.4	44.0	85.6	45.5	85.5	45.7	84.8	47.0	84.0	48.5	83.2	49.9	83.1	50.0	97
98	87.3	44.5	86.5	46.0	86.4	46.2	85.7	47.5	84.9	49.0	84.1	50.4	84.0	50.5	98
99	88.2	44.9	87.4	46.5	87.3	46.7	86.6	48.0	85.7	49.5	84.9	50.9	84.9	51.0	99
100	89.1	45.4	88.3	46.9	88.2	47.1	87.5	48.5	86.6	50.0	85.8	51.4	85.7	51.5	100
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	63 Deg.		62 Deg.		51 Point.		61 Deg.		60 Deg.		51 Point.		59 Deg.		

17C	32 D. g.		33 Deg.		3 Points		34 Deg.		35 Deg.		36 Deg.		37 Point.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.8	00.9	00.8	00.5	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	1
2	01.7	01.1	01.7	01.1	01.7	01.1	01.6	01.1	01.6	01.1	01.6	01.2	01.6	01.2	2
3	02.5	01.6	02.5	01.6	02.5	01.7	02.5	01.7	02.5	01.7	02.4	01.8	02.4	01.8	3
4	03.4	02.1	03.4	02.2	03.3	02.2	03.3	02.2	03.3	02.3	03.2	02.4	03.2	02.4	4
5	04.2	02.6	04.2	02.7	04.2	02.8	04.1	02.8	04.1	02.9	04.0	02.9	04.0	03.0	5
6	05.1	03.2	05.0	03.3	05.0	03.3	05.0	03.3	04.9	03.4	04.8	03.5	04.8	03.6	6
7	05.9	03.7	05.9	03.8	05.8	03.9	05.8	03.9	05.7	04.0	05.7	04.1	05.6	04.2	7
8	06.8	04.2	06.7	04.4	06.6	04.4	06.6	04.5	06.6	04.6	06.5	04.7	06.4	04.8	8
9	07.6	04.8	07.5	04.9	07.5	05.0	07.5	05.0	07.4	05.2	07.3	05.3	07.2	05.4	9
10	08.5	05.3	08.4	05.4	08.3	05.6	08.3	05.6	08.2	05.7	08.1	05.9	08.0	06.0	10
11	09.3	05.8	09.2	06.0	09.1	06.1	09.1	06.2	09.0	06.3	08.9	06.5	08.8	06.6	11
12	10.2	06.4	10.1	06.5	10.0	06.7	09.9	06.7	09.8	06.9	09.7	07.0	09.6	07.1	12
13	11.0	06.9	10.9	07.1	10.8	07.2	10.8	07.3	10.6	07.5	10.5	07.6	10.4	07.7	13
14	11.9	07.4	11.7	07.6	11.6	07.8	11.6	07.8	11.5	08.0	11.3	08.2	11.2	08.3	14
15	12.7	07.9	12.6	08.2	12.5	08.3	12.4	08.3	12.3	08.6	12.1	08.8	12.0	08.9	15
16	13.6	08.5	13.4	08.7	13.3	08.9	13.3	08.9	13.1	09.2	12.9	09.4	12.9	09.5	16
17	14.4	09.0	14.3	09.3	14.1	09.4	14.1	09.5	13.9	09.8	13.8	10.0	13.7	10.1	17
18	15.3	09.5	15.1	09.8	15.0	10.0	14.9	10.1	14.7	10.3	14.6	10.6	14.5	10.7	18
19	16.1	10.1	15.9	10.3	15.8	10.6	15.8	10.6	15.6	10.9	15.4	11.2	15.3	11.3	19
20	17.0	10.6	16.8	10.9	16.6	11.1	16.6	11.2	16.4	11.5	16.2	11.8	16.1	11.9	20
21	17.8	11.1	17.6	11.4	17.5	11.7	17.4	11.7	17.2	12.0	17.0	12.3	16.9	12.5	21
22	18.7	11.7	18.5	12.0	18.3	12.2	18.2	12.3	18.0	12.6	17.8	12.9	17.7	13.1	22
23	19.5	12.2	19.3	12.5	19.1	12.8	19.1	12.9	18.8	13.2	18.6	13.5	18.5	13.7	23
24	20.4	12.7	20.1	13.1	20.0	13.3	19.9	13.4	19.7	13.8	19.4	14.1	19.3	14.3	24
25	21.2	13.2	21.0	13.6	20.8	13.9	20.7	14.0	20.5	14.3	20.2	14.7	20.1	14.9	25
26	22.0	13.9	21.8	14.2	21.6	14.4	21.6	14.5	21.3	14.9	21.0	15.3	20.9	15.5	26
27	22.9	14.3	22.6	14.7	22.4	15.0	22.4	15.1	22.1	15.5	21.8	15.9	21.7	16.1	27
28	23.7	14.8	23.5	15.2	23.3	15.6	23.2	15.6	22.9	16.1	22.7	16.5	22.5	16.7	28
29	24.6	15.4	24.3	15.8	24.1	16.1	24.0	16.2	23.8	16.6	23.5	17.0	23.3	17.3	29
30	25.4	15.9	25.2	16.3	24.9	16.7	24.9	16.8	24.6	17.2	24.3	17.6	24.1	17.9	30
31	26.3	16.4	26.0	16.9	25.8	17.2	25.7	17.3	25.4	17.8	25.1	18.2	24.9	18.5	31
32	27.1	17.0	26.8	17.4	26.6	17.8	26.5	17.9	26.2	18.4	25.9	18.8	25.7	19.1	32
33	28.0	17.5	27.7	18.0	27.4	18.3	27.4	18.5	27.0	18.9	26.7	19.4	26.5	19.7	33
34	28.8	18.0	28.5	18.5	28.3	18.9	28.2	19.0	27.9	19.5	27.5	20.0	27.3	20.3	34
35	29.7	18.5	29.4	19.1	29.1	19.4	29.0	19.6	28.7	20.1	28.3	20.6	28.1	20.8	35
36	30.5	19.1	30.2	19.6	29.9	20.0	29.8	20.1	29.5	20.6	29.1	21.2	28.9	21.4	36
37	31.4	19.6	31.0	20.1	30.8	20.6	30.7	20.7	30.3	21.2	29.9	21.7	29.7	22.0	37
38	32.2	20.1	31.9	20.7	31.6	21.1	31.5	21.2	31.1	21.8	30.7	22.3	30.5	22.6	38
39	33.1	20.7	32.7	21.2	32.4	21.7	32.3	21.8	32.0	22.4	31.6	22.9	31.3	23.2	39
40	33.9	21.2	33.6	21.8	33.3	22.2	33.2	22.4	32.8	22.9	32.4	23.5	32.1	23.8	40
41	34.8	21.7	34.4	22.3	34.1	22.8	34.0	22.9	33.6	23.5	33.2	24.1	32.9	24.4	41
42	35.6	22.3	35.2	22.9	34.9	23.3	34.8	23.5	34.4	24.1	34.0	24.7	33.7	25.0	42
43	36.5	22.8	36.1	23.4	35.8	23.9	35.6	24.0	35.2	24.7	34.8	25.3	34.5	25.6	43
44	37.3	23.3	36.9	24.0	36.6	24.4	36.5	24.6	36.0	25.2	35.6	25.9	35.3	26.2	44
45	38.2	23.8	37.7	24.5	37.4	25.0	37.3	25.2	36.9	25.8	36.4	26.5	36.1	26.8	45
46	39.0	24.4	38.6	25.1	38.2	25.5	38.1	25.7	37.7	26.4	37.2	27.0	36.9	27.4	46
47	39.9	24.9	39.4	25.6	39.1	26.1	39.0	26.3	38.5	27.0	38.0	27.6	37.7	28.0	47
48	40.7	25.4	40.3	26.1	39.9	26.7	39.8	26.8	39.3	27.5	38.8	28.2	38.6	28.6	48
49	41.6	26.0	41.1	26.7	40.7	27.2	40.6	27.4	40.1	28.1	39.6	28.8	39.2	29.2	49
50	42.4	26.5	41.9	27.2	41.6	27.8	41.4	28.0	41.0	28.7	40.4	29.4	40.2	29.8	50
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	58 Deg.		57 Deg.		5 Points		56 Deg.		55 Deg.		54 Deg.		47 Point.		

Of LATITUDE and DEPARTURE.

17

Diff.	32 Deg.		33 Deg.		3 Points		34 Deg.		35 Deg.		36 Deg.		37 Point.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	43.2	27.0	42.8	27.8	42.4	28.3	42.3	28.5	41.8	29.3	41.3	30.0	41.0	30.4	51
52	44.1	27.6	43.6	28.3	43.2	28.9	43.1	29.1	42.6	29.8	42.1	30.6	41.8	31.0	52
53	44.9	28.1	44.5	28.9	44.1	29.4	43.9	29.6	43.4	30.4	42.9	31.2	42.6	31.6	53
54	45.8	28.6	45.3	29.4	44.9	30.0	44.8	30.2	44.2	31.0	43.7	31.7	43.4	32.2	54
55	46.6	29.1	46.1	30.0	45.7	30.6	45.6	30.8	45.1	31.5	44.5	32.3	44.2	32.8	55
56	47.5	29.7	47.0	30.5	46.6	31.1	46.4	31.3	45.9	32.1	45.3	32.9	45.0	33.4	56
57	48.3	30.2	47.8	31.0	47.4	31.7	47.3	31.9	46.7	32.7	46.1	33.5	45.8	34.0	57
58	49.2	30.7	48.6	31.6	48.2	32.2	48.1	32.4	47.5	33.3	46.9	34.1	46.6	34.5	58
59	50.0	31.3	49.5	32.1	49.1	32.8	48.9	33.0	48.3	33.8	47.7	34.7	47.4	35.1	59
60	50.9	31.8	50.3	32.7	49.9	33.3	49.7	33.6	49.2	34.4	48.5	35.3	48.2	35.7	60
61	51.7	32.3	51.2	33.2	50.7	33.9	50.6	34.1	50.0	35.0	49.3	35.9	49.0	36.3	61
62	52.6	32.8	52.0	33.8	51.6	34.4	51.4	34.7	50.8	35.6	50.2	36.4	49.8	36.9	62
63	53.4	33.4	52.8	34.3	52.4	35.0	52.2	35.2	51.6	36.1	51.0	37.0	50.6	37.5	63
64	54.3	33.9	53.7	34.9	53.2	35.6	53.1	35.8	52.4	36.7	51.8	37.6	51.4	38.1	64
65	55.1	34.4	54.5	35.4	54.0	36.1	53.9	36.3	53.2	37.3	52.6	38.2	52.2	38.7	65
66	56.0	35.0	55.4	35.9	54.9	36.7	54.7	36.9	54.1	37.9	53.4	38.8	53.0	39.3	66
67	56.8	35.5	56.2	36.5	55.7	37.2	55.5	37.5	54.9	38.4	54.2	39.4	53.8	39.9	67
68	57.7	36.0	57.0	37.0	56.5	37.8	56.4	38.0	55.7	39.0	55.0	40.0	54.6	40.5	68
69	58.5	36.6	57.9	37.6	57.4	38.3	57.2	38.6	56.5	39.6	55.8	40.6	55.4	41.1	69
70	59.4	37.1	58.7	38.1	58.2	38.9	58.0	39.1	57.3	40.2	56.6	41.1	56.2	41.7	70
71	60.2	37.6	59.5	38.7	59.0	39.4	58.9	39.7	58.2	40.7	57.4	41.7	57.0	42.3	71
72	61.1	38.2	60.4	39.2	59.9	40.0	59.7	40.3	59.0	41.3	58.2	42.3	57.8	42.9	72
73	61.9	38.7	61.2	39.8	60.7	40.6	60.5	40.8	59.8	41.9	59.1	42.9	58.6	43.5	73
74	62.8	39.2	62.1	40.3	61.5	41.1	61.3	41.4	60.6	42.4	59.9	43.5	59.4	44.1	74
75	63.6	39.7	62.9	40.8	62.4	41.7	62.2	41.9	61.4	43.0	60.7	44.1	60.2	44.7	75
76	64.4	40.3	63.7	41.4	63.2	42.2	63.0	42.5	62.3	43.6	61.5	44.7	61.0	45.3	76
77	65.3	40.8	64.6	41.9	64.0	42.8	63.8	43.1	63.1	44.2	62.3	45.3	61.8	45.9	77
78	66.1	41.3	65.4	42.5	64.9	43.3	64.7	43.6	63.9	44.7	63.1	45.8	62.6	46.5	78
79	66.9	41.9	66.3	43.0	65.7	43.9	65.5	44.2	64.7	45.3	63.9	46.4	63.5	47.1	79
80	67.8	42.4	67.1	43.6	66.5	44.4	66.3	44.7	65.5	45.9	64.7	47.0	64.3	47.7	80
81	68.7	42.9	67.9	44.1	67.4	45.0	67.1	45.3	66.4	46.5	65.5	47.6	65.1	48.3	81
82	69.5	43.4	68.8	44.7	68.2	45.6	68.0	45.9	67.2	47.0	66.3	48.2	65.9	48.8	82
83	70.4	44.0	69.6	45.2	69.0	46.1	68.8	46.4	68.0	47.6	67.1	48.8	66.7	49.4	83
84	71.2	44.5	70.5	45.7	69.8	46.7	69.6	47.0	68.8	48.2	68.0	49.4	67.5	50.0	84
85	72.1	45.0	71.3	46.3	70.7	47.2	70.5	47.5	69.6	48.8	68.8	50.0	68.3	50.6	85
86	72.9	45.6	72.1	46.8	71.5	47.8	71.3	48.1	70.5	49.3	69.6	50.5	69.1	51.2	86
87	73.8	46.1	73.0	47.4	72.3	48.3	72.1	48.6	71.3	49.9	70.4	51.1	69.9	51.8	87
88	74.6	46.6	73.8	47.9	73.2	48.9	73.0	49.2	72.1	50.5	71.2	51.7	70.7	52.4	88
89	75.5	47.2	74.6	48.5	74.0	49.4	73.8	49.8	72.9	51.0	72.0	52.3	71.5	53.0	89
90	76.3	47.7	75.5	49.0	74.8	50.0	74.6	50.3	73.7	51.6	72.8	52.9	72.3	53.6	90
91	77.2	48.2	76.3	49.6	75.7	50.6	75.4	50.9	74.5	52.2	73.6	53.5	73.1	54.2	91
92	78.0	48.7	77.2	50.1	76.5	51.1	76.3	51.4	75.4	52.8	74.4	54.1	73.9	54.8	92
93	78.9	49.3	78.0	50.6	77.3	51.7	77.1	52.0	76.2	53.3	75.2	54.7	74.7	55.4	93
94	79.7	49.8	78.8	51.2	78.2	52.2	77.9	52.6	77.0	53.9	76.0	55.3	75.5	56.0	94
95	80.6	50.3	79.7	51.7	79.0	52.8	78.8	53.1	77.8	54.5	76.9	55.8	76.3	56.6	95
96	81.4	50.9	80.5	52.3	79.8	53.3	79.6	53.7	78.6	55.1	77.7	56.4	77.1	57.2	96
97	82.3	51.4	81.4	52.8	80.7	53.9	80.4	54.2	79.5	55.6	78.5	57.0	77.9	57.8	97
98	83.1	51.9	82.2	53.4	81.5	54.4	81.2	54.8	80.3	56.2	79.3	57.6	78.7	58.4	98
99	84.0	52.5	83.0	53.9	82.3	55.0	82.1	55.4	81.1	56.8	80.1	58.2	79.5	59.0	99
100	84.8	53.0	83.9	54.5	83.1	55.6	82.9	55.9	81.9	57.4	80.9	58.8	80.3	59.6	100
Diff.	58 Deg.		57 Deg.		5 Points		56 Deg.		55 Deg.		54 Deg.		4½ Point.		Diff.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	

A TABLE of DIFFERENCE

Dist.	37 Deg.		38 Deg.		39 Deg.		3 ¹ / ₂ Point.		40 Deg.		41 Deg.		42 Deg.		Dist.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.6	00.8	00.7	00.7	00.7	1
2	01.6	01.2	01.6	01.2	01.6	01.3	01.5	01.3	01.5	01.3	01.5	01.3	01.5	01.3	2
3	02.4	01.8	02.4	01.8	02.3	01.9	02.3	01.9	02.3	01.9	02.3	02.0	02.2	02.0	3
4	03.2	02.4	03.2	02.5	03.1	02.5	03.1	02.5	03.1	02.6	03.0	02.6	03.0	02.7	4
5	04.0	03.0	03.9	03.1	03.9	03.1	03.9	03.2	03.8	03.2	03.8	03.3	03.7	03.3	5
6	04.8	03.6	04.7	03.7	04.7	03.8	04.6	03.8	04.6	03.9	04.5	03.9	04.5	04.0	6
7	05.6	04.2	05.5	04.3	05.4	04.4	05.4	04.4	05.4	04.5	05.3	04.6	05.2	04.7	7
8	06.4	04.8	06.3	04.9	06.2	05.0	06.2	05.1	06.1	05.1	06.0	05.2	05.9	05.4	8
9	07.2	05.4	07.1	05.5	07.0	05.7	07.0	05.7	06.9	05.8	06.8	05.9	06.7	06.0	9
10	08.0	06.0	07.9	06.2	07.8	06.3	07.7	06.3	07.7	06.4	07.5	06.6	07.4	06.7	10
11	08.8	06.6	08.7	06.8	08.5	06.9	08.5	07.0	08.4	07.1	08.3	07.2	08.2	07.4	11
12	09.6	07.2	09.5	07.4	09.3	07.6	09.3	07.6	09.2	07.7	09.1	07.9	08.9	08.0	12
13	10.4	07.8	10.2	08.0	10.1	08.2	10.0	08.2	10.0	08.4	08.8	08.5	09.7	08.7	13
14	11.2	08.4	11.0	08.6	10.9	08.8	10.8	08.9	10.7	09.0	10.6	09.2	10.4	09.4	14
15	12.0	09.0	11.8	09.2	11.7	09.4	11.6	09.5	11.5	09.6	11.3	09.8	11.1	10.0	15
16	12.8	09.6	12.6	09.9	12.4	10.1	12.4	10.1	12.3	10.3	12.1	10.5	11.9	10.7	16
17	13.6	10.2	13.4	10.5	13.2	10.7	13.1	10.8	13.0	10.9	12.8	11.2	12.6	11.4	17
18	14.4	10.8	14.2	11.1	14.0	11.3	13.9	11.4	13.8	11.6	13.6	11.8	13.4	12.0	18
19	15.2	11.4	15.0	11.7	14.8	12.0	14.7	12.1	14.6	12.2	14.3	12.5	14.1	12.7	19
20	16.0	12.0	15.8	12.3	15.5	12.6	15.5	12.7	15.3	12.9	15.1	13.1	14.9	13.4	20
21	16.8	12.6	16.5	12.9	16.3	13.2	16.2	13.3	16.1	13.5	15.8	13.8	15.6	14.0	21
22	17.6	13.2	17.3	13.5	17.1	13.8	17.0	14.0	16.9	14.1	16.6	14.4	16.3	14.7	22
23	18.4	13.8	18.1	14.2	17.9	14.5	17.8	14.6	17.6	14.8	17.4	15.1	17.1	15.4	23
24	19.2	14.4	18.9	14.8	18.6	15.1	18.6	15.2	18.4	15.4	18.1	15.7	17.8	16.1	24
25	20.0	15.0	19.7	15.4	19.4	15.7	19.3	15.9	19.1	16.1	18.9	16.4	18.6	16.7	25
26	20.8	15.6	20.5	16.0	20.2	16.4	20.1	16.5	19.9	16.7	19.6	17.1	19.3	17.4	26
27	21.6	16.2	21.3	16.6	21.0	17.0	20.9	17.1	20.7	17.4	20.4	17.7	20.1	18.1	27
28	22.4	16.8	22.1	17.2	21.8	17.6	21.6	17.8	21.4	18.0	21.1	18.4	20.8	18.7	28
29	23.2	17.5	22.9	17.9	22.5	18.2	22.4	18.4	22.2	18.6	21.9	19.0	21.5	19.4	29
30	24.0	18.1	23.6	18.5	23.3	18.9	23.2	19.0	23.0	19.3	22.6	19.7	22.3	20.1	30
31	24.8	18.7	24.4	19.1	24.1	19.5	24.0	19.7	23.7	19.9	23.4	20.3	23.0	20.7	31
32	25.6	19.3	25.2	19.7	24.9	20.1	24.7	20.3	24.5	20.6	24.1	21.0	23.8	21.4	32
33	26.4	19.9	26.0	20.3	25.6	20.8	25.5	20.9	25.3	21.2	24.9	21.7	24.5	22.1	33
34	27.2	20.5	26.8	20.9	26.4	21.4	26.3	21.6	26.0	21.9	25.7	22.3	25.3	22.7	34
35	28.0	21.1	27.6	21.5	27.2	22.0	27.1	22.2	26.8	22.5	26.4	23.0	26.0	23.4	35
36	28.7	21.7	28.4	22.2	28.0	22.7	27.8	22.8	27.6	23.1	27.2	23.6	26.8	24.1	36
37	29.5	22.3	29.2	22.8	28.8	23.3	28.6	23.5	28.3	23.8	27.9	24.3	27.5	24.8	37
38	30.3	22.9	29.9	23.4	29.5	23.9	29.4	24.1	29.1	24.4	28.7	24.9	28.2	25.4	38
39	31.1	23.5	30.7	24.0	30.3	24.5	30.1	24.7	29.9	25.1	29.4	25.6	29.0	26.1	39
40	31.9	24.1	31.5	24.6	31.1	25.2	30.9	25.4	30.6	25.7	30.2	26.2	29.7	26.8	40
41	32.7	24.7	32.3	25.2	31.9	25.8	31.7	26.0	31.4	26.4	30.9	26.9	30.5	27.4	41
42	33.5	25.3	33.1	25.9	32.6	26.4	32.5	26.6	32.2	27.0	31.7	27.6	31.2	28.1	42
43	34.3	25.9	33.9	26.5	33.4	27.1	33.2	27.3	32.9	27.6	32.5	28.2	32.0	28.8	43
44	35.1	26.5	34.7	27.1	34.2	27.7	34.0	27.9	33.7	28.3	33.2	28.9	32.7	29.4	44
45	35.9	27.1	35.5	27.7	35.0	28.3	34.8	28.5	34.5	28.9	34.0	29.5	33.4	30.1	45
46	36.7	27.7	36.2	28.3	35.7	28.9	35.6	29.2	35.2	29.6	34.7	30.2	34.2	30.8	46
47	37.5	28.3	37.0	28.9	36.5	29.6	36.3	29.8	36.0	30.2	35.5	30.8	34.9	31.4	47
48	38.3	28.9	37.8	29.6	37.3	30.2	37.1	30.5	36.8	30.9	36.2	31.5	35.7	32.1	48
49	39.1	29.5	38.6	30.2	38.1	30.8	37.9	31.1	37.5	31.5	37.0	32.1	36.4	32.8	49
50	39.9	30.1	39.4	30.8	38.9	31.5	38.6	31.7	38.3	32.1	37.7	32.8	37.2	33.5	50
Dist.	Dep. Lat.		Dep. Lat.		Dep. Lat.		Dep. Lat.		Dep. Lat.		Dep. Lat.		Dep. Lat.		Dist.
	53 Deg.		52 Deg.		51 Deg.		4 ¹ / ₂ Point.		50 Deg.		49 Deg.		48 Deg.		

Diff.	37 Deg.		38 Deg.		39 Deg.		40 Point.		40 Deg.		41 Deg.		42 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	40.7	30.7	40.2	31.4	39.6	32.1	39.4	32.4	39.1	32.8	38.5	33.1	38.1	34.1	51
52	41.5	31.3	41.0	32.0	40.4	32.7	40.2	33.0	39.8	33.4	39.2	34.1	39.2	34.8	52
53	42.3	31.9	41.8	32.6	41.2	33.4	41.0	33.6	40.6	34.1	40.0	34.8	40.4	35.5	53
54	43.1	32.5	42.6	33.2	42.0	34.0	41.7	34.3	41.4	34.7	40.8	35.5	41.2	36.1	54
55	43.9	33.1	43.3	33.9	42.7	34.6	42.5	34.9	42.2	35.4	41.5	36.1	42.0	36.8	55
56	44.7	33.7	44.1	34.5	43.5	35.2	43.3	35.5	42.9	36.0	42.3	36.7	42.8	37.5	56
57	45.5	34.3	44.9	35.1	44.3	35.9	44.1	36.2	43.7	36.6	43.0	37.4	43.4	38.1	57
58	46.3	34.9	45.7	35.7	45.1	36.5	44.8	36.8	44.4	37.3	43.8	38.1	44.1	38.8	58
59	47.1	35.5	46.5	36.3	45.8	37.1	45.6	37.4	45.2	37.9	44.5	38.7	44.8	39.5	59
60	47.9	36.1	47.3	36.9	46.6	37.8	46.4	38.1	46.0	38.6	45.3	39.4	45.6	40.1	60
61	48.7	36.7	48.1	37.6	47.4	38.4	47.2	38.7	46.7	39.2	46.0	40.0	46.3	40.8	61
62	49.5	37.3	48.9	38.2	48.2	39.0	47.9	39.3	47.5	39.9	46.8	40.7	47.1	41.5	62
63	50.3	37.9	49.6	38.8	49.0	39.6	48.7	40.0	48.3	40.5	47.5	41.3	47.8	42.2	63
64	51.1	38.5	50.4	39.4	49.7	40.3	49.5	40.6	49.0	41.1	48.3	42.0	48.6	42.8	64
65	51.9	39.1	51.2	40.0	50.5	40.9	50.2	41.2	49.8	41.8	49.1	42.6	48.3	43.5	65
66	52.7	39.7	52.0	40.6	51.3	41.5	51.0	41.9	50.6	42.4	49.8	43.3	49.0	44.2	66
67	53.5	40.3	52.8	41.3	52.1	42.2	51.8	42.5	51.3	43.1	50.6	44.0	49.8	44.8	67
68	54.3	40.9	53.6	41.9	52.8	42.8	52.6	43.1	52.1	43.7	51.3	44.6	50.5	45.5	68
69	55.1	41.5	54.4	42.5	53.6	43.4	53.3	43.8	52.9	44.4	52.1	45.3	51.3	46.2	69
70	55.9	42.1	55.2	43.1	54.4	44.1	54.1	44.4	53.6	45.0	52.8	45.9	52.0	46.8	70
71	56.7	42.7	55.9	43.7	55.2	44.7	54.9	45.0	54.4	45.6	53.6	46.6	52.8	47.5	71
72	57.5	43.3	56.7	44.3	56.0	45.3	55.7	45.7	55.2	46.3	54.3	47.2	53.5	48.2	72
73	58.3	43.9	57.5	44.9	56.7	45.9	56.4	46.3	55.9	46.9	55.1	47.9	54.2	48.8	73
74	59.1	44.5	58.3	45.6	57.5	46.6	57.2	46.9	56.7	47.6	55.8	48.6	55.0	49.5	74
75	59.9	45.1	59.1	46.2	58.3	47.2	58.0	47.6	57.5	48.2	56.6	49.2	55.7	50.2	75
76	60.7	45.7	59.9	46.8	59.1	47.8	58.7	48.2	58.2	48.9	57.4	49.9	56.5	50.9	76
77	61.5	46.3	60.7	47.4	59.8	48.5	59.5	48.8	59.0	49.5	58.1	50.5	57.2	51.5	77
78	62.3	46.9	61.5	48.0	60.6	49.1	60.3	49.5	59.7	50.1	58.9	51.2	58.0	52.2	78
79	63.1	47.5	62.3	48.6	61.4	49.7	61.1	50.1	60.5	50.8	59.6	51.8	58.7	52.9	79
80	63.9	48.1	63.0	49.3	62.2	50.3	61.8	50.8	61.3	51.4	60.4	52.5	59.4	53.5	80
81	64.7	48.7	63.8	49.9	62.9	51.0	62.6	51.4	62.0	52.1	61.1	53.1	60.2	54.2	81
82	65.5	49.3	64.6	50.5	63.7	51.6	63.4	52.0	62.8	52.7	61.9	53.8	60.9	54.9	82
83	66.3	49.9	65.4	51.1	64.5	52.2	64.2	52.7	63.6	53.4	62.6	54.5	61.7	55.5	83
84	67.1	50.6	66.2	51.7	65.3	52.9	64.9	53.3	64.3	54.0	63.4	55.1	62.4	56.2	84
85	67.9	51.2	67.0	52.3	66.1	53.5	65.7	53.9	65.1	54.6	64.1	55.8	63.2	56.9	85
86	68.7	51.8	67.8	52.9	66.8	54.1	66.5	54.6	65.9	55.3	64.9	56.4	63.9	57.5	86
87	69.5	52.4	68.6	53.6	67.6	54.7	67.3	55.2	66.6	55.9	65.7	57.1	64.6	58.2	87
88	70.3	53.0	69.3	54.2	68.4	55.4	68.0	55.8	67.4	56.6	66.4	57.7	65.4	58.9	88
89	71.1	53.6	70.1	54.8	69.2	56.0	68.8	56.5	68.2	57.2	67.2	58.4	66.1	59.6	89
90	71.9	54.2	70.9	55.4	69.9	56.6	69.6	57.1	68.9	57.9	67.9	59.0	66.9	60.2	90
91	72.7	54.8	71.7	56.0	70.7	57.3	70.3	57.7	69.7	58.5	68.7	59.7	67.6	60.9	91
92	73.5	55.4	72.5	56.6	71.5	57.9	71.1	58.4	70.5	59.1	69.4	60.4	68.4	61.6	92
93	74.3	56.0	73.3	57.3	72.3	58.5	71.9	59.0	71.2	59.8	70.2	61.0	69.1	62.2	93
94	75.1	56.6	74.1	57.9	73.0	59.2	72.7	59.6	72.0	60.4	70.9	61.7	69.9	62.9	94
95	75.9	57.2	74.9	58.5	73.8	59.8	73.4	60.3	72.8	61.1	71.7	62.3	70.6	63.6	95
96	76.7	57.8	75.6	59.1	74.6	60.4	74.2	60.9	73.5	61.7	72.5	63.0	71.3	64.2	96
97	77.5	58.4	76.4	59.7	75.4	61.0	75.0	61.5	74.3	62.4	73.2	63.6	72.1	64.9	97
98	78.3	59.0	77.2	60.3	76.2	61.7	75.8	62.2	75.1	63.0	74.0	64.3	72.8	65.6	98
99	79.1	59.6	78.0	61.0	76.9	62.3	76.5	62.8	75.8	63.6	74.7	65.0	73.6	66.2	99
100	79.9	60.2	78.8	61.6	77.7	62.9	77.3	63.4	76.6	64.3	75.5	65.6	74.3	66.9	100
Diff.	53 Deg.		52 Deg.		51 Deg.		49 Point.		50 Deg.		49 Deg.		48 Deg.		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	

A TABLE of DIFFERENCE

Diff.	3 $\frac{1}{2}$ Point.		43 Deg.		44 Deg.		4 Points		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
1	00.7	00.7	00.7	00.7	00.7	00.7	00.7	00.7	1
2	01.5	01.3	01.5	01.4	01.4	01.4	01.4	01.4	2
3	02.2	02.0	02.2	02.0	02.2	02.1	02.1	02.1	3
4	03.0	02.7	03.0	02.7	02.9	02.8	02.8	02.8	4
5	03.7	03.4	03.7	03.4	03.6	03.5	03.5	03.5	5
6	04.4	04.0	04.4	04.1	04.3	04.2	04.2	04.2	6
7	05.2	04.7	05.1	04.8	05.0	04.9	04.9	04.9	7
8	05.9	05.4	05.9	05.5	05.8	05.6	05.7	05.7	8
9	06.7	06.0	06.6	06.1	06.5	06.3	06.4	06.4	9
10	07.4	06.7	07.3	06.8	07.2	06.9	07.1	07.1	10
11	08.2	07.4	08.0	07.5	07.9	07.6	07.8	07.8	11
12	08.9	08.1	08.8	08.2	08.6	08.3	08.5	08.5	12
13	09.6	08.7	09.5	08.9	09.3	09.0	09.2	09.2	13
14	10.4	09.4	10.2	09.5	10.1	09.7	09.9	09.9	14
15	11.1	10.1	11.0	10.2	10.8	10.4	10.6	10.6	15
16	11.9	10.7	11.7	10.9	11.5	11.1	11.3	11.3	16
17	12.6	11.4	12.4	11.6	12.2	11.8	12.0	12.0	17
18	13.3	12.1	13.2	12.3	12.9	12.5	12.7	12.7	18
19	14.1	12.8	13.9	13.0	13.7	13.2	13.4	13.4	19
20	14.8	13.4	14.6	13.6	14.4	13.9	14.1	14.1	20
21	15.6	14.1	15.4	14.3	15.1	14.6	14.8	14.8	21
22	16.3	14.8	16.1	15.0	15.8	15.3	15.6	15.6	22
23	17.0	15.4	16.8	15.7	16.5	16.0	16.3	16.3	23
24	17.8	16.1	17.6	16.4	17.3	16.7	17.0	17.0	24
25	18.5	16.8	18.3	17.1	18.0	17.4	17.7	17.7	25
26	19.3	17.5	19.0	17.7	18.7	18.1	18.4	18.4	26
27	20.0	18.1	19.7	18.4	19.4	18.8	19.1	19.1	27
28	20.7	18.8	20.5	19.1	20.1	19.5	19.8	19.8	28
29	21.5	19.5	21.2	19.8	20.9	20.1	20.5	20.5	29
30	22.2	20.1	21.9	20.5	21.6	20.8	21.2	21.2	30
31	23.0	20.8	22.7	21.1	22.3	21.5	21.9	21.9	31
32	23.7	21.5	23.4	21.8	23.0	22.2	22.6	22.6	32
33	24.5	22.2	24.1	22.5	23.7	22.9	23.3	23.3	33
34	25.2	22.8	24.9	23.2	24.5	23.6	24.0	24.0	34
35	25.9	23.5	25.6	23.9	25.2	24.3	24.7	24.7	35
36	26.7	24.2	26.3	24.6	25.9	25.0	25.5	25.5	36
37	27.4	24.8	27.0	25.2	26.6	25.7	26.2	26.2	37
38	28.2	25.5	27.8	25.9	27.3	26.4	26.9	26.9	38
39	28.9	26.2	28.5	26.6	28.1	27.1	27.6	27.6	39
40	29.6	26.9	29.3	27.3	28.8	27.8	28.3	28.3	40
41	30.4	27.5	30.0	28.0	29.5	28.5	29.0	29.0	41
42	31.1	28.2	30.7	28.6	30.2	29.2	29.7	29.7	42
43	31.8	28.9	31.4	29.3	30.9	29.9	30.4	30.4	43
44	32.6	29.5	32.2	30.0	31.6	30.6	31.1	31.1	44
45	33.3	30.2	32.9	30.7	32.4	31.3	31.8	31.8	45
46	34.1	30.9	33.6	31.4	33.1	32.0	32.5	32.5	46
47	34.8	31.6	34.4	32.1	33.8	32.6	33.2	33.2	47
48	35.6	32.2	35.1	32.7	34.5	33.3	33.9	33.9	48
49	36.3	32.9	35.8	33.4	35.2	34.0	34.6	34.6	49
50	37.0	33.6	36.6	34.1	36.0	34.7	35.4	35.4	50
Diff.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Diff.
	4 $\frac{1}{2}$ Point.		47 Deg.		46 Deg.		4 Points		

Diff.	3 ^d Point.		4 ^d Deg.		44 Deg.		4 P ints		Diff.
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
51	37.8	34.3	37.3	34.8	36.7	35.4	36.1	36.1	51
52	38.5	34.9	38.0	35.5	37.4	36.1	36.8	36.8	52
53	39.3	35.6	38.8	36.1	38.1	36.8	37.5	37.5	53
54	40.0	36.3	39.5	36.8	38.8	37.5	38.2	38.2	54
55	40.8	36.9	40.2	37.5	39.6	38.2	38.9	38.9	55
56	41.5	37.6	41.0	38.2	40.3	38.9	39.6	39.6	56
57	42.2	38.3	41.7	38.9	41.0	39.6	40.3	40.3	57
58	43.0	39.0	42.4	39.6	41.7	40.3	41.0	41.0	58
59	43.7	39.6	43.2	40.2	42.4	41.0	41.7	41.7	59
60	44.5	40.3	43.9	40.9	43.2	41.7	42.4	42.4	60
61	45.2	41.0	44.6	41.6	43.9	42.4	43.1	43.1	61
62	45.9	41.6	45.3	42.3	44.6	43.1	43.8	43.8	62
63	46.7	42.3	46.1	43.0	45.3	43.8	44.5	44.5	63
64	47.4	43.0	46.8	43.6	46.0	44.5	45.3	45.3	64
65	48.2	43.7	47.5	44.3	46.8	45.2	46.0	46.0	65
66	48.9	44.3	48.3	45.0	47.5	45.8	46.7	46.7	66
67	49.6	45.0	49.0	45.7	48.2	46.5	47.4	47.4	67
68	50.4	45.7	49.7	46.4	48.9	47.2	48.1	48.1	68
69	51.1	46.3	50.5	47.1	49.6	47.9	48.8	48.8	69
70	51.9	47.0	51.2	47.7	50.4	48.6	49.5	49.5	70
71	52.6	47.7	51.9	48.4	51.1	49.3	50.2	50.2	71
72	53.4	48.4	52.7	49.1	51.8	50.0	50.9	50.9	72
73	54.1	49.0	53.4	49.8	52.5	50.7	51.6	51.6	73
74	54.8	49.7	54.1	50.5	53.2	51.4	52.3	52.3	74
75	55.6	50.4	54.9	51.1	53.9	52.1	53.0	53.0	75
76	56.3	51.0	55.6	51.8	54.7	52.8	53.7	53.7	76
77	57.1	51.7	56.3	52.5	55.4	53.5	54.4	54.4	77
78	57.8	52.4	57.0	53.2	56.1	54.2	55.2	55.2	78
79	58.5	53.1	57.8	53.9	56.8	54.9	55.9	55.9	79
80	59.3	53.7	58.5	54.6	57.5	55.6	56.6	56.6	80
81	60.0	54.4	59.2	55.2	58.3	56.3	57.3	57.3	81
82	60.8	55.1	60.0	55.9	59.0	57.0	58.0	58.0	82
83	61.5	55.7	60.7	56.6	59.7	57.7	58.7	58.7	83
84	62.2	56.4	61.4	57.3	60.4	58.4	59.4	59.4	84
85	63.0	57.1	62.2	58.0	61.1	59.0	60.1	60.1	85
86	63.7	57.8	62.9	58.7	61.9	59.7	60.8	60.8	86
87	64.5	58.4	63.6	59.3	62.6	60.4	61.5	61.5	87
88	65.2	59.1	64.4	60.0	63.3	61.1	62.2	62.2	88
89	65.9	59.8	65.1	60.7	64.0	61.8	62.9	62.9	89
90	66.7	60.4	65.8	61.4	64.7	62.5	63.6	63.6	90
91	67.4	61.1	66.6	62.1	65.5	63.2	64.3	64.3	91
92	68.2	61.8	67.3	62.7	66.2	63.9	65.1	65.1	92
93	68.9	62.5	68.0	63.4	66.9	64.6	65.8	65.8	93
94	69.7	63.1	68.8	64.1	67.6	65.3	66.5	66.5	94
95	70.4	63.8	69.5	64.8	68.3	66.0	67.2	67.2	95
96	71.1	64.5	70.2	65.5	69.1	66.7	67.9	67.9	96
97	71.9	65.1	70.9	66.2	69.8	67.4	68.6	68.6	97
98	72.6	65.8	71.7	66.8	70.5	68.1	69.3	69.3	98
99	73.4	66.5	72.4	67.5	71.2	68.8	70.0	70.0	99
100	74.1	67.2	73.1	68.2	71.9	69.5	70.7	70.7	100
Diff.	4 th Point.		47 Deg.		46 Deg.		4 Points		Diff.
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	



A TABLE of MERIDIONAL PARTS.

L.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	L.
Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.
0	0	60.0	120.0	180.1	240.3	300.4	360.7	421.1	481.6	542.1	603.1	664.1	725.3	786.8	848.5	910.5	0
1	1.0	61.0	121.0	181.1	241.2	301.4	361.7	422.1	482.6	543.1	604.1	665.1	726.4	787.9	849.5	911.5	1
2	2.0	62.0	122.0	182.1	242.2	302.4	362.7	423.1	483.6	544.1	605.1	666.1	727.4	788.9	850.5	912.6	2
3	3.0	63.0	123.0	183.1	243.2	303.4	363.7	424.1	484.6	545.1	606.1	667.1	728.4	789.9	851.6	913.6	3
4	4.0	64.0	124.0	184.1	244.2	304.4	364.7	425.1	485.6	546.1	607.1	668.1	729.4	790.9	852.6	914.6	4
5	5.0	65.0	125.0	185.1	245.2	305.4	365.7	426.1	486.6	547.1	608.1	669.1	730.5	792.0	853.7	915.7	5
6	6.0	66.0	126.0	186.1	246.2	306.4	366.7	427.1	487.6	548.1	609.1	670.1	731.5	793.0	854.7	916.7	6
7	7.0	67.0	127.0	187.1	247.2	307.4	367.7	428.1	488.6	549.1	610.1	671.1	732.5	794.0	855.7	917.7	7
8	8.0	68.0	128.0	188.1	248.2	308.4	368.7	429.1	489.6	550.1	611.1	672.1	733.5	795.0	856.8	918.8	8
9	9.0	69.0	129.0	189.1	249.2	309.4	369.7	430.1	490.7	551.1	612.1	673.1	734.6	796.1	857.8	919.8	9
10	10.0	70.0	130.0	190.1	250.2	310.4	370.7	431.1	491.7	552.1	613.1	674.1	735.6	797.1	858.8	920.8	10
11	11.0	71.0	131.0	191.1	251.2	311.4	371.7	432.1	492.7	553.1	614.1	675.1	736.6	798.1	859.8	921.8	11
12	12.0	72.0	132.0	192.1	252.2	312.4	372.7	433.1	493.7	554.1	615.1	676.1	737.6	799.1	860.8	922.8	12
13	13.0	73.0	133.0	193.1	253.2	313.4	373.7	434.1	494.7	555.1	616.1	677.1	738.7	800.2	861.8	923.9	13
14	14.0	74.0	134.0	194.1	254.2	314.4	374.7	435.1	495.7	556.1	617.1	678.1	739.7	801.2	862.8	925.0	14
15	15.0	75.0	135.0	195.1	255.2	315.4	375.8	436.1	496.7	557.1	618.1	679.1	740.7	802.2	863.1	926.0	15
16	16.0	76.0	136.0	196.1	256.2	316.5	376.8	437.1	497.7	558.1	619.1	680.1	741.7	803.2	864.1	927.0	16
17	17.0	77.0	137.0	197.1	257.2	317.5	377.8	438.1	498.7	559.1	620.1	681.1	742.8	804.3	865.1	928.1	17
18	18.0	78.0	138.0	198.1	258.2	318.5	378.8	439.1	499.8	560.1	621.1	682.1	743.8	805.3	866.2	929.1	18
19	19.0	79.0	139.0	199.1	259.3	319.5	379.8	440.1	500.8	561.1	622.1	683.1	744.8	806.3	867.2	930.1	19
20	20.0	80.0	140.0	200.1	260.3	320.5	380.8	441.1	501.8	562.1	623.1	684.1	745.8	807.3	868.2	931.2	20
21	21.0	81.0	141.0	201.1	261.3	321.5	381.8	442.1	502.8	563.1	624.1	685.1	746.9	808.4	869.3	932.2	21
22	22.0	82.0	142.0	202.1	262.3	322.5	382.8	443.1	503.8	564.1	625.1	686.1	747.9	809.4	870.3	933.2	22
23	23.0	83.0	143.0	203.1	263.3	323.5	383.8	444.1	504.8	565.1	626.1	687.1	748.9	810.4	871.3	934.3	23
24	24.0	84.0	144.0	204.1	264.3	324.5	384.8	445.1	505.8	566.1	627.1	688.1	749.9	811.4	872.3	935.3	24
25	25.0	85.0	145.0	205.1	265.3	325.5	385.8	446.1	506.8	567.1	628.1	689.1	751.0	812.5	873.4	936.3	25
26	26.0	86.0	146.0	206.1	266.3	326.5	386.8	447.1	507.8	568.1	629.1	690.1	752.0	813.5	874.4	937.3	26
27	27.0	87.0	147.0	207.1	267.3	327.5	387.8	448.1	508.9	569.1	630.1	691.1	753.0	814.5	875.5	938.4	27
28	28.0	88.0	148.1	208.1	268.3	328.5	388.8	449.1	509.9	570.1	631.1	692.1	754.0	815.5	876.5	939.4	28
29	29.0	89.0	149.1	209.1	269.3	329.5	389.8	450.1	510.9	571.1	632.1	693.1	755.1	816.6	877.6	940.5	29
30	30.0	90.0	150.1	210.1	270.3	330.5	390.8	451.1	512.0	572.1	633.1	694.1	756.1	817.6	878.6	941.5	30
31	31.0	91.0	151.1	211.1	271.3	331.5	391.8	452.1	513.0	573.1	634.1	695.1	757.1	818.6	879.6	942.5	31
32	32.0	92.0	152.1	212.1	272.3	332.5	392.8	453.1	514.0	574.1	635.1	696.1	758.1	819.6	880.6	943.5	32
33	33.0	93.0	153.1	213.1	273.3	333.5	393.8	454.1	515.0	575.1	636.1	697.1	759.1	820.7	881.7	944.6	33
34	34.0	94.0	154.1	214.1	274.3	334.5	394.8	455.1	516.0	576.1	637.1	698.1	760.2	821.7	882.7	945.6	34
35	35.0	95.0	155.1	215.1	275.3	335.5	395.8	456.1	517.0	577.1	638.1	699.1	761.2	822.7	883.7	946.7	35
36	36.0	96.0	156.1	216.1	276.3	336.5	396.8	457.1	518.0	578.1	639.1	700.8	762.2	823.7	884.8	947.7	36
37	37.0	97.0	157.1	217.1	277.3	337.5	397.8	458.1	519.0	579.1	640.1	701.8	763.3	824.8	885.8	948.7	37
38	38.0	98.0	158.1	218.2	278.3	338.5	398.8	459.1	520.0	580.1	641.1	702.3	764.3	825.8	886.8	949.8	38
39	39.0	99.0	159.1	219.2	279.3	339.6	399.9	460.1	521.0	581.1	642.1	703.3	765.3	826.8	887.9	950.8	39
40	40.0	100.0	160.1	220.2	280.3	340.6	400.9	461.1	522.0	582.1	643.1	704.9	766.3	827.9	888.9	951.9	40
41	41.0	101.0	161.1	221.2	281.3	341.6	401.9	462.1	523.0	583.1	644.1	705.9	767.4	828.9	889.9	952.9	41
42	42.0	102.0	162.1	222.2	282.3	342.6	402.9	463.1	524.0	584.1	645.1	706.9	768.4	829.9	890.9	953.9	42
43	43.0	103.0	163.1	223.2	283.3	343.6	403.9	464.1	525.0	585.1	646.1	707.9	769.4	831.0	891.9	955.0	43
44	44.0	104.0	164.1	224.2	284.3	344.6	404.9	465.1	526.0	586.1	647.1	708.9	770.4	832.0	892.9	956.0	44
45	45.0	105.0	165.1	225.2	285.3	345.6	405.9	466.1	527.1	587.1	648.1	710.0	771.5	833.0	893.1	957.1	45
46	46.0	106.0	166.1	226.2	286.3	346.6	406.9	467.1	528.1	588.1	649.1	711.0	772.5	834.1	894.1	958.1	46
47	47.0	107.0	167.1	227.2	287.3	347.6	407.9	468.1	529.1	589.1	650.1	712.0	773.5	835.1	895.1	959.2	47
48	48.0	108.0	168.1	228.2	288.3	348.6	408.9	469.1	530.1	590.1	651.1	713.0	774.5	836.1	896.1	960.2	48
49	49.0	109.0	169.1	229.2	289.3	349.6	410.0	470.1	531.1	591.1	652.1	714.1	775.6	837.2	897.2	961.3	49
50	50.0	110.0	170.1	230.3	290.3	350.6	411.0	471.1	532.1	592.1	653.1	715.1	776.6	838.2	898.2	962.3	50
51	51.0	111.0	171.1	231.2	291.3	351.6	412.0	472.1	533.1	593.1	654.1	716.1	777.6	839.2	899.2	963.4	51
52	52.0	112.0	172.1	232.2	292.4	352.6	413.0	473.1	534.1	594.1	655.1	717.1	778.6	840.3	900.3	964.4	52
53	53.0	113.0	173.1	233.2	293.4	353.6	414.0	474.1	535.1	595.1	656.1	718.1	779.7	841.3	901.3	965.5	53
54	54.0	114.0	174.1	234.2	294.4	354.6	415.0	475.1	536.1	596.1	657.1	719.1	780.7	842.3	902.3	966.5	54
55	55.0	115.0	175.1	235.2	295.4	355.6	416.0	476.1	537.1	597.1	658.1	720.0	781.7	843.4	903.4	967.6	55
56	56.0	116.0	176.1	236.2	296.4	356.6	417.0	477.1	538.1	598.1	659.1	721.0	782.7	844.4	904.4	968.6	56
57	57.0	117.0	177.1	237.2	297.4	357.6	418.0	478.1	539.1	599.1	660.1	722.1	783.7	845.4	905.4	969.6	57
58	58.0	118.0	178.1	238.2	298.4	358.6	419.0	479.1	540.1	600.1	661.1	723.1	784.7	846.5	906.4	970.7	58
59	59.0	119.0	179.1	239.2	299.4	359.6	420.0	480.1	541.1	601.1	662.1	724.1	785.7	847.5	907.5	971.7	59
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.
L.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	L.

A Table of Meridional Parts.

L.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	L.
M.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M.
0	972.8	1035.3	1098.2	1161.5	1225.1	1289.2	1353.7	1418.7	1484.1	1550.0	1616.5	1683.6	1751.2	1819.5	0
1	973.8	1036.9	1099.3	1163.2	1227.2	1291.6	1356.8	1422.7	1489.1	1556.1	1623.7	1691.8	1760.3	1829.1	1
2	974.8	1037.4	1100.0	1164.6	1229.3	1294.3	1360.2	1427.3	1495.7	1563.7	1632.3	1701.4	1770.9	1840.7	2
3	975.9	1038.4	1101.4	1166.7	1231.5	1296.5	1363.0	1431.9	1501.3	1570.3	1640.0	1710.2	1780.8	1851.7	3
4	976.9	1039.5	1102.4	1168.7	1233.9	1299.4	1365.5	1435.0	1504.9	1574.8	1645.5	1716.7	1788.2	1860.0	4
5	978.0	1040.5	1103.5	1169.8	1236.4	1302.5	1370.5	1440.5	1510.5	1580.5	1652.0	1724.0	1796.5	1869.5	5
6	979.0	1041.6	1104.5	1170.8	1239.1	1305.6	1374.2	1445.1	1515.1	1585.1	1657.5	1730.0	1803.5	1877.5	6
7	980.0	1042.6	1105.6	1172.8	1241.6	1308.7	1377.8	1449.7	1519.7	1589.7	1662.0	1735.5	1809.0	1882.5	7
8	981.1	1043.7	1106.6	1174.8	1244.1	1311.8	1381.3	1453.2	1523.2	1593.2	1666.5	1740.0	1814.5	1889.5	8
9	982.1	1044.7	1107.7	1176.8	1246.7	1314.9	1384.3	1456.7	1526.7	1596.7	1670.0	1744.5	1819.0	1894.5	9
10	983.2	1045.8	1108.7	1178.7	1249.3	1318.0	1387.3	1460.2	1530.2	1600.2	1674.0	1749.0	1824.0	1900.0	10
11	984.2	1046.8	1109.8	1180.7	1251.8	1321.0	1390.3	1463.7	1533.7	1603.7	1678.5	1753.5	1828.5	1905.5	11
12	985.2	1047.9	1110.8	1182.7	1254.9	1324.0	1393.4	1467.2	1536.7	1606.7	1681.5	1757.5	1833.5	1911.5	12
13	986.3	1048.9	1111.9	1184.7	1257.9	1327.0	1396.5	1470.7	1540.2	1610.2	1685.5	1761.5	1837.5	1917.5	13
14	987.3	1049.9	1112.9	1186.7	1260.9	1330.0	1399.6	1474.2	1543.7	1613.7	1689.0	1766.5	1843.5	1923.5	14
15	988.4	1051.0	1114.0	1188.7	1263.9	1333.0	1402.6	1477.7	1547.2	1616.7	1692.5	1772.5	1849.5	1929.5	15
16	989.4	1052.0	1115.0	1190.8	1266.9	1336.0	1405.6	1480.7	1550.2	1620.2	1696.5	1777.5	1854.5	1935.5	16
17	990.4	1053.1	1116.1	1192.8	1269.9	1339.0	1408.6	1483.7	1553.7	1623.7	1700.0	1782.5	1861.5	1941.5	17
18	991.5	1054.1	1117.1	1194.8	1272.9	1342.0	1411.6	1486.7	1556.7	1626.7	1703.5	1786.5	1866.5	1947.5	18
19	992.5	1055.2	1118.2	1196.8	1275.9	1345.0	1414.6	1489.7	1559.7	1629.7	1706.5	1789.5	1871.5	1953.5	19
20	993.6	1056.2	1119.2	1198.7	1278.9	1348.0	1417.6	1492.7	1562.7	1632.7	1709.5	1792.5	1877.5	1959.5	20
21	994.6	1057.3	1120.3	1200.7	1281.9	1351.0	1420.6	1495.7	1565.7	1635.7	1712.5	1795.5	1883.5	1965.5	21
22	995.6	1058.3	1121.3	1202.7	1284.9	1354.0	1423.6	1498.7	1568.7	1638.7	1715.5	1798.5	1887.5	1971.5	22
23	996.7	1059.3	1122.4	1204.7	1287.9	1357.0	1426.6	1501.7	1571.7	1641.7	1718.5	1801.5	1891.5	1977.5	23
24	997.7	1060.4	1123.4	1206.7	1290.9	1360.0	1429.6	1504.7	1574.7	1644.7	1721.5	1804.5	1894.5	1983.5	24
25	998.8	1061.4	1124.5	1208.7	1293.9	1363.0	1432.6	1507.7	1577.7	1647.7	1724.5	1807.5	1897.5	1989.5	25
26	999.8	1062.5	1125.5	1210.7	1296.9	1366.0	1435.6	1510.7	1580.7	1650.7	1727.5	1810.5	1900.5	1995.5	26
27	1000.8	1063.5	1126.6	1212.7	1299.9	1369.0	1438.6	1513.7	1583.7	1653.7	1730.5	1813.5	1903.5	2001.5	27
28	1001.9	1064.6	1127.6	1214.7	1302.9	1372.0	1441.6	1516.7	1586.7	1656.7	1733.5	1816.5	1906.5	2007.5	28
29	1002.9	1065.6	1128.7	1216.7	1305.9	1375.0	1444.6	1519.7	1589.7	1659.7	1736.5	1819.5	1909.5	2013.5	29
30	1004.0	1066.7	1129.7	1218.7	1308.9	1378.0	1447.6	1522.7	1592.7	1662.7	1739.5	1822.5	1912.5	2019.5	30
31	1005.0	1067.7	1130.8	1220.7	1311.9	1381.0	1450.6	1525.7	1595.7	1665.7	1742.5	1825.5	1915.5	2025.5	31
32	1006.1	1068.8	1131.8	1222.7	1314.9	1384.0	1453.6	1528.7	1598.7	1668.7	1745.5	1828.5	1918.5	2031.5	32
33	1007.1	1069.8	1132.9	1224.7	1317.9	1387.0	1456.6	1531.7	1601.7	1671.7	1748.5	1831.5	1921.5	2037.5	33
34	1008.1	1070.9	1134.0	1226.7	1320.9	1390.0	1459.6	1534.7	1604.7	1674.7	1751.5	1834.5	1924.5	2043.5	34
35	1009.2	1072.0	1135.1	1228.7	1323.9	1393.0	1462.6	1537.7	1607.7	1677.7	1754.5	1837.5	1927.5	2049.5	35
36	1010.2	1073.0	1136.1	1230.7	1326.9	1396.0	1465.6	1540.7	1610.7	1680.7	1757.5	1840.5	1930.5	2055.5	36
37	1011.3	1074.1	1137.2	1232.7	1329.9	1399.0	1468.6	1543.7	1613.7	1683.7	1760.5	1843.5	1933.5	2061.5	37
38	1012.3	1075.1	1138.2	1234.7	1332.9	1402.0	1471.6	1546.7	1616.7	1686.7	1763.5	1846.5	1936.5	2067.5	38
39	1013.4	1076.2	1139.3	1236.7	1335.9	1405.0	1474.6	1549.7	1619.7	1689.7	1766.5	1849.5	1939.5	2073.5	39
40	1014.4	1077.2	1140.3	1238.7	1338.9	1408.0	1477.6	1552.7	1622.7	1692.7	1769.5	1852.5	1942.5	2079.5	40
41	1015.4	1078.3	1141.4	1240.7	1341.9	1411.0	1480.6	1555.7	1625.7	1695.7	1772.5	1855.5	1945.5	2085.5	41
42	1016.5	1079.3	1142.4	1242.7	1344.9	1414.0	1483.6	1558.7	1628.7	1698.7	1775.5	1858.5	1948.5	2091.5	42
43	1017.5	1080.4	1143.5	1244.7	1347.9	1417.0	1486.6	1561.7	1631.7	1701.7	1778.5	1861.5	1951.5	2097.5	43
44	1018.6	1081.4	1144.6	1246.7	1350.9	1420.0	1489.6	1564.7	1634.7	1704.7	1781.5	1864.5	1954.5	2103.5	44
45	1019.6	1082.5	1145.6	1248.7	1353.9	1423.0	1492.6	1567.7	1637.7	1707.7	1784.5	1867.5	1957.5	2109.5	45
46	1020.6	1083.5	1146.7	1250.7	1356.9	1426.0	1495.6	1570.7	1640.7	1710.7	1787.5	1870.5	1960.5	2115.5	46
47	1021.7	1084.6	1147.7	1252.7	1359.9	1429.0	1498.6	1573.7	1643.7	1713.7	1790.5	1873.5	1963.5	2121.5	47
48	1022.7	1085.6	1148.8	1254.7	1362.9	1432.0	1501.6	1576.7	1646.7	1716.7	1793.5	1876.5	1966.5	2127.5	48
49	1023.8	1086.7	1149.8	1256.7	1365.9	1435.0	1504.6	1579.7	1649.7	1719.7	1796.5	1879.5	1969.5	2133.5	49
50	1024.8	1087.7	1150.9	1258.7	1368.9	1438.0	1507.6	1582.7	1652.7	1722.7	1799.5	1882.5	1972.5	2139.5	50
51	1025.9	1088.8	1151.9	1260.7	1371.9	1441.0	1510.6	1585.7	1655.7	1725.7	1802.5	1885.5	1975.5	2145.5	51
52	1026.9	1089.8	1153.0	1262.7	1374.9	1444.0	1513.6	1588.7	1658.7	1728.7	1805.5	1888.5	1978.5	2151.5	52
53	1028.0	1090.9	1154.1	1264.7	1377.9	1447.0	1516.6	1591.7	1661.7	1731.7	1808.5	1891.5	1981.5	2157.5	53
54	1029.0	1091.9	1155.1	1266.7	1380.9	1450.0	1519.6	1594.7	1664.7	1734.7	1811.5	1894.5	1984.5	2163.5	54
55	1030.1	1093.0	1156.2	1268.7	1383.9	1453.0	1522.6	1597.7	1667.7	1737.7	1814.5	1897.5	1987.5	2169.5	55
56	1031.1	1094.0	1157.2	1270.7	1386.9	1456.0	1525.6	1600.7	1670.7	1740.7	1817.5	1900.5	1990.5	2175.5	56
57	1032.2	1095.1	1158.3	1272.7	1389.9	1459.0	1528.6	1603.7	1673.7	1743.7	1820.5	1903.5	1993.5	2181.5	57
58	1033.2	1096.1	1159.3	1274.7	1392.9	1462.0	1531.6	1606.7	1676.7	1746.7	1823.5	1906.5	1996.5	2187.5	58
59	1034.3	1097.2	1160.4	1276.7	1395.9	1465.0	1534.6	1609.7	1679.7	1749.7	1826.5	1909.5	1999.5	2193.5	59
60	1035.3	1098.2	1161.4	1278.7	1398.9	1468.0	1537.6	1612.7	1682.7	1752.7	1829.5	1912.5	2002.5	2200.0	60

A Table of Meridional Parts.

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L.	30	31	32	33	34	35	36	37	38	39	40	41	42	L.
M.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Nin.	Min.	Min.	Min.	Min.	Min.	M.
0	1888.4	1958.1	2028.4	2099.6	2171.5	2244.3	2318.0	2392.7	2468.3	2545.0	2622.7	2701.6	2781.7	0
1	1889.5	1959.2	2029.6	2100.7	2172.7	2245.5	2319.3	2394.0	2469.6	2546.2	2624.0	2702.9	2783.1	1
2	1890.7	1960.4	2030.7	2101.9	2173.9	2246.8	2320.5	2395.2	2470.8	2547.5	2625.3	2704.1	2784.4	2
3	1891.9	1961.6	2031.9	2103.1	2175.1	2248.0	2321.7	2396.4	2472.1	2548.8	2626.6	2705.6	2785.6	3
4	1893.0	1962.7	2033.1	2104.3	2176.3	2249.2	2323.0	2397.7	2473.4	2550.1	2627.9	2706.9	2787.1	4
5	1894.2	1963.9	2034.3	2105.5	2177.5	2250.4	2324.2	2398.9	2474.6	2551.4	2629.2	2708.3	2788.4	5
6	1895.3	1965.0	2035.5	2106.7	2178.7	2251.6	2325.4	2400.2	2475.9	2552.7	2630.5	2709.6	2789.4	6
7	1896.5	1966.2	2036.7	2107.9	2180.0	2252.9	2326.7	2401.4	2477.1	2554.0	2631.9	2710.9	2791.2	7
8	1897.8	1967.4	2037.8	2109.2	2181.2	2254.1	2327.9	2402.7	2478.5	2555.3	2633.2	2712.2	2792.5	8
9	1898.8	1968.5	2039.0	2110.3	2182.4	2255.3	2329.2	2403.9	2479.7	2556.6	2634.5	2713.6	2793.3	9
10	1899.9	1969.7	2040.2	2111.5	2183.6	2256.5	2330.4	2404.2	2481.0	2557.8	2635.8	2714.9	2795.1	10
11	1901.1	1970.9	2041.4	2112.7	2184.8	2257.8	2331.6	2405.4	2482.3	2559.1	2637.1	2716.2	2796.5	11
12	1902.3	1972.0	2042.6	2113.9	2186.0	2259.0	2332.9	2407.7	2483.5	2560.4	2638.4	2717.5	2797.9	12
13	1903.4	1973.2	2043.8	2115.1	2187.2	2260.2	2334.1	2409.0	2484.8	2561.7	2639.7	2718.9	2799.3	13
14	1904.6	1974.4	2044.9	2116.3	2188.4	2261.4	2335.3	2410.2	2486.1	2563.0	2641.0	2720.2	2800.6	14
15	1905.7	1975.6	2046.1	2117.5	2189.6	2262.7	2336.6	2411.5	2487.4	2564.3	2642.3	2721.5	2802.0	15
16	1906.9	1976.8	2047.3	2118.7	2190.8	2263.9	2337.8	2412.7	2488.6	2565.6	2643.6	2722.9	2803.3	16
17	1908.1	1977.9	2048.5	2119.8	2192.0	2265.1	2339.0	2414.0	2489.9	2566.9	2644.9	2724.2	2804.7	17
18	1909.2	1979.1	2049.7	2121.0	2193.3	2266.3	2340.3	2415.2	2491.2	2568.2	2646.3	2725.5	2806.0	18
19	1910.4	1980.3	2050.8	2122.2	2194.5	2267.6	2341.5	2416.5	2492.5	2569.5	2647.6	2726.9	2807.4	19
20	1911.5	1981.4	2052.0	2123.4	2195.7	2268.8	2342.8	2417.8	2493.7	2570.7	2648.9	2728.2	2808.7	20
21	1912.7	1982.6	2053.2	2124.6	2196.9	2270.0	2344.0	2419.0	2495.0	2572.0	2650.2	2729.5	2810.1	21
22	1913.8	1983.7	2054.4	2125.8	2198.1	2271.2	2345.3	2420.3	2496.3	2573.3	2651.5	2730.8	2811.4	22
23	1915.0	1984.9	2055.6	2127.0	2199.3	2272.5	2346.5	2421.5	2497.6	2574.6	2652.8	2732.2	2812.8	23
24	1916.2	1986.1	2056.8	2128.2	2200.5	2273.7	2347.8	2422.8	2498.8	2575.9	2654.1	2733.5	2814.1	24
25	1917.3	1987.3	2058.0	2129.4	2201.7	2274.9	2349.0	2424.0	2500.1	2577.2	2655.4	2734.8	2815.5	25
26	1918.5	1988.4	2059.2	2130.6	2203.0	2276.1	2350.2	2425.3	2501.4	2578.5	2656.8	2736.2	2816.8	26
27	1919.6	1989.6	2060.3	2131.8	2204.2	2277.4	2351.5	2426.5	2502.7	2579.8	2658.1	2737.5	2818.2	27
28	1920.8	1990.8	2061.5	2133.0	2205.4	2278.6	2352.7	2427.8	2503.9	2581.1	2659.4	2738.8	2819.5	28
29	1921.9	1991.0	2062.7	2134.2	2206.6	2279.8	2354.0	2429.1	2505.2	2582.4	2660.7	2740.2	2820.9	29
30	1923.1	1993.1	2063.9	2135.4	2207.8	2281.0	2355.2	2430.3	2506.5	2583.7	2662.0	2741.5	2822.3	30
31	1924.3	1994.3	2065.1	2136.6	2209.0	2282.3	2356.5	2431.6	2507.8	2585.0	2663.3	2742.9	2823.6	31
32	1925.4	1995.5	2066.2	2137.8	2210.2	2283.5	2357.7	2432.9	2509.0	2586.3	2664.6	2744.2	2825.0	32
33	1926.6	1996.6	2067.4	2139.0	2211.4	2284.7	2358.9	2434.1	2510.3	2587.6	2666.0	2745.5	2826.3	33
34	1927.8	1997.8	2068.6	2140.2	2212.7	2286.0	2360.2	2435.4	2511.6	2588.9	2667.3	2746.9	2827.7	34
35	1928.9	1999.0	2069.8	2141.4	2213.9	2287.2	2361.4	2436.7	2512.9	2590.2	2668.6	2748.2	2829.0	35
36	1930.1	2000.2	2071.0	2142.6	2215.1	2288.4	2362.7	2437.9	2514.2	2591.5	2669.9	2749.5	2830.4	36
37	1931.3	2001.3	2072.2	2143.8	2216.3	2289.7	2363.9	2439.2	2515.4	2592.8	2671.2	2750.9	2831.8	37
38	1932.4	2002.5	2073.4	2145.0	2217.5	2290.9	2365.2	2440.4	2516.7	2594.1	2672.5	2752.2	2833.1	38
39	1933.6	2003.7	2074.6	2146.2	2218.7	2292.1	2366.4	2441.7	2518.0	2595.4	2673.9	2753.5	2834.5	39
40	1934.7	2004.9	2075.7	2147.4	2219.9	2293.3	2367.7	2443.0	2519.3	2596.7	2675.1	2754.8	2835.8	40
41	1935.9	2006.0	2076.9	2148.6	2221.2	2294.6	2368.9	2444.2	2520.6	2598.0	2676.5	2756.2	2837.2	41
42	1937.1	2007.2	2078.1	2149.8	2222.4	2295.8	2370.2	2445.5	2521.8	2599.3	2677.8	2757.6	2838.6	42
43	1938.2	2008.4	2079.3	2151.0	2223.6	2297.0	2371.4	2446.8	2523.1	2600.6	2679.1	2759.0	2839.9	43
44	1939.4	2009.6	2080.5	2152.2	2224.8	2298.3	2372.7	2448.0	2524.4	2601.9	2680.5	2760.2	2841.3	44
45	1940.5	2010.7	2081.7	2153.4	2226.0	2299.5	2373.9	2449.3	2525.7	2603.2	2681.8	2761.5	2842.6	45
46	1941.7	2011.9	2082.9	2154.6	2227.2	2300.7	2375.2	2450.6	2527.0	2604.5	2683.1	2762.9	2844.0	46
47	1942.8	2013.1	2084.1	2155.8	2228.5	2302.0	2376.4	2451.8	2528.3	2605.8	2684.4	2764.1	2845.4	47
48	1944.0	2014.3	2085.3	2157.0	2229.7	2303.2	2377.7	2453.1	2529.6	2607.1	2685.7	2765.6	2846.7	48
49	1945.2	2015.4	2086.5	2158.2	2230.9	2304.4	2378.9	2454.3	2530.8	2608.4	2687.1	2766.9	2848.1	49
50	1946.4	2016.6	2087.7	2159.4	2232.1	2305.7	2380.1	2455.6	2532.1	2609.7	2688.4	2768.1	2849.5	50
51	1947.5	2017.8	2088.9	2160.7	2233.3	2306.9	2381.4	2456.9	2533.4	2611.0	2690.7	2769.6	2850.8	51
52	1948.7	2019.0	2090.1	2161.9	2234.6	2308.1	2382.6	2458.1	2534.7	2612.3	2692.0	2771.0	2852.2	52
53	1949.9	2020.2	2091.3	2163.1	2235.8	2309.4	2383.9	2459.4	2536.0	2613.6	2693.3	2772.3	2853.6	53
54	1951.0	2021.3	2092.5	2164.3	2237.0	2310.6	2385.1	2460.7	2537.2	2614.9	2694.7	2773.7	2854.9	54
55	1952.2	2022.5	2093.7	2165.5	2238.2	2311.8	2386.4	2462.0	2538.5	2616.2	2696.0	2775.0	2856.3	55
56	1953.4	2023.7	2094.9	2166.7	2239.4	2313.1	2387.6	2463.3	2539.8	2617.5	2697.3	2776.4	2857.7	56
57	1954.5	2024.9	2096.1	2167.9	2240.7	2314.3	2388.9	2464.6	2541.1	2618.8	2698.6	2777.7	2859.1	57
58	1955.7	2026.0	2097.3	2169.1	2241.9	2315.5	2390.2	2465.8	2542.4	2620.1	2699.9	2779.0	2860.5	58
59	1956.9	2027.2	2098.5	2170.3	2243.1	2316.7	2391.4	2467.0	2543.7	2621.4	2700.3	2780.4	2861.8	59
L.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	L.
30	31	32	33	34	35	36	37	38	39	40	41	42		

A Table of Meridional Parts.

L.	43	44	45	46	47	48	49	50	51	52	53	54	55	L.
M.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M.
0	2863.1	2945.7	3030.0	3115.6	3202.8	3291.6	3382.1	3474.5	3568.8	3665.2	3763.8	3864.7	3968.0	0
1	2864.5	2947.2	3031.4	3117.0	3204.2	3293.1	3383.6	3476.1	3570.4	3666.9	3765.5	3866.4	3969.7	1
2	2865.8	2948.6	3032.8	3118.5	3205.7	3294.6	3385.2	3477.6	3572.0	3668.5	3767.1	3868.1	3971.5	2
3	2867.2	2950.0	3034.2	3119.9	3207.2	3296.1	3386.7	3479.2	3573.6	3670.1	3768.8	3869.8	3973.2	3
4	2868.5	2951.4	3035.6	3121.4	3208.6	3297.5	3388.2	3480.7	3575.2	3671.7	3770.4	3871.5	3975.0	4
5	2870.0	2952.8	3037.0	3122.8	3210.1	3299.0	3389.7	3482.3	3576.8	3673.4	3772.1	3873.2	3976.7	5
6	2871.3	2954.2	3038.4	3124.2	3211.6	3300.5	3391.3	3483.9	3578.4	3675.0	3773.8	3874.9	3978.5	6
7	2872.7	2955.6	3039.8	3125.7	3213.0	3302.0	3392.8	3485.4	3580.0	3676.6	3775.4	3876.6	3980.2	7
8	2874.1	2957.0	3041.3	3127.1	3214.5	3303.5	3394.3	3487.0	3581.6	3678.2	3777.1	3878.3	3981.8	8
9	2875.4	2958.4	3042.7	3128.6	3216.0	3305.0	3395.9	3488.5	3583.2	3679.9	3778.8	3880.0	3983.7	9
10	2876.8	2959.8	3044.1	3130.0	3217.4	3306.5	3397.4	3490.1	3584.8	3681.5	3780.4	3882.7	3985.5	10
11	2878.2	2961.1	3045.5	3131.5	3218.9	3308.0	3398.9	3491.7	3586.4	3683.1	3782.1	3884.3	3987.2	11
12	2879.5	2962.5	3046.9	3132.9	3220.4	3309.5	3400.4	3493.2	3588.0	3684.8	3783.8	3885.1	3988.9	12
13	2880.9	2963.9	3048.4	3134.3	3221.9	3311.0	3402.0	3494.8	3589.5	3686.4	3785.5	3886.8	3990.7	13
14	2882.3	2965.3	3049.8	3135.8	3223.3	3312.5	3403.5	3496.3	3591.1	3688.0	3787.1	3888.6	3992.5	14
15	2883.7	2966.7	3051.2	3137.2	3224.8	3314.0	3405.0	3497.9	3592.7	3689.7	3788.8	3890.3	3994.2	15
16	2885.0	2968.1	3052.6	3138.7	3226.3	3315.5	3406.6	3499.5	3594.3	3691.3	3790.5	3892.0	3996.0	16
17	2886.4	2969.5	3054.1	3140.1	3227.7	3317.0	3408.1	3501.0	3595.9	3692.9	3792.1	3893.7	3997.7	17
18	2887.8	2970.9	3055.5	3141.6	3229.2	3318.5	3409.6	3502.6	3597.5	3694.6	3793.8	3895.8	3999.5	18
19	2889.2	2972.3	3056.9	3143.0	3230.7	3320.0	3411.2	3504.2	3599.1	3696.2	3795.5	3897.1	4001.3	19
20	2890.5	2973.7	3058.3	3144.5	3232.2	3321.5	3412.7	3505.7	3600.7	3697.8	3797.2	3898.8	4003.0	20
21	2891.9	2975.1	3059.7	3145.9	3233.6	3323.1	3414.2	3507.3	3602.3	3699.5	3798.7	3900.5	4004.8	21
22	2893.3	2976.5	3061.2	3147.4	3235.1	3324.6	3415.8	3508.9	3603.9	3701.1	3800.5	3902.3	4006.5	22
23	2894.7	2977.9	3062.6	3148.8	3236.6	3326.1	3417.3	3510.5	3605.5	3702.7	3802.2	3904.0	4008.3	23
24	2896.0	2979.3	3064.0	3150.3	3238.1	3327.6	3418.8	3512.0	3607.1	3704.4	3803.9	3905.7	4010.0	24
25	2897.4	2980.7	3065.4	3151.7	3239.5	3329.1	3420.4	3513.6	3608.7	3706.0	3805.5	3907.4	4011.8	25
26	2898.8	2982.1	3066.9	3153.2	3241.0	3330.6	3421.9	3515.1	3610.3	3707.7	3807.2	3909.1	4013.6	26
27	2900.2	2983.5	3068.3	3154.6	3242.5	3332.1	3423.5	3516.7	3611.9	3709.3	3808.9	3910.9	4015.3	27
28	2901.5	2984.9	3069.7	3156.1	3244.0	3333.6	3425.0	3518.3	3613.6	3710.9	3810.6	3912.6	4017.1	28
29	2902.9	2986.3	3071.1	3157.5	3245.5	3335.1	3426.5	3519.8	3615.2	3712.6	3812.3	3914.3	4018.9	29
30	2904.3	2987.7	3072.6	3159.0	3246.9	3336.6	3428.1	3521.4	3616.8	3714.2	3813.9	3916.0	4020.6	30
31	2905.7	2989.1	3074.0	3160.4	3248.4	3338.1	3429.6	3523.0	3618.4	3715.9	3815.6	3917.7	4022.4	31
32	2907.1	2990.5	3075.4	3161.9	3249.9	3339.6	3431.2	3524.6	3620.0	3717.5	3817.3	3919.5	4024.2	32
33	2908.4	2991.9	3076.9	3163.3	3251.4	3341.1	3432.7	3526.1	3621.6	3719.2	3819.0	3921.2	4026.9	33
34	2909.7	2993.3	3078.3	3164.8	3252.9	3342.7	3434.2	3527.7	3623.2	3720.8	3820.7	3922.5	4028.7	34
35	2911.2	2994.7	3079.7	3166.2	3254.4	3344.2	3435.8	3529.3	3624.8	3722.4	3822.3	3924.6	4029.5	35
36	2912.6	2996.1	3081.1	3167.7	3255.8	3345.7	3437.3	3530.9	3626.4	3724.1	3824.0	3926.4	4031.3	36
37	2914.0	2997.5	3082.6	3169.1	3257.3	3347.2	3438.9	3532.4	3628.0	3725.7	3825.7	3928.1	4033.0	37
38	2915.3	2998.9	3084.0	3170.6	3258.8	3348.7	3440.0	3534.0	3629.6	3727.4	3827.4	3929.8	4034.8	38
39	2916.7	3000.3	3085.4	3172.1	3260.3	3350.1	3441.0	3535.6	3631.3	3729.0	3829.1	3931.5	4036.6	39
40	2918.1	3001.8	3086.9	3173.5	3261.8	3351.7	3443.5	3537.2	3632.9	3730.7	3830.8	3933.3	4038.3	40
41	2919.5	3003.2	3088.3	3175.0	3263.3	3353.2	3445.0	3538.8	3634.5	3732.3	3832.5	3935.0	4040.1	41
42	2920.9	3004.6	3089.7	3176.4	3264.7	3354.8	3446.6	3540.3	3636.1	3734.0	3834.2	3936.7	4041.9	42
43	2922.3	3006.0	3091.2	3177.9	3266.2	3356.3	3448.1	3541.9	3637.7	3735.6	3835.8	3938.5	4043.6	43
44	2923.6	3007.4	3092.6	3179.3	3267.7	3357.8	3449.7	3543.5	3639.3	3737.3	3837.5	3940.2	4045.4	44
45	2925.0	3008.8	3094.0	3180.8	3269.2	3359.3	3451.2	3545.1	3640.9	3738.9	3839.2	3941.9	4047.2	45
46	2926.4	3010.2	3095.5	3182.3	3270.7	3360.8	3452.8	3546.7	3642.5	3740.6	3840.9	3943.7	4049.0	46
47	2927.8	3011.6	3096.9	3183.7	3272.2	3362.3	3454.3	3548.2	3644.2	3742.2	3842.6	3945.4	4050.8	47
48	2929.2	3013.0	3098.3	3185.2	3273.7	3363.9	3455.9	3549.8	3645.8	3743.9	3844.3	3947.1	4052.5	48
49	2930.6	3014.4	3099.8	3186.6	3275.2	3365.4	3457.4	3551.4	3647.4	3745.6	3846.0	3948.9	4054.3	49
50	2932.0	3015.8	3101.2	3188.1	3276.6	3366.9	3459.0	3553.0	3649.0	3747.2	3847.7	3950.6	4056.1	50
51	2933.3	3017.2	3102.6	3189.6	3278.1	3368.4	3460.5	3554.6	3650.6	3748.9	3849.4	3952.3	4057.9	51
52	2934.7	3018.7	3104.1	3191.0	3279.6	3369.9	3462.1	3556.1	3652.3	3750.5	3851.1	3954.1	4059.7	52
53	2936.1	3020.1	3105.6	3192.5	3281.1	3371.5	3463.6	3557.7	3653.9	3752.2	3852.8	3955.8	4061.4	53
54	2937.5	3021.5	3107.0	3194.0	3282.6	3373.0	3465.2	3559.3	3655.5	3753.8	3854.5	3957.6	4063.2	54
55	2938.9	3022.9	3108.4	3195.4	3284.1	3374.5	3466.7	3560.9	3657.1	3755.5	3856.2	3959.3	4065.0	55
56	2940.3	3024.3	3109.8	3196.9	3285.6	3376.0	3468.3	3562.5	3658.7	3757.2	3857.9	3961.0	4066.8	56
57	2941.7	3025.7	3111.2	3198.4	3287.1	3377.6	3469.8	3564.1	3660.4	3758.9	3859.6	3962.8	4068.6	57
58	2943.1	3027.1	3112.7	3199.8	3288.6	3379.1	3471.4	3565.7	3662.0	3760.5	3861.3	3964.5	4070.4	58
59	2944.4	3028.5	3114.1	3201.3	3290.1	3380.6	3473.0	3567.3	3663.6	3762.2	3863.0	3966.3	4072.2	59
N.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	N.
L.	43	44	45	46	47	48	49	50	51	52	53	54	55	L.

A Table of Meridional Parts.

27

L.	56	57	58	59	60	61	62	63	64	65	66	67	68	M.
M.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M.
0	4073.9	4182.7	4294.3	4409.2	4527.4	4649.3	4775.0	4905.0	5039.5	5178.8	5323.6	5474.0	5630.0	1
1	4075.7	4184.5	4296.2	4411.1	4529.4	4651.3	4777.1	4907.2	5041.7	5181.2	5326.0	5476.4	5632.5	2
2	4077.5	4186.3	4298.1	4413.1	4531.4	4653.4	4779.3	4909.4	5044.0	5183.6	5328.5	5478.9	5634.7	3
3	4079.3	4188.2	4300.0	4415.0	4533.4	4655.5	4781.4	4911.6	5046.3	5186.0	5330.9	5481.7	5636.8	4
4	4081.1	4190.0	4301.9	4417.0	4535.4	4657.5	4783.5	4913.8	5048.6	5188.3	5333.4	5484.3	5638.5	5
5	4082.9	4191.8	4303.8	4418.9	4537.4	4659.6	4785.7	4916.0	5050.9	5190.7	5335.0	5486.9	5641.4	6
6	4084.7	4193.7	4305.7	4420.8	4539.4	4661.7	4787.8	4918.2	5053.2	5193.1	5337.2	5489.4	5643.6	7
7	4086.5	4195.5	4307.6	4422.8	4541.4	4663.7	4790.0	4920.4	5055.5	5195.4	5340.8	5492.0	5646.0	8
8	4088.3	4197.4	4309.5	4424.7	4543.4	4665.8	4792.1	4922.6	5057.7	5197.8	5343.3	5494.6	5648.3	9
9	4090.1	4199.2	4311.4	4426.7	4545.4	4667.9	4794.2	4924.8	5060.0	5200.2	5345.7	5497.1	5650.0	10
10	4091.9	4201.1	4313.2	4428.6	4547.5	4669.9	4796.4	4927.1	5062.3	5202.6	5348.2	5499.7	5651.6	11
11	4093.7	4202.9	4315.1	4430.6	4549.5	4672.0	4798.5	4929.3	5064.6	5205.0	5350.7	5502.3	5653.1	12
12	4095.5	4204.7	4317.0	4432.5	4551.5	4674.1	4800.7	4931.5	5066.9	5207.3	5353.2	5504.9	5654.7	13
13	4097.3	4206.6	4318.9	4434.5	4553.5	4676.2	4802.8	4933.7	5069.2	5209.7	5355.6	5507.5	5656.7	14
14	4099.1	4208.4	4320.8	4436.4	4555.5	4678.2	4804.9	4935.9	5071.5	5212.1	5358.1	5510.0	5658.4	15
15	4100.9	4210.3	4322.7	4438.4	4557.5	4680.3	4807.1	4938.1	5073.8	5214.5	5360.6	5512.6	5661.1	16
16	4102.7	4212.1	4324.6	4440.4	4559.5	4682.4	4809.2	4940.4	5076.1	5216.9	5363.1	5515.2	5663.8	17
17	4104.5	4214.0	4326.5	4442.3	4561.5	4684.5	4811.4	4942.6	5078.4	5219.3	5365.6	5517.8	5666.5	18
18	4106.3	4215.8	4328.4	4444.3	4563.6	4686.6	4813.5	4944.8	5080.7	5221.7	5368.1	5520.4	5669.2	19
19	4108.1	4217.7	4330.3	4446.2	4565.6	4688.6	4815.7	4947.0	5083.0	5224.1	5370.5	5523.0	5671.9	20
20	4109.9	4219.5	4332.2	4448.2	4567.6	4690.7	4817.8	4949.3	5085.3	5226.5	5373.0	5525.6	5674.6	21
21	4111.7	4221.4	4334.1	4450.2	4569.6	4692.8	4819.0	4951.5	5087.7	5228.9	5375.5	5528.2	5677.3	22
22	4113.5	4223.2	4336.1	4452.1	4571.6	4694.9	4821.2	4953.7	5090.0	5231.3	5378.0	5530.8	5679.0	23
23	4115.3	4225.1	4338.0	4454.1	4573.7	4697.0	4823.3	4956.0	5092.3	5233.7	5380.5	5533.4	5681.8	24
24	4117.1	4227.0	4339.9	4456.0	4575.7	4699.1	4825.5	4958.2	5094.6	5236.1	5383.0	5536.0	5684.5	25
25	4118.9	4228.8	4341.8	4458.0	4577.7	4701.2	4827.6	4960.4	5096.9	5238.5	5385.5	5538.6	5687.2	26
26	4120.7	4230.7	4343.7	4460.0	4579.7	4703.2	4829.8	4962.7	5099.2	5240.9	5388.0	5541.2	5689.9	27
27	4122.5	4232.5	4345.6	4461.9	4581.8	4705.3	4831.9	4964.9	5101.5	5243.3	5390.5	5543.8	5692.6	28
28	4124.3	4234.4	4347.5	4463.9	4583.8	4707.4	4834.1	4967.1	5103.9	5245.7	5393.0	5546.4	5695.3	29
29	4126.1	4236.2	4349.4	4465.8	4585.8	4709.5	4836.3	4969.4	5106.2	5248.1	5395.5	5549.0	5697.9	30
30	4127.9	4238.1	4351.3	4467.8	4587.8	4711.6	4838.4	4971.6	5108.5	5250.5	5398.0	5551.6	5700.6	31
31	4129.7	4240.0	4353.3	4469.8	4589.9	4713.7	4840.6	4973.9	5110.8	5252.9	5400.5	5554.2	5703.3	32
32	4131.5	4241.8	4355.2	4471.8	4591.9	4715.8	4842.8	4976.1	5113.1	5255.3	5403.0	5556.8	5706.0	33
33	4133.3	4243.7	4357.1	4473.8	4593.9	4717.9	4845.0	4978.3	5115.5	5257.7	5405.6	5559.5	5708.7	34
34	4135.1	4245.6	4359.0	4475.7	4596.0	4720.0	4847.1	4980.6	5117.8	5260.1	5408.1	5562.1	5711.4	35
35	4136.9	4247.4	4360.9	4477.7	4598.0	4722.1	4849.3	4982.8	5120.1	5262.6	5410.6	5564.7	5714.1	36
36	4138.8	4249.3	4362.8	4479.7	4600.1	4724.2	4851.5	4985.1	5122.5	5265.0	5413.1	5567.3	5716.8	37
37	4140.6	4251.2	4364.8	4481.7	4602.1	4726.3	4853.6	4987.3	5124.8	5267.4	5415.6	5569.9	5719.5	38
38	4142.5	4253.0	4366.7	4483.6	4604.1	4728.4	4855.8	4989.6	5127.1	5269.8	5418.1	5572.6	5722.2	39
39	4144.3	4254.9	4368.6	4485.6	4606.2	4730.5	4857.9	4991.8	5129.5	5272.3	5420.7	5575.2	5724.9	40
40	4146.1	4256.8	4370.5	4487.6	4608.2	4732.6	4860.1	4994.1	5131.8	5274.7	5423.2	5577.8	5727.6	41
41	4147.9	4258.6	4372.5	4489.6	4610.3	4734.7	4862.3	4996.3	5134.1	5277.1	5425.7	5580.5	5730.3	42
42	4149.7	4260.5	4374.4	4491.6	4612.3	4736.9	4864.5	4998.6	5136.5	5279.5	5428.2	5583.1	5733.0	43
43	4151.5	4262.4	4376.3	4493.5	4614.3	4739.0	4866.7	5000.9	5138.8	5282.0	5430.7	5585.7	5735.7	44
44	4153.4	4264.3	4378.2	4495.5	4616.4	4741.1	4868.9	5003.1	5141.2	5284.4	5433.3	5588.4	5738.4	45
45	4155.2	4266.1	4380.2	4497.5	4618.4	4743.2	4871.1	5005.4	5143.5	5286.8	5435.8	5591.0	5741.1	46
46	4157.0	4268.0	4382.1	4499.5	4620.5	4745.3	4873.3	5007.6	5145.9	5289.3	5438.4	5593.7	5743.8	47
47	4158.8	4269.0	4384.0	4501.5	4622.5	4747.4	4875.4	5009.9	5148.2	5291.7	5440.9	5596.3	5746.5	48
48	4160.7	4271.8	4385.9	4503.5	4624.6	4749.5	4878.6	5012.2	5150.6	5294.2	5443.5	5599.0	5749.3	49
49	4162.5	4273.6	4387.9	4505.5	4626.6	4751.7	4880.8	5014.4	5152.9	5296.6	5446.0	5601.6	5752.0	50
50	4164.3	4275.5	4389.8	4507.5	4628.7	4753.8	4883.0	5016.7	5155.3	5299.0	5448.5	5604.3	5754.8	51
51	4166.1	4277.4	4391.7	4509.4	4630.7	4755.9	4885.2	5019.0	5157.6	5301.5	5451.1	5606.9	5757.6	52
52	4168.0	4279.3	4393.7	4511.4	4632.8	4758.0	4887.4	5021.2	5160.0	5303.9	5453.6	5609.6	5760.3	53
53	4169.8	4281.1	4395.6	4513.4	4634.8	4760.1	4889.6	5023.5	5162.3	5306.4	5456.2	5612.2	5763.1	54
54	4171.7	4282.0	4397.5	4515.4	4636.9	4762.3	4891.8	5025.8	5164.7	5308.8	5458.7	5614.9	5765.9	55
55	4173.5	4284.0	4399.5	4517.4	4638.9	4764.4	4894.0	5028.1	5167.0	5311.3	5461.3	5617.5	5768.7	56
56	4175.3	4286.8	4401.4	4519.4	4641.0	4766.5	4896.2	5030.3	5169.4	5313.7	5463.8	5620.2	5771.5	57
57	4177.2	4288.7	4403.4	4521.4	4643.1	4768.6	4898.4	5032.6	5171.8	5316.2	5466.4	5622.9	5774.3	58
58	4179.0	4290.6	4405.3	4523.4	4645.1	4770.8	4900.6	5034.9	5174.1	5318.6	5468.9	5625.5	5777.0	59
59	4180.8	4292.5	4407.2	4525.4	4647.2	4772.9	4902.8	5037.2	5176.5	5321.1	5471.5	5628.3	5779.8	60
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
L.	56	57	58	59	60	61	62	63	64	65	66	67	68	L

L.	69	70	71	72	73	74	75	76	77	78	79	80	81	L.
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
0	5794.6	5966.0	6145.7	6334.9	6534.5	6745.7	6970.3	7210.1	7467.2	7744.6	8045.7	8375.3	8739.1	0
1	5797.4	5968.8	6148.5	6338.1	6537.9	6749.4	6974.2	7214.2	7471.7	7749.4	8051.0	8381.0	8745.5	1
2	5800.2	5971.5	6151.9	6341.4	6541.3	6753.0	6978.1	7218.3	7476.1	7754.2	8056.2	8386.8	8751.9	2
3	5803.0	5974.7	6155.0	6344.6	6544.7	6756.6	6980.9	7221.6	7480.6	7759.0	8061.5	8392.6	8757.3	3
4	5805.8	5977.7	6158.0	6347.8	6548.2	6760.3	6985.8	7226.6	7485.0	7763.2	8066.8	8398.3	8762.8	4
5	5808.6	5980.6	6161.1	6351.1	6551.6	6763.9	6989.7	7230.3	7489.5	7768.7	8072.0	8404.1	8771.2	5
6	5811.4	5983.5	6164.2	6354.4	6555.0	6767.6	6993.6	7234.9	7494.0	7773.5	8077.3	8409.9	8777.7	6
7	5814.2	5986.5	6167.3	6357.8	6558.5	6771.2	6997.5	7239.1	7498.5	7778.4	8082.6	8415.8	8784.1	7
8	5817.0	5989.4	6170.4	6360.9	6561.9	6774.9	7001.3	7243.3	7502.9	7783.2	8087.9	8421.6	8790.6	8
9	5819.8	5992.4	6173.5	6364.1	6565.4	6778.5	7005.3	7247.5	7507.4	7788.1	8093.2	8427.4	8797.1	9
10	5822.6	5995.3	6176.6	6367.4	6568.8	6782.2	7009.4	7251.6	7511.9	7793.0	8098.5	8433.3	8803.6	10
11	5825.4	5998.3	6179.7	6370.6	6572.3	6785.8	7013.4	7255.8	7516.4	7797.8	8103.8	8439.1	8810.1	11
12	5828.2	6001.2	6182.8	6373.9	6575.7	6789.5	7017.0	7260.0	7520.9	7802.7	8109.2	8445.0	8816.6	12
13	5831.0	6004.2	6185.9	6377.2	6579.2	6793.2	7020.9	7264.2	7525.4	7807.6	8114.5	8450.9	8823.2	13
14	5833.9	6007.1	6189.0	6380.5	6582.6	6796.9	7024.8	7268.4	7530.0	7812.5	8119.8	8456.8	8829.7	14
15	5836.7	6010.1	6192.1	6383.7	6586.1	6800.5	7028.7	7272.6	7534.5	7817.4	8125.2	8462.6	8836.1	15
16	5839.5	6013.0	6195.2	6387.0	6589.5	6804.2	7032.7	7276.8	7539.0	7822.3	8130.6	8468.6	8842.8	16
17	5842.3	6016.0	6198.3	6390.3	6593.0	6807.9	7036.6	7281.0	7543.6	7827.2	8135.9	8474.7	8849.4	17
18	5845.2	6019.0	6201.4	6393.6	6596.5	6811.6	7040.5	7285.2	7548.1	7832.2	8141.3	8480.4	8856.0	18
19	5848.0	6021.9	6204.6	6396.9	6600.0	6815.8	7044.5	7289.4	7552.7	7837.1	8146.7	8486.3	8862.6	19
20	5850.8	6024.9	6207.7	6400.2	6603.4	6819.0	7048.4	7293.7	7557.2	7842.0	8152.1	8492.3	8869.3	20
21	5853.7	6027.9	6210.8	6403.5	6606.9	6822.7	7052.4	7297.9	7561.8	7847.0	8157.5	8498.2	8875.9	21
22	5856.5	6030.8	6213.9	6406.8	6610.4	6826.4	7056.3	7302.1	7566.3	7851.9	8162.9	8504.2	8882.6	22
23	5859.3	6033.8	6217.1	6410.1	6613.9	6830.1	7060.3	7306.4	7570.9	7856.9	8168.3	8510.2	8889.2	23
24	5862.2	6036.8	6220.2	6413.4	6617.4	6833.8	7064.2	7310.6	7575.5	7861.9	8173.7	8516.2	8895.9	24
25	5865.0	6039.8	6223.3	6416.7	6620.9	6837.6	7068.2	7314.9	7580.1	7866.8	8179.2	8522.2	8902.6	25
26	5867.9	6042.7	6226.7	6420.0	6624.4	6841.3	7072.2	7319.1	7584.7	7871.8	8184.6	8528.2	8909.3	26
27	5870.7	6045.7	6229.6	6423.3	6627.9	6845.0	7076.2	7323.4	7589.3	7876.8	8190.1	8534.2	8916.0	27
28	5873.5	6048.7	6232.7	6426.6	6631.4	6848.7	7080.1	7327.7	7593.9	7881.8	8195.5	8540.2	8922.7	28
29	5876.4	6051.7	6235.9	6429.9	6635.0	6852.5	7084.1	7332.0	7598.3	7886.8	8201.0	8546.2	8929.5	29
30	5879.3	6054.7	6239.0	6433.2	6638.5	6856.2	7088.1	7336.2	7603.1	7891.8	8206.5	8552.3	8936.2	30
31	5882.1	6057.7	6242.2	6436.6	6642.0	6860.0	7092.1	7340.5	7607.7	7896.8	8212.0	8558.4	8943.0	31
32	5885.0	6060.7	6245.3	6439.9	6645.5	6863.7	7096.1	7344.8	7612.3	7901.9	8217.5	8564.4	8949.8	32
33	5887.8	6063.7	6248.5	6443.2	6649.1	6867.5	7100.1	7349.1	7617.0	7906.9	8223.0	8570.5	8956.6	33
34	5890.7	6066.7	6251.7	6446.6	6652.6	6871.2	7104.1	7353.4	7621.6	7911.9	8228.5	8576.6	8963.4	34
35	5893.6	6069.7	6254.8	6449.9	6656.1	6875.0	7108.2	7357.7	7626.3	7917.0	8234.1	8582.7	8970.2	35
36	5896.4	6072.7	6258.0	6453.3	6659.7	6878.7	7112.2	7362.0	7630.9	7922.1	8239.6	8588.9	8977.1	36
37	5899.3	6075.7	6261.2	6456.6	6663.2	6882.5	7116.2	7366.4	7635.6	7927.1	8245.1	8595.0	8983.9	37
38	5902.2	6078.8	6264.4	6460.0	6666.8	6886.3	7120.2	7370.7	7640.2	7932.2	8250.7	8601.1	8990.8	38
39	5905.1	6081.8	6267.5	6463.3	6670.3	6890.1	7124.3	7375.0	7644.9	7937.3	8256.3	8607.3	8997.7	39
40	5907.9	6084.8	6270.7	6466.7	6673.9	6893.8	7128.3	7379.4	7649.6	7942.4	8261.8	8613.5	9004.6	40
41	5910.8	6087.8	6273.9	6470.0	6677.4	6897.6	7132.3	7383.7	7654.3	7947.5	8267.4	8619.6	9011.5	41
42	5913.7	6090.8	6277.1	6473.4	6681.0	6901.4	7136.4	7388.0	7659.0	7952.6	8273.0	8625.8	9018.4	42
43	5916.6	6093.9	6280.3	6476.8	6684.6	6905.3	7140.4	7392.4	7663.7	7957.7	8278.6	8632.0	9025.4	43
44	5919.5	6096.9	6283.5	6480.1	6688.1	6909.0	7144.5	7396.8	7668.4	7962.8	8284.2	8638.2	9032.4	44
45	5922.4	6099.9	6286.6	6483.5	6691.7	6912.8	7148.6	7401.1	7673.1	7967.8	8289.9	8644.5	9039.1	45
46	5925.2	6103.0	6289.8	6486.9	6695.4	6916.6	7152.6	7405.5	7677.8	7972.1	8295.5	8650.7	9045.8	46
47	5928.1	6106.0	6293.0	6490.3	6699.9	6920.4	7156.7	7409.9	7682.6	7976.8	8301.1	8656.9	9052.5	47
48	5931.0	6109.1	6296.2	6493.6	6704.2	6924.2	7160.8	7414.2	7687.3	7981.4	8306.8	8663.2	9059.3	48
49	5933.9	6112.1	6299.4	6497.0	6708.0	6928.1	7164.9	7418.6	7692.0	7986.5	8312.4	8669.5	9067.1	49
50	5936.8	6115.1	6302.7	6500.4	6711.9	6931.9	7169.0	7423.0	7696.8	7991.7	8318.1	8675.7	9074.4	50
51	5939.7	6118.2	6305.9	6503.8	6715.3	6935.7	7173.0	7427.4	7701.5	7996.9	8323.8	8682.0	9081.4	51
52	5942.6	6121.2	6309.1	6507.2	6718.8	6939.5	7177.1	7431.8	7706.3	8002.0	8329.4	8688.3	9088.5	52
53	5945.5	6124.3	6312.3	6510.6	6722.4	6943.4	7181.2	7436.2	7711.0	8007.2	8335.1	8694.6	9095.6	53
54	5948.5	6127.4	6315.5	6514.0	6726.0	6947.2	7185.3	7440.6	7715.8	8012.4	8340.8	8701.0	9102.7	54
55	5951.4	6130.4	6318.7	6517.4	6729.6	6951.1	7189.5	7445.0	7720.6	8017.6	8346.6	8707.3	9109.8	55
56	5954.2	6133.5	6322.0	6520.8	6733.2	6954.0	7193.6	7449.5	7725.4	8022.8	8352.3	8713.6	9116.9	56
57	5957.1	6136.5	6325.2	6524.2	6737.4	6958.8	7197.7	7453.9	7730.2	8028.0	8358.0	8719.0	9124.0	57
58	5960.1	6139.6	6328.4	6527.6	6738.5	6962.6	7201.8	7458.3	7735.0	8033.3	8363.7	8724.3	9131.2	58
59	5963.0	6142.7	6331.7	6531.0	6742.1	6966.5	7205.9	7462.8	7739.8	8038.5	8369.5	8729.7	9138.4	59
M	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M
L.	60	70	71	72	73	74	75	76	77	78	79	80	81	L.

A Table of Meridional Parts.

29

L.	81	83	84	85	86	87	88	89	M.
M.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	Min.	M.
0	9145.6	9605.9	10137.0	10764.7	11432.6	12151.3	12916.0	13729.8	0
1	9152.7	9614.1	10146.6	10766.2	11447.0	12164.1	12924.5	13737.5	1
2	9159.9	9621.4	10156.2	10767.7	11461.4	12176.7	12932.9	13745.4	2
3	9167.2	9628.6	10165.8	10769.3	11475.9	12189.0	12941.3	13753.1	3
4	9174.4	9635.9	10175.4	10770.8	11490.4	12201.5	12949.7	13760.7	4
5	9181.6	9643.2	10185.1	10772.4	11504.9	12213.9	12958.0	13768.3	5
6	9188.9	9650.5	10194.8	10773.9	11519.4	12226.3	12966.4	13775.9	6
7	9196.1	9657.8	10204.6	10775.5	11533.9	12238.7	12974.7	13783.5	7
8	9203.5	9665.1	10214.4	10777.0	11548.4	12251.1	12983.1	13791.1	8
9	9210.8	9672.4	10224.2	10778.6	11562.9	12263.5	12991.4	13798.7	9
10	9218.1	9679.7	10234.0	10780.1	11577.4	12275.9	13000.0	13806.3	10
11	9225.4	9687.0	10243.8	10781.7	11591.9	12288.3	13008.3	13813.9	11
12	9232.8	9694.3	10253.7	10783.2	11606.4	12300.7	13016.7	13821.5	12
13	9240.2	9701.6	10263.6	10784.8	11620.9	12313.1	13025.0	13829.1	13
14	9247.6	9708.9	10273.4	10786.3	11635.4	12325.5	13033.4	13836.7	14
15	9255.0	9716.2	10283.3	10787.9	11649.9	12337.9	13041.7	13844.3	15
16	9262.4	9723.5	10293.2	10789.4	11664.4	12350.3	13050.1	13851.9	16
17	9269.8	9730.8	10303.1	10791.0	11678.9	12362.7	13058.4	13859.5	17
18	9277.1	9738.1	10313.0	10792.5	11693.4	12375.1	13066.8	13867.1	18
19	9284.5	9745.4	10322.9	10794.1	11707.9	12387.5	13075.1	13874.7	19
20	9291.9	9752.7	10332.8	10795.6	11722.4	12399.9	13083.5	13882.3	20
21	9299.3	9760.0	10342.7	10797.2	11736.9	12412.3	13091.8	13889.9	21
22	9306.7	9767.3	10352.6	10798.7	11751.4	12424.7	13100.2	13897.5	22
23	9314.1	9774.6	10362.5	10800.3	11765.9	12437.1	13108.5	13905.1	23
24	9321.5	9781.9	10372.4	10801.8	11780.4	12449.5	13116.9	13912.7	24
25	9328.9	9789.2	10382.3	10803.4	11794.9	12461.9	13125.2	13920.3	25
26	9336.3	9796.5	10392.2	10804.9	11809.4	12474.3	13133.6	13927.9	26
27	9343.7	9803.8	10402.1	10806.5	11823.9	12486.7	13141.9	13935.5	27
28	9351.1	9811.1	10412.0	10808.0	11838.4	12499.1	13150.3	13943.1	28
29	9358.5	9818.4	10421.9	10809.6	11852.9	12511.5	13158.6	13950.7	29
30	9365.9	9825.7	10431.8	10811.1	11867.4	12523.9	13167.0	13958.3	30
31	9373.3	9833.0	10441.7	10812.7	11881.9	12536.3	13175.3	13965.9	31
32	9380.7	9840.3	10451.6	10814.2	11896.4	12548.7	13183.7	13973.5	32
33	9388.1	9847.6	10461.5	10815.8	11910.9	12561.1	13192.0	13981.1	33
34	9395.5	9854.9	10471.4	10817.3	11925.4	12573.5	13200.4	13988.7	34
35	9402.9	9862.2	10481.3	10818.9	11939.9	12585.9	13208.7	13996.3	35
36	9410.3	9869.5	10491.2	10820.4	11954.4	12598.3	13217.1	14003.9	36
37	9417.7	9876.8	10501.1	10822.0	11968.9	12610.7	13225.4	14011.5	37
38	9425.1	9884.1	10511.0	10823.5	11983.4	12623.1	13233.8	14019.1	38
39	9432.5	9891.4	10520.9	10825.1	11997.9	12635.5	13242.1	14026.7	39
40	9439.9	9898.7	10530.8	10826.6	12012.4	12647.9	13250.5	14034.3	40
41	9447.3	9906.0	10540.7	10828.2	12026.9	12660.3	13258.8	14041.9	41
42	9454.7	9913.3	10550.6	10829.7	12041.4	12672.7	13267.2	14049.5	42
43	9462.1	9920.6	10560.5	10831.3	12055.9	12685.1	13275.5	14057.1	43
44	9469.5	9927.9	10570.4	10832.8	12070.4	12697.5	13283.9	14064.7	44
45	9476.9	9935.2	10580.3	10834.4	12084.9	12709.9	13292.2	14072.3	45
46	9484.3	9942.5	10590.2	10835.9	12099.4	12722.3	13300.6	14079.9	46
47	9491.7	9949.8	10600.1	10837.5	12113.9	12734.7	13308.9	14087.5	47
48	9499.1	9957.1	10610.0	10839.0	12128.4	12747.1	13317.3	14095.1	48
49	9506.5	9964.4	10619.9	10840.6	12142.9	12759.5	13325.6	14102.7	49
50	9513.9	9971.7	10629.8	10842.1	12157.4	12771.9	13334.0	14110.3	50
51	9521.3	9979.0	10639.7	10843.7	12171.9	12784.3	13342.3	14117.9	51
52	9528.7	9986.3	10649.6	10845.2	12186.4	12796.7	13350.7	14125.5	52
53	9536.1	9993.6	10659.5	10846.8	12200.9	12809.1	13359.0	14133.1	53
54	9543.5	10000.9	10669.4	10848.3	12215.4	12821.5	13367.4	14140.7	54
55	9550.9	10008.2	10679.3	10850.0	12229.9	12833.9	13375.7	14148.3	55
56	9558.3	10015.5	10689.2	10851.5	12244.4	12846.3	13384.1	14155.9	56
57	9565.7	10022.8	10699.1	10853.1	12258.9	12858.7	13392.4	14163.5	57
58	9573.1	10030.1	10709.0	10854.6	12273.4	12871.1	13400.8	14171.1	58
59	9580.5	10037.4	10718.9	10856.2	12287.9	12883.5	13409.1	14178.7	59
60	9587.9	10044.7	10728.8	10857.7	12302.4	12895.9	13417.5	14186.3	60
61	9595.3	10052.0	10738.7	10859.3	12316.9	12908.3	13425.8	14193.9	61
62	9602.7	10059.3	10748.6	10860.8	12331.4	12920.7	13434.2	14201.5	62
63	9610.1	10066.6	10758.5	10862.4	12345.9	12933.1	13442.5	14209.1	63
64	9617.5	10073.9	10768.4	10863.9	12360.4	12945.5	13450.9	14216.7	64
65	9624.9	10081.2	10778.3	10865.5	12374.9	12957.9	13459.2	14224.3	65
66	9632.3	10088.5	10788.2	10867.0	12389.4	12970.3	13467.6	14231.9	66
67	9639.7	10095.8	10798.1	10868.6	12403.9	12982.7	13475.9	14239.5	67
68	9647.1	10103.1	10808.0	10870.1	12418.4	12995.1	13484.3	14247.1	68
69	9654.5	10110.4	10817.9	10871.7	12432.9	13007.5	13492.6	14254.7	69
70	9661.9	10117.7	10827.8	10873.2	12447.4	13019.9	13501.0	14262.3	70
71	9669.3	10125.0	10837.7	10874.8	12461.9	13032.3	13509.3	14269.9	71
72	9676.7	10132.3	10847.6	10876.3	12476.4	13044.7	13517.7	14277.5	72
73	9684.1	10139.6	10857.5	10877.9	12490.9	13057.1	13526.0	14285.1	73
74	9691.5	10146.9	10867.4	10879.4	12505.4	13069.5	13534.4	14292.7	74
75	9698.9	10154.2	10877.3	10881.0	12519.9	13081.9	13542.7	14300.3	75
76	9706.3	10161.5	10887.2	10882.5	12534.4	13094.3	13551.1	14307.9	76
77	9713.7	10168.8	10897.1	10884.1	12548.9	13106.7	13559.4	14315.5	77
78	9721.1	10176.1	10907.0	10885.6	12563.4	13119.1	13567.8	14323.1	78
79	9728.5	10183.4	10916.9	10887.2	12577.9	13131.5	13576.1	14330.7	79
80	9735.9	10190.7	10926.8	10888.7	12592.4	13143.9	13584.5	14338.3	80
81	9743.3	10198.0	10936.7	10890.3	12606.9	13156.3	13592.8	14345.9	81
82	9750.7	10205.3	10946.6	10891.8	12621.4	13168.7	13601.2	14353.5	82
83	9758.1	10212.6	10956.5	10893.4	12635.9	13181.1	13609.5	14361.1	83
84	9765.5	10219.9	10966.4	10894.9	12650.4	13193.5	13617.9	14368.7	84
85	9772.9	10227.2	10976.3	10896.5	12664.9	13205.9	13626.2	14376.3	85
86	9780.3	10234.5	10986.2	10898.0	12679.4	13218.3	13634.6	14383.9	86
87	9787.7	10241.8	10996.1	10900.0	12693.9	13230.7	13642.9	14391.5	87
88	9795.1	10249.1	11006.0	10901.5	12708.4	13243.1	13651.3	14399.1	88
89	9802.5	10256.4	11015.9	10903.0	12722.9	13255.5	13659.6	14406.7	89
90	9809.9	10263.7	11025.8	10904.6	12737.4	13267.9	13668.0	14414.3	90
91	9817.3	10271.0	11035.7	10906.1	12751.9	13280.3	13676.3	14421.9	91
92	9824.7	10278.3	11045.6	10907.7	12766.4	13292.7	13684.7	14429.5	92
93	9832.1	10285.6	11055.5	10909.2	12780.9	13305.1	13693.0	14437.1	93
94	9839.5	10292.9	11065.4	10910.8	12795.4	13317.5	13701.4	14444.7	94
95	9846.9	10300.2	11075.3	10912.3	12809.9	13329.9	13709.7	14452.3	95
96	9854.3	10307.5	11085.2	10913.9	12824.4	13342.3	13718.1	14459.9	96
97	9861.7	10314.8	11095.1	10915.4	12838.9	13354.7	13726.4	14467.5	97
98	9869.1	10322.1	11105.0	10917.0	12853.4	13367.1	13734.8	14475.1	98
99	9876.5	10329.4	11114.9	10918.5	12867.9	13379.5	13743.1	14482.7	99
100	9883.9	10336.7	11124.8	10920.1	12882.4	13391.9	13751.5	14490.3	100

A T A B L E O F L O G A R I T H M S

For Numbers increasing in their Natural Order from Unite
to 10000.

Num.	Logarith.	Num.	Logarith.	Num.	Logarith.
1	0.000000	34	1.531479	67	1.826075
2	0.301030	35	1.544068	68	1.832509
3	0.477121	36	1.556302	69	1.838849
4	0.602060	37	1.568202	70	1.845098
5	0.698970	38	1.579784	71	1.851258
6	0.778151	39	1.591065	72	1.857332
7	0.845098	40	1.602060	73	1.863323
8	0.903090	41	1.612784	74	1.869232
9	0.954242	42	1.623249	75	1.875061
10	1.000000	43	1.633468	76	1.880814
11	1.041393	44	1.643453	77	1.886491
12	1.079181	45	1.653212	78	1.892095
13	1.113943	46	1.662758	79	1.897627
14	1.146128	47	1.672098	80	1.903090
15	1.176091	48	1.681241	81	1.908485
16	1.204120	49	1.690196	82	1.913814
17	1.230449	50	1.698970	83	1.919078
18	1.255272	51	1.707570	84	1.924279
19	1.278754	52	1.716003	85	1.929419
20	1.301030	53	1.724276	86	1.934498
21	1.322219	54	1.732394	87	1.939519
22	1.342423	55	1.740363	88	1.944483
23	1.361728	56	1.748188	89	1.949390
24	1.380211	57	1.755875	90	1.954242
25	1.397940	58	1.763428	91	1.959041
26	1.414973	59	1.770852	92	1.963788
27	1.431364	60	1.778151	93	1.968483
28	1.447158	61	1.785330	94	1.973128
29	1.462398	62	1.792392	95	1.977724
30	1.477121	63	1.799340	96	1.982271
31	1.491362	64	1.806180	97	1.986772
32	1.505150	65	1.812913	98	1.991226
33	1.518514	66	1.819544	99	1.995635

LOGARITHMS.

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No.	0	1	2	3	4	5	6	7	8	9
100	000000	000434	000868	001301	001734	002166	002598	003029	003460	003890
101	004321	004751	005180	005609	006038	006466	006894	007321	007748	008174
102	008600	009026	009451	009876	010300	010724	011147	011570	011993	012415
103	012837	013259	013680	014100	014520	014940	015359	015779	016197	016614
104	017033	017451	017868	018284	018700	019117	019532	019947	020361	020775
105	021189	021603	022016	022428	022841	023252	023662	024075	024486	024896
106	025306	025715	026124	026533	026942	027350	027757	028164	028571	028978
107	029384	029789	030195	030600	031004	031408	031812	032216	032619	033022
108	033424	033826	034227	034628	035029	035430	035830	036229	036629	037028
109	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998
110	041393	041787	042182	042575	042969	043362	043755	044148	044540	044931
111	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830
112	049218	049606	049993	050380	050766	051152	051538	051924	052309	052694
113	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524
114	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320
115	060698	061072	061445	061829	062206	062582	062958	063333	063708	064083
116	064458	064832	065206	065580	065953	066326	066698	067071	067443	067814
117	068186	068557	068928	069298	069668	070037	070407	070776	071145	071514
118	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182
119	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426
121	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004
122	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552
123	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071
124	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562
125	096910	097257	097604	097951	098297	098644	098990	099335	099681	100026
126	100370	100715	101059	101403	101747	102090	102434	102777	103119	103462
127	103804	104145	104487	104828	105169	105510	105851	106191	106531	106870
128	107210	107549	107888	108227	108565	108903	109241	109578	109916	110253
129	110590	110926	111261	111598	111934	112270	112605	112940	113275	113609
130	113943	114277	114611	114944	115278	115610	115943	116276	116608	116940
131	117271	117603	117934	118265	118595	118926	119256	119586	119915	120245
132	120574	120903	121231	121560	121888	122216	122543	122871	123198	123525
133	123852	124178	124504	124830	125156	125481	125806	126131	126456	126781
134	127105	127429	127752	128076	128399	128722	129045	129368	129690	130012
135	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219
136	133539	133858	134177	134496	134814	135133	135451	135768	136086	136403
137	136721	137037	137354	137670	137987	138303	138618	138934	139249	139564
138	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702
139	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911
141	149219	149527	149835	150142	150449	150756	151063	151370	151678	151982
142	152288	152594	152900	153205	153510	153815	154119	154424	154728	155032
143	155336	155640	155943	156246	156549	156852	157154	157457	157759	158061
144	158362	158664	158965	159266	159567	159868	160168	160468	160769	161068
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No	0	1	2	3	4	5	6	7	8	9
145	161368	161667	161967	162266	162564	162863	163161	163459	163757	164055
146	164353	164650	164947	165244	165541	165838	166134	166430	166726	167022
147	167317	167613	167908	168203	168497	168792	169086	169380	169674	169968
148	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895
149	173186	173477	173769	174060	174351	174641	174932	175222	175512	175802
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689
151	178977	179264	179552	179839	180126	180413	180699	180986	181272	181558
152	181844	182129	182415	182700	182985	183270	183554	183839	184123	184407
153	184691	184975	185259	185542	185825	186108	186391	186674	186956	187239
154	187521	187803	188084	188366	188647	188928	189209	189490	189771	190051
155	190332	190612	190892	191171	191451	191730	192010	192289	192567	192846
156	193125	193403	193681	193959	194237	194514	194792	195069	195346	195623
157	195900	196176	196452	196729	197005	197281	197556	197831	198107	198382
158	198657	198932	199206	199481	199755	200029	200303	200577	200850	201124
159	201397	201670	201943	202216	202488	202761	203033	203305	203577	203848
160	204120	204391	204662	204933	205204	205475	205745	206016	206286	206556
161	206826	207095	207365	207634	207903	208172	208441	208710	208978	209247
162	209515	209783	210051	210318	210586	210853	211120	211388	211654	211921
163	212188	212454	212720	212986	213252	213518	213783	214049	214314	214579
164	214844	215109	215373	215638	215902	216166	216430	216694	216957	217221
165	217484	217747	218010	218273	218535	218798	219060	219322	219584	219846
166	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456
167	222716	222976	223236	223496	223755	224015	224274	224533	224792	225051
168	225309	225568	225826	226084	226342	226600	226858	227115	227372	227630
169	227887	228143	228400	228657	228913	229170	229426	229682	229938	230193
170	230449	230704	230960	231215	231470	231724	231979	232233	232488	232742
171	232996	233250	233504	233757	234011	234264	234517	234770	235023	235276
172	235528	235781	236033	236285	236537	236789	237041	237292	237544	237795
173	238046	238297	238548	238799	239049	239299	239550	239800	240050	240299
174	240549	240799	241048	241297	241546	241795	242044	242293	242541	242790
175	243038	243286	243534	243782	244030	244277	244524	244772	245019	245266
176	245513	245759	246006	246252	246499	246745	246991	247236	247482	247728
177	247973	248219	248464	248709	248954	249198	249443	249687	249932	250176
178	250420	250664	250908	251151	251395	251638	251881	252125	252367	252610
179	252853	253096	253338	253580	253822	254064	254306	254548	254790	255031
180	255272	255514	255755	255996	256236	256477	256718	256958	257198	257439
181	257679	257918	258158	258398	258637	258877	259116	259355	259594	259833
182	260071	260310	260548	260787	261025	261263	261501	261738	261976	262214
183	262451	262688	262925	263162	263399	263636	263873	264109	264345	264581
184	264818	265054	265290	265525	265761	265996	266232	266467	266702	266937
185	267172	267406	267641	267875	268110	268344	268578	268812	269046	269279
186	269513	269746	269980	270213	270446	270679	270912	271144	271377	271609
187	271842	272074	272306	272538	272770	273001	273233	273464	273696	273927
188	274158	274389	274620	274850	275081	275311	275542	275772	276002	276232
189	276462	276691	276921	277151	277380	277609	277838	278067	278296	278525
	0	1	2	3	4	5	6	7	8	9

LOGARITHMS.

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No.	0	1	2	3	4	5	6	7	8	9
190	278754	278982	279210	279439	279667	279895	280123	280351	280578	280806
191	281033	281261	281488	281715	281942	282169	282395	282622	282849	283075
192	283301	283527	283753	283979	284205	284431	284656	284882	285107	285332
193	285557	285782	286007	286232	286456	286681	286905	287130	287354	287578
194	287805	288025	288249	288473	288696	288920	289143	289366	289589	289811
196	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034
195	292256	292478	292699	292920	293141	293362	293583	293804	294025	294246
197	294466	294687	294907	295127	295347	295567	295787	296007	296226	296446
198	296665	296884	297104	297323	297542	297760	297979	298198	298416	298635
199	298853	299071	299289	299507	299725	299943	300160	300378	300595	300813
200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980
201	303196	303412	303628	303844	304059	304275	304490	304706	304921	305136
202	305351	305566	305781	305996	306210	306425	306639	306854	307068	307282
203	307496	307710	307924	308137	308351	308564	308778	308991	309204	309417
204	309630	309843	310056	310268	310481	310693	310906	311118	311330	311542
205	311754	311966	312177	312389	312600	312812	313023	313234	313445	313656
206	313867	314078	314289	314499	314710	314920	315130	315340	315550	315760
207	315970	316180	316390	316599	316809	317018	317227	317436	317645	317854
208	318063	318272	318481	318689	318898	319106	319314	319523	319730	319938
209	320146	320354	320562	320769	320977	321184	321391	321598	321805	322011
210	322219	322426	322633	322839	323046	323252	323458	323664	323871	324077
211	324282	324488	324694	324899	325105	325310	325516	325721	325926	326131
212	326336	326541	326745	326950	327154	327359	327563	327767	327972	328176
213	328380	328583	328787	328991	329194	329398	329601	329804	330008	330211
214	330414	330617	330819	331022	331225	331427	331629	331832	332034	332236
215	332438	332640	332842	333044	333246	333447	333649	333850	334051	334253
216	334454	334655	334856	335056	335257	335458	335658	335859	336059	336259
217	336460	336660	336860	337060	337260	337459	337659	337858	338058	338257
218	338456	338656	338855	339054	339253	339451	339650	339849	340047	340246
219	340444	340642	340840	341039	341237	341434	341632	341830	342028	342225
220	342423	342620	342817	343014	343212	343409	343605	343802	343999	344196
221	344392	344589	344785	344981	345178	345374	345570	345766	345961	346157
222	346353	346549	346744	346939	347135	347330	347525	347720	347915	348110
223	348305	348500	348694	348889	349083	349277	349472	349666	349860	350054
224	350248	350442	350636	350829	351023	351216	351500	351693	351886	352079
225	352182	352375	352568	352761	352954	353146	353339	353532	353724	353916
226	354108	354301	354493	354684	354876	355068	355260	355451	355643	355834
227	356026	356217	356408	356599	356790	356981	357172	357363	357554	357744
228	357935	358125	358316	358506	358696	358886	359076	359266	359456	359646
229	359835	360025	360215	360404	360593	360783	360972	361161	361350	361539
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424
231	363612	363800	363988	364176	364363	364551	364739	364926	365113	365301
232	365488	365675	365862	366049	366236	366423	366609	366796	366983	367169
233	367353	367542	367728	367915	368101	368287	368473	368659	368844	369030
234	369216	369401	369587	369772	369958	370143	370328	370513	370698	370883
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No.	0	1	2	3	4	5	6	7	8	9
235	371068	371253	371437	371622	371806	371991	372175	372360	372544	372728
236	372912	373096	373280	373464	373647	373831	374015	374198	374382	374565
237	374748	374931	375115	375298	375481	375664	375846	376029	376213	376394
238	376577	376759	376942	377124	377306	377488	377670	377852	378034	378216
239	378398	378580	378761	378943	379124	379305	379487	379668	379849	380030
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837
241	382017	382197	382377	382557	382737	382917	383097	383277	383456	383636
242	383815	383995	384174	384353	384533	384712	384891	385070	385249	385427
243	385606	385785	385964	386142	386321	386499	386677	386855	387034	387212
244	387390	387568	387746	387923	388101	388279	388456	388634	388811	388989
245	389166	389343	389520	389697	389874	390051	390228	390405	390582	390758
246	390935	391112	391288	391464	391641	391817	391993	392169	392345	392521
247	392697	392873	393048	393224	393400	393575	393751	393926	394101	394276
248	394452	394627	394802	394977	395152	395326	395501	395676	395850	396025
249	396199	396374	396548	396722	396896	397070	397245	397418	397592	397766
250	397940	398114	398287	398461	398634	398808	398981	399154	399327	399501
251	399674	399847	400020	400192	400365	400538	400711	400883	401056	401228
252	401400	401573	401745	401917	402089	402261	402433	402605	402777	402949
253	403120	403292	403464	403635	403807	403978	404149	404320	404492	404663
254	404834	405005	405175	405346	405517	405688	405858	406029	406199	406369
255	406540	406710	406881	407051	407221	407391	407561	407731	407900	408070
256	408240	408410	408579	408749	408918	409087	409257	409426	409595	409764
257	409933	410102	410271	410440	410608	410777	410946	411114	411283	411451
258	411620	411788	411956	412124	412292	412460	412628	412796	412964	413132
259	413300	413467	413635	413802	413970	414137	414305	414472	414639	414806
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474
261	416640	416807	416973	417139	417306	417472	417638	417804	417970	418135
262	418301	418467	418633	418798	418964	419129	419295	419460	419625	419791
263	419956	420121	420286	420451	420616	420781	420945	421110	421275	421439
264	421604	421768	421933	422097	422261	422426	422590	422754	422918	423082
265	423246	423410	423573	423737	423901	424064	424228	424391	424555	424718
266	424882	425045	425208	425371	425534	425697	425860	426023	426186	426349
267	426511	426674	426836	426999	427161	427324	427486	427648	427811	427973
268	428135	428297	428459	428621	428782	428944	429106	429268	429429	429591
269	429752	429914	430075	430236	430398	430559	430720	430881	431042	431203
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809
271	432969	433129	433289	433450	433610	433770	433930	434090	434249	434409
272	434569	434728	434888	435048	435207	435366	435526	435685	435844	436003
273	436163	436322	436481	436640	436798	436957	437116	437275	437433	437592
274	437751	437909	438067	438226	438384	438542	438700	438859	439017	439175
275	439333	439491	439648	439806	439964	440122	440279	440437	440594	440752
276	440909	441066	441224	441381	441538	441695	441852	442009	442166	442323
277	442480	442636	442793	442950	443106	443263	443419	443576	443732	443888
278	444045	444201	444357	444513	444669	444825	444981	445137	445293	445448
279	445604	445760	445915	446071	446226	446382	446537	446692	446848	447003
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281	448706	448861	449015	449170	449324	449478	449633	449787	449941	450095
282	450249	450403	450557	450711	450865	451018	451172	451326	451479	451633
283	451786	451940	452093	452247	452400	452553	452706	452859	453012	453165
284	453318	453471	453624	453777	453930	454082	454235	454387	454540	454692
285	454845	454997	455149	455302	455454	455606	455758	455910	456062	456214
286	456366	456518	456670	456821	456973	457125	457276	457428	457579	457730
287	457882	458033	458184	458336	458487	458638	458789	458940	459091	459242
288	459392	459543	459694	459845	459995	460146	460296	460447	460597	460748
289	460898	461048	461198	461348	461498	461649	461799	461948	462098	462248
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744
291	463893	464042	464191	464340	464489	464639	464787	464936	465085	465234
292	465383	465532	465680	465829	465977	466126	466274	466423	466571	466719
293	466868	467016	467164	467312	467460	467608	467756	467904	468052	468200
294	468347	468495	468643	468790	468938	469085	469233	469380	469527	469675
295	469822	469969	470116	470263	470410	470557	470704	470851	470998	471145
296	471292	471438	471585	471732	471878	472025	472171	472317	472464	472610
297	472756	472903	473049	473195	473341	473487	473633	473779	473925	474070
298	474216	474362	474508	474653	474799	474944	475090	475235	475381	475526
299	475671	475816	475962	476107	476252	476397	476542	476687	476832	476976
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422
301	478566	478711	478855	478999	479143	479287	479431	479575	479719	479863
302	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299
303	481443	481586	481729	481872	482016	482159	482302	482445	482588	482731
304	482874	483016	483159	483302	483445	483587	483730	483872	484015	484157
305	484300	484442	484584	484727	484869	485011	485153	485295	485437	485579
306	485721	485863	486005	486147	486289	486430	486572	486714	486855	486997
307	487138	487280	487421	487563	487704	487845	487986	488127	488269	488410
308	488551	488692	488833	488973	489114	489255	489396	489537	489677	489818
309	489958	490099	490239	490380	490520	490661	490801	490941	491081	491222
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621
311	492760	492900	493040	493179	493319	493458	493597	493737	493876	494015
312	494155	494294	494433	494572	494711	494850	494989	495128	495267	495406
313	495544	495683	495822	495960	496099	496237	496376	496514	496653	496791
314	496930	497068	497206	497344	497482	497621	497759	497897	498035	498173
315	498311	498448	498586	498724	498862	498999	499137	499275	499412	499550
316	499687	499824	499962	500099	500236	500374	500511	500648	500785	500922
317	501059	501196	501333	501470	501607	501744	501880	502017	502154	502290
318	502427	502564	502700	502837	502973	503109	503246	503382	503518	503654
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323	509202	509337	509471	509606	509740	509874	510008	510143	510277	510411
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326	513218	513351	513484	513617	513750	513883	514015	514149	514282	514415
327	514548	514680	514813	514946	515079	515211	515344	515476	515609	515741
328	515874	516006	516139	516271	516403	516535	516668	516800	516932	517064
329	517196	517328	517460	517592	517724	517855	517987	518119	518251	518382
330	518514	518645	518777	518909	519040	519171	519303	519434	519565	519697
331	519828	519959	520090	520221	520352	520483	520614	520745	520876	521007
332	521138	521269	521400	521530	521661	521792	521922	522053	522183	522314
333	522444	522575	522705	522835	522966	523096	523226	523356	523486	523616
334	523746	523876	524006	524136	524266	524396	524526	524656	524785	524915
335	525045	525174	525304	525433	525563	525692	525822	525951	526081	526210
336	526339	526468	526598	526727	526856	526985	527114	527243	527372	527501
337	527630	527759	527888	528016	528145	528274	528402	528531	528660	528788
338	528917	529045	529174	529302	529430	529559	529687	529815	529943	530072
339	530200	530328	530456	530584	530712	530840	530968	531095	531223	531351
340	531479	531607	531734	531862	531989	532117	532245	532372	532500	532627
341	532754	532882	533009	533136	533263	533391	533518	533645	533772	533899
342	534026	534153	534280	534407	534534	534661	534787	534914	535041	535167
343	535294	535421	535547	535674	535800	535927	536053	536179	536306	536432
344	536558	536685	536811	536937	537063	537189	537315	537441	537567	537693
345	537819	537945	538071	538197	538322	538448	538574	538699	538825	538951
346	539076	539202	539327	539452	539578	539703	539828	539954	540079	540204
347	540329	540455	540580	540705	540830	540955	541080	541205	541330	541454
348	541579	541704	541829	541953	542078	542203	542327	542452	542576	542701
349	542825	542950	543074	543199	543323	543447	543571	543696	543820	543944
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183
351	545307	545431	545554	545678	545802	545925	546049	546172	546296	546419
352	546543	546666	546789	546913	547036	547159	547282	547405	547528	547652
353	547775	547898	548021	548144	548266	548389	548512	548635	548759	548881
354	549003	549126	549249	549371	549494	549616	549739	549861	549984	550106
355	550228	550351	550473	550595	550717	550840	550962	551084	551206	551328
356	551450	551572	551694	551816	551938	552059	552181	552303	552425	552546
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358	553883	554004	554126	554247	554368	554489	554610	554731	554852	554973
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362	558709	558828	558948	559068	559188	559308	559428	559548	559667	559787
363	559907	560026	560146	560265	560385	560504	560624	560743	560863	560982
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365	562293	562412	562531	562650	562768	562887	563006	563125	563244	563362
366	563481	563600	563718	563837	563955	564074	564192	564311	564429	564548
367	564666	564784	564903	565021	565139	565257	565375	565494	565612	565730
368	565848	565966	566084	566202	566320	566437	566555	566673	566791	566909
369	567026	567144	567262	567379	567497	567614	567732	567849	567967	568084
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372	570543	570660	570776	570893	571010	571126	571243	571359	571476	571592
373	571709	571825	571942	572058	572174	572291	572407	572523	572639	572755
374	572872	572988	573104	573220	573336	573452	573568	573684	573800	573915
375	574031	574147	574263	574379	574494	574610	574726	574841	574957	575072
376	575188	575303	575419	575534	575650	575765	575880	575996	576111	576226
377	576341	576456	576572	576687	576802	576917	577032	577147	577262	577377
378	577492	577607	577721	577836	577951	578066	578181	578295	578410	578525
379	578639	578754	578868	578983	579097	579212	579326	579441	579555	579669
380	579784	579898	580012	580126	580240	580355	580469	580583	580697	580811
381	580925	581039	581153	581267	581381	581495	581608	581722	581836	581950
382	582063	582177	582291	582404	582518	582631	582745	582858	582972	583085
383	583199	583312	583425	583539	583652	583765	583879	583992	584105	584218
384	584331	584444	584557	584670	584783	584896	585009	585122	585235	585348
385	585461	585573	585686	585799	585912	586024	586137	586250	586363	586475
386	586587	586700	586812	586925	587037	587149	587262	587374	587486	587599
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388	588832	588944	589055	589167	589279	589391	589503	589614	589726	589838
389	589950	590061	590173	590284	590396	590507	590619	590730	590842	590953
390	591065	591176	591287	591398	591510	591621	591732	591843	591955	592066
391	592177	592288	592399	592510	592621	592732	592843	592954	593064	593175
392	593286	593397	593508	593618	593729	593840	593950	594061	594171	594282
393	594392	594503	594613	594724	594834	594945	595055	595165	595276	595386
394	595496	595606	595717	595827	595937	596047	596157	596267	596377	596487
395	596597	596707	596817	596927	597037	597146	597256	597366	597476	597585
396	597695	597805	597914	598024	598134	598243	598353	598462	598572	598681
397	598790	598900	599009	599119	599228	599337	599446	599556	599665	599774
398	599883	599992	600101	600210	600319	600428	600537	600646	600755	600864
399	600973	601082	601190	601299	601408	601517	601625	601734	601843	601951
400	602060	602168	602277	602386	602494	602602	602711	602819	602928	603036
401	603144	603253	603361	603469	603577	603685	603794	603902	604010	604118
402	604226	604334	604442	604550	604658	604766	604874	604982	605089	605197
403	605305	605413	605520	605628	605736	605843	605951	606059	606166	606274
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405	607455	607562	607669	607777	607884	607991	608098	608205	608312	608419
406	608526	608633	608740	608847	608954	609060	609176	609281	609387	609494
407	609594	609701	609808	609914	610021	610128	610234	610341	610447	610554
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409	611723	611829	611936	612042	612148	612254	612360	612467	612572	612678
410	612784	612890	612996	613102	613207	613313	613419	613525	613631	613736
411	613842	613947	614053	614159	614264	614370	614475	614581	614686	614792
412	614897	615003	615108	615213	615319	615424	615529	615634	615740	615845
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417	620136	620240	620344	620448	620552	620656	620760	620864	620968	621072
418	621176	621280	621384	621488	621592	621695	621799	621903	622007	622110
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420	623249	623353	623456	623559	623663	623766	623869	623972	624076	624179
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422	625312	625415	625518	625621	625724	625827	625929	626032	626135	626238
423	626340	626443	626546	626648	626751	626853	626956	627058	627161	627263
424	627366	627468	627571	627673	627775	627878	627980	628082	628184	628287
425	628389	628491	628593	628695	628797	628900	629002	629104	629206	629308
426	629410	629511	629613	629715	629817	629919	630021	630123	630224	630326
427	630428	630529	630631	630733	630834	630936	631038	631139	631241	631342
428	631444	631545	631647	631748	631849	631951	632052	632153	632255	632356
429	632457	632558	632660	632761	632862	632963	633064	633165	633266	633367
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431	634477	634578	634679	634779	634880	634981	635081	635182	635283	635383
432	635484	635584	635685	635785	635886	635986	636086	636187	636287	636388
433	636488	636588	636688	636789	636889	636989	637089	637189	637289	637390
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436	639486	639586	639686	639785	639885	639984	640084	640183	640283	640382
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443	646404	646502	646600	646698	646796	646894	646991	647089	647187	647285
444	647383	647481	647578	647676	647774	647872	647969	648067	648165	648262
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447	650307	650405	650502	650599	650696	650793	650890	650987	651084	651181
448	651278	651375	651472	651569	651666	651762	651859	651956	652053	652150
449	652246	652343	652440	652536	652633	652730	652826	652923	653019	653116
450	653212	653309	653405	653502	653598	653695	653791	653888	653984	654080
451	654176	654273	654369	654465	654562	654658	654754	654850	654946	655042
452	655133	655234	655331	655427	655523	655619	655714	655810	655906	656002
453	656098	656194	656290	656386	656481	656577	656673	656769	656864	656960
454	657056	657151	657247	657343	657438	657534	657629	657725	657820	657916
455	658011	658107	658202	658298	658393	658488	658584	658679	658774	658870
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457	659916	660010	660103	660201	660296	660391	660486	660581	660676	660771
458	660865	660960	661055	661150	661245	661339	661434	661529	661623	661718
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462	664642	664736	664830	664924	665018	665112	665206	665299	665393	665487
463	665581	665675	665769	665862	665956	666050	666143	666237	666331	666424
464	666518	666612	666705	666799	666892	666986	667079	667173	667266	667359
465	667453	667546	667640	667733	667826	667920	668013	668106	668199	668293
466	668386	668479	668572	668665	668758	668852	668945	669038	669131	669224
467	669317	669410	669503	669596	669689	669782	669874	669967	670060	670153
468	670246	670339	670431	670524	670617	670710	670802	670895	670988	671080
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484	684845	684935	685025	685114	685204	685294	685383	685473	685563	685652
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866	937518	937568	937618	937668	937718	937769	937819	937869	937919	937969
867	938019	938069	938119	938169	938219	938269	938319	938370	938420	938470
868	938520	938570	938620	938670	938720	938770	938820	938870	938920	938970
869	939020	939070	939120	939170	939220	939270	939319	939369	939419	939469
870	939519	939569	939619	939669	939719	939769	939819	939868	939918	939968
871	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467
872	940516	940566	940616	940666	940716	940765	940815	940865	940915	940964
873	941014	941064	941114	941163	941213	941263	941313	941362	941412	941462
874	941511	941561	941611	941660	941710	941760	941809	941859	941909	941958
875	942008	942058	942107	942157	942206	942256	942306	942355	942405	942454
876	942504	942554	942603	942653	942702	942752	942801	942851	942900	942950
877	943000	943049	943099	943148	943198	943247	943297	943346	943396	943445
878	943494	943544	943593	943643	943692	943742	943791	943841	943890	943939
879	943989	944038	944088	944137	944186	944236	944285	944335	944384	944433
880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927
881	944976	945025	945074	945124	945173	945222	945272	945321	945370	945419
882	945469	945518	945567	945616	945665	945715	945764	945813	945862	945911
883	945961	946010	946059	946108	946157	946207	946256	946305	946354	946403
884	946452	946501	946550	946600	946649	946698	946747	946796	946845	946894
885	946943	946992	947041	947090	947139	947189	947238	947287	947336	947385
886	947434	947483	947532	947581	947630	947679	947728	947777	947826	947875
887	947924	947973	948021	948070	948119	948168	948217	948266	948315	948364
888	948413	948462	948511	948560	948608	948657	948706	948755	948804	948853
889	948902	948951	948999	949048	949097	949146	949195	949244	949292	949341
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894	951337	951386	951435	951483	951532	951580	951629	951677	951726	951774
895	951823	951872	951920	951969	952017	952066	952114	952163	952211	952259
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898	953276	953325	953373	953421	953470	953518	953566	953615	953663	953711
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901	954725	954773	954821	954869	954918	954966	955014	955062	955110	955158
902	955206	955255	955303	955351	955399	955447	955495	955543	955591	955640
903	955688	955736	955784	955832	955880	955928	955976	956024	956072	956120
904	956168	956216	956264	956312	956360	956409	956457	956505	956553	956601
905	956649	956697	956744	956792	956840	956888	956936	956984	957032	957080
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907	957607	957655	957703	957751	957799	957847	957894	957942	957990	958038
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916	961895	961943	961990	962038	962085	962132	962180	962227	962275	962322
917	962369	962417	962464	962511	962559	962606	962653	962701	962748	962796
918	962843	962890	962937	962985	963032	963079	963126	963174	963221	963268
919	963315	963363	963410	963457	963504	963552	963599	963646	963693	963741
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212
921	964260	964307	964354	964401	964448	964495	964542	964590	964637	964684
922	964731	964778	964825	964872	964919	964966	965013	965060	965108	965155
923	965202	965249	965296	965343	965390	965437	965484	965531	965578	965625
924	965672	965719	965766	965813	965860	965907	965954	966001	966048	966095
925	966142	966189	966236	966283	966329	966376	966423	966470	966517	966564
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940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543
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942	974051	974097	974143	974189	974235	974281	974327	974373	974420	974466
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951	978180	978226	978272	978317	978363	978409	978454	978500	978546	978591
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953	979093	979138	979184	979229	979275	979321	979366	979412	979457	979503
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958	981365	981411	981456	981501	981547	981592	981637	981683	981728	981773
959	981819	981864	981909	981954	982000	982045	982090	982135	982181	982226
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961	982723	982769	982814	982859	982904	982949	982994	983040	983085	983130
962	983175	983220	983265	983310	983356	983401	983446	983491	983536	983581
963	983626	983671	983716	983762	983807	983852	983897	983942	983987	984032
964	984077	984122	984167	984212	984257	984302	984347	984392	984437	984482
965	984527	984572	984617	984662	984707	984752	984797	984842	984887	984932
966	984977	985022	985067	985112	985157	985202	985247	985292	985337	985382
967	985426	985471	985516	985561	985606	985651	985696	985741	985786	985830
968	985875	985920	985965	986010	986055	986100	986144	986189	986234	986279
969	986324	986369	986413	986458	986503	986548	986593	986637	986682	986727
970	986772	986816	986861	986906	986951	986995	987040	987085	987130	987174
971	987219	987264	987309	987353	987398	987443	987487	987532	987577	987622
972	987666	987711	987756	987800	987845	987890	987934	987979	988024	988068
973	988113	988157	988202	988247	988291	988336	988381	988425	988470	988514
974	988559	988603	988648	988693	988737	988782	988826	988871	988915	988960
975	989005	989049	989094	989138	989183	989227	989272	989316	989361	989405
976	989450	989494	989539	989583	989628	989672	989717	989761	989806	989850
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981	991669	991713	991757	991802	991846	991890	991934	991979	992023	992067
982	992111	992156	992200	992244	992288	992333	992377	992421	992465	992509
983	992553	992598	992642	992686	992730	992774	992818	992863	992907	992951
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985	993437	993481	993524	993568	993613	993657	993701	993745	993789	993833
986	993877	993921	993965	994009	994053	994097	994141	994185	994229	994273
987	994317	994361	994405	994449	994493	994537	994581	994625	994669	994713
988	994757	994801	994845	994889	994933	994977	995021	995064	995108	995152
989	995196	995240	995284	995328	995372	995416	995460	995504	995547	995591
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030
991	996074	996117	996161	996205	996249	996293	996336	996380	996424	996468
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994	997386	997430	997474	997517	997561	997605	997648	997692	997736	997779
995	997823	997867	997910	997954	997998	998041	998085	998128	998172	998216
996	998259	998303	998346	998390	998434	998477	998521	998564	998608	998652
997	998695	998739	998782	998826	998869	998913	998956	999000	999043	999087
998	999130	999174	999218	999261	999305	999348	999392	999435	999478	999522
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A TRIANGULAR CANON LOGARITHMICAL: or, A TABLE of ARITHMETICAL, TANGENTS and SECANTS, the Radius 10.000000; and to every Degree of the QUADRANT.

D. Degr.										L. Degr.									
Sine					Tang.					Sec. nt.					Sine				
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A Table of Artificial Sines, Tangents and Secants.

2 Degrees.

M	Sine.	Tang.	Secant.
0			
1	8.5428 9	6.999735	11.456916
2	8.54622	6.999731	11.456916
3	8.54995	6.999726	11.456916
4	8.55359	6.999722	11.456916
5	8.55704	6.999717	11.456916
6	8.56040	6.999712	11.456916
7	8.56399	6.999708	11.456916
8	8.56741	6.999704	11.456916
9	8.57086	6.999700	11.456916
10	8.57424	6.999696	11.456916
11	8.57766	6.999692	11.456916
12	8.58102	6.999688	11.456916
13	8.58441	6.999684	11.456916
14	8.58772	6.999680	11.456916
15	8.59106	6.999676	11.456916
16	8.59439	6.999672	11.456916
17	8.59775	6.999668	11.456916
18	8.60103	6.999664	11.456916
19	8.60439	6.999660	11.456916
20	8.60768	6.999656	11.456916
21	8.61094	6.999652	11.456916
22	8.61423	6.999648	11.456916
23	8.61749	6.999644	11.456916
24	8.62074	6.999640	11.456916
25	8.62399	6.999636	11.456916
26	8.62724	6.999632	11.456916
27	8.63049	6.999628	11.456916
28	8.63374	6.999624	11.456916
29	8.63699	6.999620	11.456916
30	8.64024	6.999616	11.456916
31	8.64349	6.999612	11.456916
32	8.64674	6.999608	11.456916
33	8.64999	6.999604	11.456916
34	8.65324	6.999600	11.456916
35	8.65649	6.999596	11.456916
36	8.65974	6.999592	11.456916
37	8.66299	6.999588	11.456916
38	8.66624	6.999584	11.456916
39	8.66949	6.999580	11.456916
40	8.67274	6.999576	11.456916
41	8.67599	6.999572	11.456916
42	8.67924	6.999568	11.456916
43	8.68249	6.999564	11.456916
44	8.68574	6.999560	11.456916
45	8.68899	6.999556	11.456916
46	8.69224	6.999552	11.456916
47	8.69549	6.999548	11.456916
48	8.69874	6.999544	11.456916
49	8.70199	6.999540	11.456916
50	8.70524	6.999536	11.456916
51	8.70849	6.999532	11.456916
52	8.71174	6.999528	11.456916
53	8.71499	6.999524	11.456916
54	8.71824	6.999520	11.456916
55	8.72149	6.999516	11.456916
56	8.72474	6.999512	11.456916
57	8.72799	6.999508	11.456916
58	8.73124	6.999504	11.456916
59	8.73449	6.999500	11.456916
60	8.73774	6.999496	11.456916

87 Degrees.

3 Degrees.

M	Sine.	Tang.	Secant.
0			
1	8.74100	6.999492	11.456916
2	8.74425	6.999488	11.456916
3	8.74750	6.999484	11.456916
4	8.75075	6.999480	11.456916
5	8.75400	6.999476	11.456916
6	8.75725	6.999472	11.456916
7	8.76050	6.999468	11.456916
8	8.76375	6.999464	11.456916
9	8.76700	6.999460	11.456916
10	8.77025	6.999456	11.456916
11	8.77350	6.999452	11.456916
12	8.77675	6.999448	11.456916
13	8.78000	6.999444	11.456916
14	8.78325	6.999440	11.456916
15	8.78650	6.999436	11.456916
16	8.78975	6.999432	11.456916
17	8.79300	6.999428	11.456916
18	8.79625	6.999424	11.456916
19	8.79950	6.999420	11.456916
20	8.80275	6.999416	11.456916
21	8.80600	6.999412	11.456916
22	8.80925	6.999408	11.456916
23	8.81250	6.999404	11.456916
24	8.81575	6.999400	11.456916
25	8.81900	6.999396	11.456916
26	8.82225	6.999392	11.456916
27	8.82550	6.999388	11.456916
28	8.82875	6.999384	11.456916
29	8.83200	6.999380	11.456916
30	8.83525	6.999376	11.456916
31	8.83850	6.999372	11.456916
32	8.84175	6.999368	11.456916
33	8.84500	6.999364	11.456916
34	8.84825	6.999360	11.456916
35	8.85150	6.999356	11.456916
36	8.85475	6.999352	11.456916
37	8.85800	6.999348	11.456916
38	8.86125	6.999344	11.456916
39	8.86450	6.999340	11.456916
40	8.86775	6.999336	11.456916
41	8.87100	6.999332	11.456916
42	8.87425	6.999328	11.456916
43	8.87750	6.999324	11.456916
44	8.88075	6.999320	11.456916
45	8.88400	6.999316	11.456916
46	8.88725	6.999312	11.456916
47	8.89050	6.999308	11.456916
48	8.89375	6.999304	11.456916
49	8.89700	6.999300	11.456916
50	8.90025	6.999296	11.456916
51	8.90350	6.999292	11.456916
52	8.90675	6.999288	11.456916
53	8.91000	6.999284	11.456916
54	8.91325	6.999280	11.456916
55	8.91650	6.999276	11.456916
56	8.91975	6.999272	11.456916
57	8.92300	6.999268	11.456916
58	8.92625	6.999264	11.456916
59	8.92950	6.999260	11.456916
60	8.93275	6.999256	11.456916

86 Degrees.

A Table of Artificial Sines, Tangents and Secants.

53

4 Degrees.

Min.	Sine	Tang.	Secant.
0	8.833384	9.018941	11.551556
1	8.833387	9.018932	11.551545
2	8.833391	9.018923	11.551534
3	8.833395	9.018914	11.551523
4	8.833397	9.018905	11.551512
5	8.833401	9.018896	11.551501
6	8.833405	9.018887	11.551490
7	8.833409	9.018878	11.551479
8	8.833413	9.018869	11.551468
9	8.833417	9.018860	11.551457
10	8.833421	9.018851	11.551446
11	8.833425	9.018842	11.551435
12	8.833429	9.018833	11.551424
13	8.833433	9.018824	11.551413
14	8.833437	9.018815	11.551402
15	8.833441	9.018806	11.551391
16	8.833445	9.018797	11.551380
17	8.833449	9.018788	11.551369
18	8.833453	9.018779	11.551358
19	8.833457	9.018770	11.551347
20	8.833461	9.018761	11.551336
21	8.833465	9.018752	11.551325
22	8.833469	9.018743	11.551314
23	8.833473	9.018734	11.551303
24	8.833477	9.018725	11.551292
25	8.833481	9.018716	11.551281
26	8.833485	9.018707	11.551270
27	8.833489	9.018698	11.551259
28	8.833493	9.018689	11.551248
29	8.833497	9.018680	11.551237
30	8.833501	9.018671	11.551226
31	8.833505	9.018662	11.551215
32	8.833509	9.018653	11.551204
33	8.833513	9.018644	11.551193
34	8.833517	9.018635	11.551182
35	8.833521	9.018626	11.551171
36	8.833525	9.018617	11.551160
37	8.833529	9.018608	11.551149
38	8.833533	9.018599	11.551138
39	8.833537	9.018590	11.551127
40	8.833541	9.018581	11.551116
41	8.833545	9.018572	11.551105
42	8.833549	9.018563	11.551094
43	8.833553	9.018554	11.551083
44	8.833557	9.018545	11.551072
45	8.833561	9.018536	11.551061
46	8.833565	9.018527	11.551050
47	8.833569	9.018518	11.551039
48	8.833573	9.018509	11.551028
49	8.833577	9.018500	11.551017
50	8.833581	9.018491	11.551006
51	8.833585	9.018482	11.550995
52	8.833589	9.018473	11.550984
53	8.833593	9.018464	11.550973
54	8.833597	9.018455	11.550962
55	8.833601	9.018446	11.550951
56	8.833605	9.018437	11.550940
57	8.833609	9.018428	11.550929
58	8.833613	9.018419	11.550918
59	8.833617	9.018410	11.550907
60	8.833621	9.018401	11.550896

85 Degrees.

5 Degrees.

Min.	Sine	Tang.	Secant.
0	8.942360	9.028344	11.559488
1	8.942363	9.028335	11.559477
2	8.942367	9.028326	11.559466
3	8.942371	9.028317	11.559455
4	8.942375	9.028308	11.559444
5	8.942379	9.028299	11.559433
6	8.942383	9.028290	11.559422
7	8.942387	9.028281	11.559411
8	8.942391	9.028272	11.559400
9	8.942395	9.028263	11.559389
10	8.942399	9.028254	11.559378
11	8.942403	9.028245	11.559367
12	8.942407	9.028236	11.559356
13	8.942411	9.028227	11.559345
14	8.942415	9.028218	11.559334
15	8.942419	9.028209	11.559323
16	8.942423	9.028200	11.559312
17	8.942427	9.028191	11.559301
18	8.942431	9.028182	11.559290
19	8.942435	9.028173	11.559279
20	8.942439	9.028164	11.559268
21	8.942443	9.028155	11.559257
22	8.942447	9.028146	11.559246
23	8.942451	9.028137	11.559235
24	8.942455	9.028128	11.559224
25	8.942459	9.028119	11.559213
26	8.942463	9.028110	11.559202
27	8.942467	9.028101	11.559191
28	8.942471	9.028092	11.559180
29	8.942475	9.028083	11.559169
30	8.942479	9.028074	11.559158
31	8.942483	9.028065	11.559147
32	8.942487	9.028056	11.559136
33	8.942491	9.028047	11.559125
34	8.942495	9.028038	11.559114
35	8.942499	9.028029	11.559103
36	8.942503	9.028020	11.559092
37	8.942507	9.028011	11.559081
38	8.942511	9.028002	11.559070
39	8.942515	9.027993	11.559059
40	8.942519	9.027984	11.559048
41	8.942523	9.027975	11.559037
42	8.942527	9.027966	11.559026
43	8.942531	9.027957	11.559015
44	8.942535	9.027948	11.559004
45	8.942539	9.027939	11.558993
46	8.942543	9.027930	11.558982
47	8.942547	9.027921	11.558971
48	8.942551	9.027912	11.558960
49	8.942555	9.027903	11.558949
50	8.942559	9.027894	11.558938
51	8.942563	9.027885	11.558927
52	8.942567	9.027876	11.558916
53	8.942571	9.027867	11.558905
54	8.942575	9.027858	11.558894
55	8.942579	9.027849	11.558883
56	8.942583	9.027840	11.558872
57	8.942587	9.027831	11.558861
58	8.942591	9.027822	11.558850
59	8.942595	9.027813	11.558839
60	8.942599	9.027804	11.558828

84 Degrees.

A Table of Artificial Sines, Tangents and Secants.

6 Degrees.

M.in.	Sine.	Tang.	Secant.
0	0.997235	0.997614	10.978380
1	0.997235	0.997601	10.978380
2	0.997235	0.997588	10.978380
3	0.997235	0.997574	10.978380
4	0.997235	0.997561	10.978380
5	0.997235	0.997547	10.978380
6	0.997235	0.997534	10.978380
7	0.997235	0.997520	10.978380
8	0.997235	0.997507	10.978380
9	0.997235	0.997493	10.978380
10	0.997235	0.997480	10.978380
11	0.997235	0.997466	10.978380
12	0.997235	0.997453	10.978380
13	0.997235	0.997439	10.978380
14	0.997235	0.997425	10.978380
15	0.997235	0.997412	10.978380
16	0.997235	0.997398	10.978380
17	0.997235	0.997385	10.978380
18	0.997235	0.997371	10.978380
19	0.997235	0.997358	10.978380
20	0.997235	0.997344	10.978380
21	0.997235	0.997331	10.978380
22	0.997235	0.997317	10.978380
23	0.997235	0.997304	10.978380
24	0.997235	0.997290	10.978380
25	0.997235	0.997277	10.978380
26	0.997235	0.997263	10.978380
27	0.997235	0.997250	10.978380
28	0.997235	0.997236	10.978380
29	0.997235	0.997223	10.978380
30	0.997235	0.997209	10.978380
31	0.997235	0.997196	10.978380
32	0.997235	0.997182	10.978380
33	0.997235	0.997169	10.978380
34	0.997235	0.997155	10.978380
35	0.997235	0.997142	10.978380
36	0.997235	0.997128	10.978380
37	0.997235	0.997115	10.978380
38	0.997235	0.997101	10.978380
39	0.997235	0.997088	10.978380
40	0.997235	0.997074	10.978380
41	0.997235	0.997061	10.978380
42	0.997235	0.997047	10.978380
43	0.997235	0.997034	10.978380
44	0.997235	0.997020	10.978380
45	0.997235	0.997007	10.978380
46	0.997235	0.996993	10.978380
47	0.997235	0.996980	10.978380
48	0.997235	0.996966	10.978380
49	0.997235	0.996953	10.978380
50	0.997235	0.996939	10.978380
51	0.997235	0.996926	10.978380
52	0.997235	0.996912	10.978380
53	0.997235	0.996899	10.978380
54	0.997235	0.996885	10.978380
55	0.997235	0.996872	10.978380
56	0.997235	0.996858	10.978380
57	0.997235	0.996845	10.978380
58	0.997235	0.996831	10.978380
59	0.997235	0.996818	10.978380
60	0.997235	0.996804	10.978380

7 Degrees.

M.in.	Sine.	Tang.	Secant.
0	0.997235	0.997614	10.978380
1	0.997235	0.997601	10.978380
2	0.997235	0.997588	10.978380
3	0.997235	0.997574	10.978380
4	0.997235	0.997561	10.978380
5	0.997235	0.997547	10.978380
6	0.997235	0.997534	10.978380
7	0.997235	0.997520	10.978380
8	0.997235	0.997507	10.978380
9	0.997235	0.997493	10.978380
10	0.997235	0.997480	10.978380
11	0.997235	0.997466	10.978380
12	0.997235	0.997453	10.978380
13	0.997235	0.997439	10.978380
14	0.997235	0.997425	10.978380
15	0.997235	0.997412	10.978380
16	0.997235	0.997398	10.978380
17	0.997235	0.997385	10.978380
18	0.997235	0.997371	10.978380
19	0.997235	0.997358	10.978380
20	0.997235	0.997344	10.978380
21	0.997235	0.997331	10.978380
22	0.997235	0.997317	10.978380
23	0.997235	0.997304	10.978380
24	0.997235	0.997290	10.978380
25	0.997235	0.997277	10.978380
26	0.997235	0.997263	10.978380
27	0.997235	0.997250	10.978380
28	0.997235	0.997236	10.978380
29	0.997235	0.997223	10.978380
30	0.997235	0.997209	10.978380
31	0.997235	0.997196	10.978380
32	0.997235	0.997182	10.978380
33	0.997235	0.997169	10.978380
34	0.997235	0.997155	10.978380
35	0.997235	0.997142	10.978380
36	0.997235	0.997128	10.978380
37	0.997235	0.997115	10.978380
38	0.997235	0.997101	10.978380
39	0.997235	0.997088	10.978380
40	0.997235	0.997074	10.978380
41	0.997235	0.997061	10.978380
42	0.997235	0.997047	10.978380
43	0.997235	0.997034	10.978380
44	0.997235	0.997020	10.978380
45	0.997235	0.997007	10.978380
46	0.997235	0.996993	10.978380
47	0.997235	0.996980	10.978380
48	0.997235	0.996966	10.978380
49	0.997235	0.996953	10.978380
50	0.997235	0.996939	10.978380
51	0.997235	0.996926	10.978380
52	0.997235	0.996912	10.978380
53	0.997235	0.996899	10.978380
54	0.997235	0.996885	10.978380
55	0.997235	0.996872	10.978380
56	0.997235	0.996858	10.978380
57	0.997235	0.996845	10.978380
58	0.997235	0.996831	10.978380
59	0.997235	0.996818	10.978380
60	0.997235	0.996804	10.978380

83 Degrees.

82 Degrees.

A Table of Artificial Sines, Tangents and Secants.

55

8 Degrees.

Min.	Sine	Tang.	Secant.
0	14.1555	9.959735	10.832107
1	14.14453	9.959735	10.832107
2	14.13349	9.959735	10.832107
3	14.12244	9.959735	10.832107
4	14.11139	9.959735	10.832107
5	14.10034	9.959735	10.832107
6	13.98929	9.959735	10.832107
7	13.97824	9.959735	10.832107
8	13.96719	9.959735	10.832107
9	13.95614	9.959735	10.832107
10	13.94509	9.959735	10.832107
11	13.93404	9.959735	10.832107
12	13.92299	9.959735	10.832107
13	13.91194	9.959735	10.832107
14	13.90089	9.959735	10.832107
15	13.88984	9.959735	10.832107
16	13.87879	9.959735	10.832107
17	13.86774	9.959735	10.832107
18	13.85669	9.959735	10.832107
19	13.84564	9.959735	10.832107
20	13.83459	9.959735	10.832107
21	13.82354	9.959735	10.832107
22	13.81249	9.959735	10.832107
23	13.80144	9.959735	10.832107
24	13.79039	9.959735	10.832107
25	13.77934	9.959735	10.832107
26	13.76829	9.959735	10.832107
27	13.75724	9.959735	10.832107
28	13.74619	9.959735	10.832107
29	13.73514	9.959735	10.832107
30	13.72409	9.959735	10.832107
31	13.71304	9.959735	10.832107
32	13.70199	9.959735	10.832107
33	13.69094	9.959735	10.832107
34	13.67989	9.959735	10.832107
35	13.66884	9.959735	10.832107
36	13.65779	9.959735	10.832107
37	13.64674	9.959735	10.832107
38	13.63569	9.959735	10.832107
39	13.62464	9.959735	10.832107
40	13.61359	9.959735	10.832107
41	13.60254	9.959735	10.832107
42	13.59149	9.959735	10.832107
43	13.58044	9.959735	10.832107
44	13.56939	9.959735	10.832107
45	13.55834	9.959735	10.832107
46	13.54729	9.959735	10.832107
47	13.53624	9.959735	10.832107
48	13.52519	9.959735	10.832107
49	13.51414	9.959735	10.832107
50	13.50309	9.959735	10.832107
51	13.49204	9.959735	10.832107
52	13.48099	9.959735	10.832107
53	13.46994	9.959735	10.832107
54	13.45889	9.959735	10.832107
55	13.44784	9.959735	10.832107
56	13.43679	9.959735	10.832107
57	13.42574	9.959735	10.832107
58	13.41469	9.959735	10.832107
59	13.40364	9.959735	10.832107
60	13.39259	9.959735	10.832107

9 Degrees.

Min.	Sine	Tang.	Secant.
0	14.1555	9.959735	10.832107
1	14.14453	9.959735	10.832107
2	14.13349	9.959735	10.832107
3	14.12244	9.959735	10.832107
4	14.11139	9.959735	10.832107
5	14.10034	9.959735	10.832107
6	13.98929	9.959735	10.832107
7	13.97824	9.959735	10.832107
8	13.96719	9.959735	10.832107
9	13.95614	9.959735	10.832107
10	13.94509	9.959735	10.832107
11	13.93404	9.959735	10.832107
12	13.92299	9.959735	10.832107
13	13.91194	9.959735	10.832107
14	13.90089	9.959735	10.832107
15	13.88984	9.959735	10.832107
16	13.87879	9.959735	10.832107
17	13.86774	9.959735	10.832107
18	13.85669	9.959735	10.832107
19	13.84564	9.959735	10.832107
20	13.83459	9.959735	10.832107
21	13.82354	9.959735	10.832107
22	13.81249	9.959735	10.832107
23	13.80144	9.959735	10.832107
24	13.79039	9.959735	10.832107
25	13.77934	9.959735	10.832107
26	13.76829	9.959735	10.832107
27	13.75724	9.959735	10.832107
28	13.74619	9.959735	10.832107
29	13.73514	9.959735	10.832107
30	13.72409	9.959735	10.832107
31	13.71304	9.959735	10.832107
32	13.70199	9.959735	10.832107
33	13.69094	9.959735	10.832107
34	13.67989	9.959735	10.832107
35	13.66884	9.959735	10.832107
36	13.65779	9.959735	10.832107
37	13.64674	9.959735	10.832107
38	13.63569	9.959735	10.832107
39	13.62464	9.959735	10.832107
40	13.61359	9.959735	10.832107
41	13.60254	9.959735	10.832107
42	13.59149	9.959735	10.832107
43	13.58044	9.959735	10.832107
44	13.56939	9.959735	10.832107
45	13.55834	9.959735	10.832107
46	13.54729	9.959735	10.832107
47	13.53624	9.959735	10.832107
48	13.52519	9.959735	10.832107
49	13.51414	9.959735	10.832107
50	13.50309	9.959735	10.832107
51	13.49204	9.959735	10.832107
52	13.48099	9.959735	10.832107
53	13.46994	9.959735	10.832107
54	13.45889	9.959735	10.832107
55	13.44784	9.959735	10.832107
56	13.43679	9.959735	10.832107
57	13.42574	9.959735	10.832107
58	13.41469	9.959735	10.832107
59	13.40364	9.959735	10.832107
60	13.39259	9.959735	10.832107

81 Degrees.

80 Degrees.

10 Degrees.

Mfn.	Sine.	Tang.	Secant.
0	9.235670	9.993351	10.753681
1	9.245786	9.993299	10.753943
2	9.251101	9.993257	10.754206
3	9.251814	9.993214	10.754470
4	9.252526	9.993172	10.754733
5	9.253237	9.993130	10.754996
6	9.253947	9.993087	10.755259
7	9.254656	9.993045	10.755522
8	9.255363	9.993003	10.755785
9	9.256069	9.992961	10.756048
10	9.256775	9.992919	10.756311
11	9.257481	9.992877	10.756574
12	9.258188	9.992835	10.756837
13	9.258893	9.992793	10.757100
14	9.259598	9.992751	10.757363
15	9.260302	9.992709	10.757626
16	9.261006	9.992667	10.757889
17	9.261710	9.992625	10.758152
18	9.262414	9.992583	10.758415
19	9.263117	9.992541	10.758678
20	9.263821	9.992500	10.758941
21	9.264524	9.992458	10.759204
22	9.265227	9.992416	10.759467
23	9.265930	9.992374	10.759730
24	9.266633	9.992332	10.760000
25	9.267336	9.992290	10.760263
26	9.268038	9.992248	10.760526
27	9.268741	9.992206	10.760789
28	9.269443	9.992164	10.761052
29	9.270145	9.992122	10.761315
30	9.270847	9.992080	10.761578
31	9.271549	9.992038	10.761841
32	9.272251	9.991996	10.762104
33	9.272953	9.991954	10.762367
34	9.273655	9.991912	10.762630
35	9.274357	9.991870	10.762893
36	9.275059	9.991828	10.763156
37	9.275761	9.991786	10.763419
38	9.276463	9.991744	10.763682
39	9.277165	9.991702	10.763945
40	9.277867	9.991660	10.764208
41	9.278569	9.991618	10.764471
42	9.279271	9.991576	10.764734
43	9.279973	9.991534	10.764997
44	9.280675	9.991492	10.765260
45	9.281377	9.991450	10.765523
46	9.282079	9.991408	10.765786
47	9.282781	9.991366	10.766049
48	9.283483	9.991324	10.766312
49	9.284185	9.991282	10.766575
50	9.284887	9.991240	10.766838
51	9.285589	9.991198	10.767101
52	9.286291	9.991156	10.767364
53	9.286993	9.991114	10.767627
54	9.287695	9.991072	10.767890
55	9.288397	9.991030	10.768153
56	9.289099	9.990988	10.768416
57	9.289801	9.990946	10.768679
58	9.290503	9.990904	10.768942
59	9.291205	9.990862	10.769205
60	9.291907	9.990820	10.769468

Sine.

Tang.

Secant.

M

79 Degrees.

11 Degrees.

Mfn.	Sine.	Tang.	Secant.
0	9.285599	9.991447	10.771348
1	9.286298	9.991405	10.771611
2	9.286997	9.991363	10.771874
3	9.287696	9.991321	10.772137
4	9.288395	9.991279	10.772400
5	9.289094	9.991237	10.772663
6	9.289793	9.991195	10.772926
7	9.290492	9.991153	10.773189
8	9.291191	9.991111	10.773452
9	9.291890	9.991069	10.773715
10	9.292589	9.991027	10.773978
11	9.293288	9.990985	10.774241
12	9.293987	9.990943	10.774504
13	9.294686	9.990901	10.774767
14	9.295385	9.990859	10.775030
15	9.296084	9.990817	10.775293
16	9.296783	9.990775	10.775556
17	9.297482	9.990733	10.775819
18	9.298181	9.990691	10.776082
19	9.298880	9.990649	10.776345
20	9.299579	9.990607	10.776608
21	9.300278	9.990565	10.776871
22	9.300977	9.990523	10.777134
23	9.301676	9.990481	10.777397
24	9.302375	9.990439	10.777660
25	9.303074	9.990397	10.777923
26	9.303773	9.990355	10.778186
27	9.304472	9.990313	10.778449
28	9.305171	9.990271	10.778712
29	9.305870	9.990229	10.778975
30	9.306569	9.990187	10.779238
31	9.307268	9.990145	10.779501
32	9.307967	9.990103	10.779764
33	9.308666	9.990061	10.780027
34	9.309365	9.990019	10.780290
35	9.310064	9.989977	10.780553
36	9.310763	9.989935	10.780816
37	9.311462	9.989893	10.781079
38	9.312161	9.989851	10.781342
39	9.312860	9.989809	10.781605
40	9.313559	9.989767	10.781868
41	9.314258	9.989725	10.782131
42	9.314957	9.989683	10.782394
43	9.315656	9.989641	10.782657
44	9.316355	9.989599	10.782920
45	9.317054	9.989557	10.783183
46	9.317753	9.989515	10.783446
47	9.318452	9.989473	10.783709
48	9.319151	9.989431	10.783972
49	9.319850	9.989389	10.784235
50	9.320549	9.989347	10.784498
51	9.321248	9.989305	10.784761
52	9.321947	9.989263	10.785024
53	9.322646	9.989221	10.785287
54	9.323345	9.989179	10.785550
55	9.324044	9.989137	10.785813
56	9.324743	9.989095	10.786076
57	9.325442	9.989053	10.786339
58	9.326141	9.989011	10.786602
59	9.326840	9.988969	10.786865
60	9.327539	9.988927	10.787128

Sine.

Tang.

Secant.

M

78 Degrees.

A Table of Artificial Sines, Tangents and Secants.

57

12 Degrees.

Mm.	Sine	Tang.	Secant.
0	9.317879	9.900164	10.672325
1	9.318473	9.900377	10.671905
2	9.319066	9.900591	10.671483
3	9.319658	9.900804	10.671060
4	9.320249	9.901017	10.670637
5	9.320839	9.901229	10.670213
6	9.321428	9.901441	10.669789
7	9.322016	9.901653	10.669364
8	9.322603	9.901864	10.668939
9	9.323189	9.902075	10.668513
10	9.323774	9.902285	10.668087
11	9.324358	9.902495	10.667661
12	9.324941	9.902704	10.667234
13	9.325523	9.902913	10.666807
14	9.326104	9.903121	10.666380
15	9.326684	9.903329	10.665952
16	9.327263	9.903536	10.665524
17	9.327841	9.903743	10.665095
18	9.328418	9.903949	10.664666
19	9.328994	9.904155	10.664236
20	9.329569	9.904360	10.663806
21	9.330143	9.904565	10.663375
22	9.330716	9.904769	10.662944
23	9.331288	9.904973	10.662512
24	9.331859	9.905176	10.662080
25	9.332429	9.905379	10.661647
26	9.332998	9.905581	10.661214
27	9.333566	9.905783	10.660780
28	9.334133	9.905984	10.660346
29	9.334699	9.906185	10.659911
30	9.335264	9.906385	10.659476
31	9.335828	9.906585	10.659040
32	9.336391	9.906784	10.658604
33	9.336953	9.906983	10.658167
34	9.337514	9.907181	10.657730
35	9.338074	9.907379	10.657292
36	9.338633	9.907576	10.656854
37	9.339191	9.907773	10.656415
38	9.339748	9.907969	10.655976
39	9.340304	9.908165	10.655536
40	9.340859	9.908360	10.655095
41	9.341413	9.908555	10.654654
42	9.341966	9.908749	10.654212
43	9.342518	9.908943	10.653770
44	9.343069	9.909136	10.653327
45	9.343619	9.909329	10.652884
46	9.344168	9.909521	10.652440
47	9.344716	9.909713	10.651996
48	9.345263	9.909904	10.651551
49	9.345809	9.910095	10.651106
50	9.346354	9.910285	10.650660
51	9.346898	9.910475	10.650214
52	9.347441	9.910664	10.649767
53	9.347983	9.910853	10.649320
54	9.348524	9.911041	10.648872
55	9.349064	9.911229	10.648424
56	9.349603	9.911416	10.647975
57	9.350141	9.911603	10.647526
58	9.350678	9.911789	10.647076
59	9.351214	9.911975	10.646626
60	9.351749	9.912160	10.646175

77 Degrees.

13 Degrees.

Mm.	Sine	Tang.	Secant.
0	9.352088	9.988724	10.631364
1	9.352635	9.988605	10.631942
2	9.353181	9.988485	10.632519
3	9.353726	9.988364	10.633095
4	9.354271	9.988243	10.633671
5	9.354815	9.988121	10.634246
6	9.355358	9.988000	10.634821
7	9.355901	9.987878	10.635395
8	9.356443	9.987756	10.635969
9	9.356984	9.987633	10.636542
10	9.357524	9.987511	10.637115
11	9.358063	9.987388	10.637688
12	9.358601	9.987265	10.638260
13	9.359138	9.987142	10.638832
14	9.359674	9.987018	10.639403
15	9.360209	9.986894	10.639974
16	9.360743	9.986770	10.640544
17	9.361276	9.986645	10.641114
18	9.361808	9.986521	10.641684
19	9.362339	9.986396	10.642253
20	9.362869	9.986271	10.642822
21	9.363398	9.986146	10.643391
22	9.363926	9.986021	10.643959
23	9.364453	9.985895	10.644527
24	9.364979	9.985769	10.645094
25	9.365504	9.985643	10.645661
26	9.366028	9.985517	10.646227
27	9.366551	9.985390	10.646793
28	9.367073	9.985264	10.647358
29	9.367594	9.985137	10.647923
30	9.368114	9.985011	10.648487
31	9.368633	9.984884	10.649051
32	9.369151	9.984757	10.649614
33	9.369668	9.984630	10.650177
34	9.370184	9.984503	10.650739
35	9.370699	9.984375	10.651301
36	9.371213	9.984248	10.651862
37	9.371726	9.984120	10.652423
38	9.372238	9.983992	10.652983
39	9.372749	9.983864	10.653543
40	9.373259	9.983736	10.654102
41	9.373768	9.983607	10.654661
42	9.374276	9.983478	10.655219
43	9.374783	9.983349	10.655777
44	9.375289	9.983220	10.656334
45	9.375794	9.983091	10.656891
46	9.376298	9.982962	10.657447
47	9.376799	9.982832	10.658003
48	9.377299	9.982703	10.658558
49	9.377798	9.982573	10.659112
50	9.378296	9.982443	10.659666
51	9.378793	9.982313	10.660219
52	9.379289	9.982183	10.660772
53	9.379784	9.982053	10.661324
54	9.380278	9.981923	10.661876
55	9.380771	9.981793	10.662427
56	9.381263	9.981662	10.662978
57	9.381754	9.981532	10.663528
58	9.382244	9.981401	10.664078
59	9.382733	9.981271	10.664627
60	9.383221	9.981140	10.665176

76 Degrees.

A Table of Artificial Sines, Tangents and Secants.

14 Degrees.

Min.	Sine.	Tang.	Secant.	Min.	Sine.	Tang.	Secant.
0	9.384675	9.986904	10.603229	10.013496	10.616325	60	0
1	9.384818	9.986873	9.997771	10.606691	10.613127	59	1
2	9.384967	9.986841	9.997545	10.602154	10.610131	58	2
3	9.385102	9.986809	9.997318	10.601977	10.610180	57	3
4	9.385607	9.986778	9.997091	10.601801	10.613222	56	4
5	9.386201	9.986747	9.996865	10.601645	10.613254	55	5
6	9.386704	9.986716	9.996639	10.600010	10.613286	54	6
7	9.387209	9.986685	9.996413	10.599476	10.613317	53	7
8	9.387810	9.986654	9.996187	10.598942	10.613349	52	8
9	9.388411	9.986623	9.995961	10.598408	10.613381	51	9
10	9.389012	9.986592	9.995735	10.597876	10.613413	50	10
11	9.389613	9.986561	9.995509	10.597344	10.613445	49	11
12	9.390214	9.986530	9.995283	10.596813	10.613477	48	12
13	9.390815	9.986500	9.995057	10.596282	10.613509	47	13
14	9.391416	9.986469	9.994831	10.595751	10.613541	46	14
15	9.392017	9.986438	9.994605	10.595222	10.613573	45	15
16	9.392618	9.986407	9.994379	10.594692	10.613605	44	16
17	9.393219	9.986376	9.994153	10.594164	10.613637	43	17
18	9.393820	9.986345	9.993927	10.593635	10.613669	42	18
19	9.394421	9.986314	9.993701	10.593108	10.613701	41	19
20	9.395022	9.986283	9.993475	10.592581	10.613734	40	20
21	9.395623	9.986252	9.993249	10.592055	10.613766	39	21
22	9.396224	9.986221	9.993023	10.591529	10.613798	38	22
23	9.396825	9.986190	9.992797	10.591003	10.613831	37	23
24	9.397426	9.986159	9.992571	10.590477	10.613863	36	24
25	9.398027	9.986128	9.992345	10.589951	10.613895	35	25
26	9.398628	9.986097	9.992119	10.589425	10.613928	34	26
27	9.399229	9.986066	9.991893	10.588899	10.613960	33	27
28	9.399830	9.986035	9.991667	10.588373	10.613993	32	28
29	9.400431	9.986004	9.991441	10.587847	10.614026	31	29
30	9.401032	9.985973	9.991215	10.587321	10.614058	30	30
31	9.401633	9.985942	9.990989	10.586795	10.614091	29	31
32	9.402234	9.985911	9.990763	10.586269	10.614124	28	32
33	9.402835	9.985880	9.990537	10.585743	10.614157	27	33
34	9.403436	9.985849	9.990311	10.585217	10.614189	26	34
35	9.404037	9.985818	9.990085	10.584691	10.614222	25	35
36	9.404638	9.985787	9.989859	10.584165	10.614255	24	36
37	9.405239	9.985756	9.989633	10.583639	10.614288	23	37
38	9.405840	9.985725	9.989407	10.583113	10.614321	22	38
39	9.406441	9.985694	9.989181	10.582587	10.614354	21	39
40	9.407042	9.985663	9.988955	10.582061	10.614387	20	40
41	9.407643	9.985632	9.988729	10.581535	10.614420	19	41
42	9.408244	9.985601	9.988503	10.581009	10.614453	18	42
43	9.408845	9.985570	9.988277	10.580483	10.614486	17	43
44	9.409446	9.985539	9.988051	10.579957	10.614519	16	44
45	9.410047	9.985508	9.987825	10.579431	10.614552	15	45
46	9.410648	9.985477	9.987599	10.578905	10.614585	14	46
47	9.411249	9.985446	9.987373	10.578379	10.614618	13	47
48	9.411850	9.985415	9.987147	10.577853	10.614651	12	48
49	9.412451	9.985384	9.986921	10.577327	10.614684	11	49
50	9.413052	9.985353	9.986695	10.576801	10.614717	10	50
51	9.413653	9.985322	9.986469	10.576275	10.614750	9	51
52	9.414254	9.985291	9.986243	10.575749	10.614783	8	52
53	9.414855	9.985260	9.986017	10.575223	10.614816	7	53
54	9.415456	9.985229	9.985791	10.574697	10.614849	6	54
55	9.416057	9.985198	9.985565	10.574171	10.614882	5	55
56	9.416658	9.985167	9.985339	10.573645	10.614915	4	56
57	9.417259	9.985136	9.985113	10.573119	10.614948	3	57
58	9.417860	9.985105	9.984887	10.572593	10.614981	2	58
59	9.418461	9.985074	9.984661	10.572067	10.615014	1	59
60	9.419062	9.985043	9.984435	10.571541	10.615047	0	60
	Sine.	Tang.	Secant.				

75 Degrees.

15 Degrees.

Min.	Sine.	Tang.	Secant.	Min.	Sine.	Tang.	Secant.
0	9.419663	9.985012	10.571015	10.615080	10.571044	60	0
1	9.420264	9.984981	10.570590	10.615054	10.571442	59	1
2	9.420865	9.984950	10.570165	10.615028	10.571840	58	2
3	9.421466	9.984919	10.569740	10.615002	10.572238	57	3
4	9.422067	9.984888	10.569315	10.614976	10.572636	56	4
5	9.422668	9.984857	10.568890	10.614950	10.573034	55	5
6	9.423269	9.984826	10.568465	10.614924	10.573432	54	6
7	9.423870	9.984795	10.568040	10.614898	10.573830	53	7
8	9.424471	9.984764	10.567615	10.614872	10.574228	52	8
9	9.425072	9.984733	10.567190	10.614846	10.574626	51	9
10	9.425673	9.984702	10.566765	10.614820	10.575024	50	10
11	9.426274	9.984671	10.566340	10.614794	10.575422	49	11
12	9.426875	9.984640	10.565915	10.614768	10.575820	48	12
13	9.427476	9.984609	10.565490	10.614742	10.576218	47	13
14	9.428077	9.984578	10.565065	10.614716	10.576616	46	14
15	9.428678	9.984547	10.564640	10.614690	10.577014	45	15
16	9.429279	9.984516	10.564215	10.614664	10.577412	44	16
17	9.429880	9.984485	10.563790	10.614638	10.577810	43	17
18	9.430481	9.984454	10.563365	10.614612	10.578208	42	18
19	9.431082	9.984423	10.562940	10.614586	10.578606	41	19
20	9.431683	9.984392	10.562515	10.614560	10.579004	40	20
21	9.432284	9.984361	10.562090	10.614534	10.579402	39	21
22	9.432885	9.984330	10.561665	10.614508	10.579800	38	22
23	9.433486	9.984300	10.561240	10.614482	10.580198	37	23
24	9.434087	9.984269	10.560815	10.614456	10.580596	36	24
25	9.434688	9.984238	10.560390	10.614430	10.580994	35	25
26	9.435289	9.984207	10.559965	10.614404	10.581392	34	26
27	9.435890	9.984176	10.559540	10.614378	10.581790	33	27
28	9.436491	9.984145	10.559115	10.614352	10.582188	32	28
29	9.437092	9.984114	10.558690	10.614326	10.582586	31	29
30	9.437693	9.984083	10.558265	10.614300	10.582984	30	30
31	9.438294	9.984052	10.557840	10.614274	10.583382	29	31
32	9.438895	9.984021	10.557415	10.614248	10.583780	28	32
33	9.439496	9.983990	10.556990	10.614222	10.584178	27	33
34	9.440097	9.983959	10.556565	10.614196	10.584576	26	34
35	9.440698	9.983928	10.556140	10.614170	10.584974	25	35
36	9.441299	9.983897	10.555715	10.614144	10.585372	24	36
37	9.441900	9.983866	10.555290	10.614118	10.585770	23	37
38	9.442501	9.983835	10.554865	10.614092	10.586168	22	38
39	9.443102	9.983804	10.554440	10.614066	10.586566	21	39
40	9.443703	9.983773	10.554015	10.614040	10.586964	20	40
41	9.444304	9.983742	10.553590	10.614014	10.587362	19	41
42	9.444905	9.983711	10.553165	10.613988	10.587760	18	42
43	9.445506	9.983680	10.552740	10.613962	10.588158	17	43
44	9.446107	9.983649	10.552315	10.613936	10.588556	16	44
45	9.446708	9.983618	10.551890	10.613910	10.588954	15	45
46	9.447309	9.983587	10.551465	10.613884	10.589352	14	46
47	9.447910	9.983556	10.551040	10.613858	10.589750	13	47
48	9.448511	9.983525	10.550615	10.613832	10.590148	12	48
49	9.449112	9.983494	10.550190	10.613806	10.590546	11	49
50	9.449713	9.983463	10.549765	10.613780	10.590944	10	50
51	9.450314	9.983432	10.549340	10.613754	10.591342	9	51
52	9.450915	9.983401	10.548915	10.613728	10.591740	8	52
53	9.451516	9.983370	10.548490	10.613702	10.592138	7	53
54	9.452117	9.983339	10.548065	10.613676	10.592536	6	54
55	9.452718	9.983308	10.547640	10.613650	10.592934	5	55
56	9.453319	9.983277	10.547215	10.613624	10.593332	4	56
57	9.453920	9.983246	10.546790	10.613598	10.593730	3	57
58	9.454521	9.983215	10.546365	10.613572	10.594128	2	58
59	9.455122	9.983184	10.545940	10.613546	10.594526	1	59
60	9.455723	9.983153	10.545515	10.613520	10.594924	0	60
	Sine.	Tang.	Secant.				

74 Degrees.

A Table of Artificial Sines, Tangents and Secants.

59

16 Degrees.

17 Degrees.

	Sine	Tang.	Secant.		Min.	Sine	Tang.	Secant.		Min.	Sine	Tang.	Secant.		Min.	Sine	Tang.	Secant.		Min.	Sine	Tang.	Secant.							
0	9.44333	9.91812	9.45746	10.54504	10.01758	10.55602	60	0	9.46595	9.98056	9.48533	10.51466	10.01444	10.51466	60	0	9.46595	9.98056	9.48533	10.51466	10.01444	10.51466	60	0	9.46595	9.98056	9.48533	10.51466	10.01444	10.51466
1	9.44377	9.91854	9.45788	10.54546	10.01799	10.55644	59	1	9.46637	9.98098	9.48575	10.51508	10.01486	10.51508	59	1	9.46637	9.98098	9.48575	10.51508	10.01486	10.51508	59	1	9.46637	9.98098	9.48575	10.51508	10.01486	10.51508
2	9.44418	9.91896	9.45830	10.54588	10.01841	10.55686	58	2	9.46679	9.98140	9.48617	10.51550	10.01528	10.51550	58	2	9.46679	9.98140	9.48617	10.51550	10.01528	10.51550	58	2	9.46679	9.98140	9.48617	10.51550	10.01528	10.51550
3	9.44460	9.91938	9.45872	10.54630	10.01883	10.55728	57	3	9.46721	9.98182	9.48659	10.51592	10.01570	10.51592	57	3	9.46721	9.98182	9.48659	10.51592	10.01570	10.51592	57	3	9.46721	9.98182	9.48659	10.51592	10.01570	10.51592
4	9.44502	9.91980	9.45914	10.54672	10.01925	10.55770	56	4	9.46763	9.98224	9.48701	10.51634	10.01612	10.51634	56	4	9.46763	9.98224	9.48701	10.51634	10.01612	10.51634	56	4	9.46763	9.98224	9.48701	10.51634	10.01612	10.51634
5	9.44544	9.92022	9.45956	10.54714	10.01967	10.55812	55	5	9.46805	9.98266	9.48743	10.51676	10.01654	10.51676	55	5	9.46805	9.98266	9.48743	10.51676	10.01654	10.51676	55	5	9.46805	9.98266	9.48743	10.51676	10.01654	10.51676
6	9.44586	9.92064	9.45998	10.54756	10.02009	10.55854	54	6	9.46847	9.98308	9.48785	10.51718	10.01696	10.51718	54	6	9.46847	9.98308	9.48785	10.51718	10.01696	10.51718	54	6	9.46847	9.98308	9.48785	10.51718	10.01696	10.51718
7	9.44628	9.92106	9.46040	10.54798	10.02051	10.55896	53	7	9.46889	9.98350	9.48827	10.51760	10.01738	10.51760	53	7	9.46889	9.98350	9.48827	10.51760	10.01738	10.51760	53	7	9.46889	9.98350	9.48827	10.51760	10.01738	10.51760
8	9.44670	9.92148	9.46082	10.54840	10.02093	10.55938	52	8	9.46931	9.98392	9.48869	10.51802	10.01780	10.51802	52	8	9.46931	9.98392	9.48869	10.51802	10.01780	10.51802	52	8	9.46931	9.98392	9.48869	10.51802	10.01780	10.51802
9	9.44712	9.92190	9.46124	10.54882	10.02135	10.55980	51	9	9.46973	9.98434	9.48911	10.51844	10.01822	10.51844	51	9	9.46973	9.98434	9.48911	10.51844	10.01822	10.51844	51	9	9.46973	9.98434	9.48911	10.51844	10.01822	10.51844
10	9.44754	9.92232	9.46166	10.54924	10.02177	10.56022	50	10	9.47015	9.98476	9.48953	10.51886	10.01864	10.51886	50	10	9.47015	9.98476	9.48953	10.51886	10.01864	10.51886	50	10	9.47015	9.98476	9.48953	10.51886	10.01864	10.51886
11	9.44796	9.92274	9.46208	10.54966	10.02219	10.56064	49	11	9.47057	9.98518	9.48995	10.51928	10.01906	10.51928	49	11	9.47057	9.98518	9.48995	10.51928	10.01906	10.51928	49	11	9.47057	9.98518	9.48995	10.51928	10.01906	10.51928
12	9.44838	9.92316	9.46250	10.55008	10.02261	10.56106	48	12	9.47099	9.98560	9.49037	10.51970	10.01948	10.51970	48	12	9.47099	9.98560	9.49037	10.51970	10.01948	10.51970	48	12	9.47099	9.98560	9.49037	10.51970	10.01948	10.51970
13	9.44880	9.92358	9.46292	10.55050	10.02303	10.56148	47	13	9.47141	9.98602	9.49079	10.52012	10.01990	10.52012	47	13	9.47141	9.98602	9.49079	10.52012	10.01990	10.52012	47	13	9.47141	9.98602	9.49079	10.52012	10.01990	10.52012
14	9.44922	9.92400	9.46334	10.55092	10.02345	10.56190	46	14	9.47183	9.98644	9.49121	10.52054	10.02032	10.52054	46	14	9.47183	9.98644	9.49121	10.52054	10.02032	10.52054	46	14	9.47183	9.98644	9.49121	10.52054	10.02032	10.52054
15	9.44964	9.92442	9.46376	10.55134	10.02387	10.56232	45	15	9.47225	9.98686	9.49163	10.52096	10.02074	10.52096	45	15	9.47225	9.98686	9.49163	10.52096	10.02074	10.52096	45	15	9.47225	9.98686	9.49163	10.52096	10.02074	10.52096
16	9.45006	9.92484	9.46418	10.55176	10.02429	10.56274	44	16	9.47267	9.98728	9.49205	10.52138	10.02116	10.52138	44	16	9.47267	9.98728	9.49205	10.52138	10.02116	10.52138	44	16	9.47267	9.98728	9.49205	10.52138	10.02116	10.52138
17	9.45048	9.92526	9.46460	10.55218	10.02471	10.56316	43	17	9.47309	9.98770	9.49247	10.52180	10.02158	10.52180	43	17	9.47309	9.98770	9.49247	10.52180	10.02158	10.52180	43	17	9.47309	9.98770	9.49247	10.52180	10.02158	10.52180
18	9.45090	9.92568	9.46502	10.55260	10.02513	10.56358	42	18	9.47351	9.98812	9.49289	10.52222	10.02200	10.52222	42	18	9.47351	9.98812	9.49289	10.52222	10.02200	10.52222	42	18	9.47351	9.98812	9.49289	10.52222	10.02200	10.52222
19	9.45132	9.92610	9.46544	10.55302	10.02555	10.56400	41	19	9.47393	9.98854	9.49331	10.52264	10.02242	10.52264	41	19	9.47393	9.98854	9.49331	10.52264	10.02242	10.52264	41	19	9.47393	9.98854	9.49331	10.52264	10.02242	10.52264
20	9.45174	9.92652	9.46586	10.55344	10.02597	10.56442	40	20	9.47435	9.98896	9.49373	10.52306	10.02284	10.52306	40	20	9.47435	9.98896	9.49373	10.52306	10.02284	10.52306	40	20	9.47435	9.98896	9.49373	10.52306	10.02284	10.52306
21	9.45216	9.92694	9.46628	10.55386	10.02639	10.56484	39	21	9.47477	9.98938	9.49415	10.52348	10.02326	10.52348	39	21	9.47477	9.98938	9.49415	10.52348	10.02326	10.52348	39	21	9.47477	9.98938	9.49415	10.52348	10.02326	10.52348
22	9.45258	9.92736	9.46670	10.55428	10.02681	10.56526	38	22	9.47519	9.98980	9.49457	10.52390	10.02368	10.52390	38	22	9.47519	9.98980	9.49457	10.52390	10.02368	10.52390	38	22	9.47519	9.98980	9.49457	10.52390	10.02368	10.52390
23	9.45300	9.92778	9.46712	10.55470	10.02723	10.56568	37	23	9.47561	9.99022	9.49499	10.52432	10.02410	10.52432	37	23	9.47561	9.99022	9.49499	10.52432	10.02410	10.52432	37	23	9.47561	9.99022	9.49499	10.52432	10.02410	10.52432
24	9.45342	9.92820	9.46754	10.55512	10.02765	10.56610	36	24	9.47603	9.99064	9.49541	10.52474	10.02452	10.52474	36	24	9.47603	9.99064	9.49541	10.52474	10.02452	10.52474	36	24	9.47603	9.99064	9.49541	10.52474	10.02452	10.52474
25	9.45384	9.92862	9.46796	10.55554	10.02807	10.56652	35	25	9.47645	9.99106	9.49583	10.52516	10.02494	10.52516	35	25	9.47645	9.99106	9.49583	10.52516	10.02494	10.52516	35	25	9.47645	9.99106	9.49583	10.52516	10.02494	10.52516
26	9.45426	9.92904	9.46838	10.55596	10.02849	10.56694	34	26	9.47687	9.99148	9.49625	10.52558	10.02536	10.52558	34	26	9.47687	9.99148	9.49625	10.52558	10.02536	10.52558	34	26	9.47687	9.99148	9.49625	10.52558	10.02536	10.52558
27	9.45468	9.92946	9.46880	10.55638	10.02891	10.56736	33	27	9.47729	9.99190	9.49667	10.52600	10.02578	10.52600	33	27	9.47729	9.99190	9.49667	10.52600	10.02578	10.52600	33	27	9.47729	9.99190	9.49667	10.52600	10.02578	10.52600
28	9.45510	9.92988	9.46922	10.55680	10.02933	10.56778	32	28	9.47771	9.99232	9.49709	10.52642	10.02620	10.52642	32	28	9.47771	9.99232	9.49709	10.52642	10.02620	10.52642	32	28	9.47771	9.99232	9.49709	10.52642	10.02620	10.52642
29	9.45552	9.93030	9.46964	10.55722	10.02975	10.56820	31	29	9.47813	9.99274	9.49751	10.52684	10.02662	10.52684	31	29	9.47813	9.99274	9.49751	10.52684	10.02662	10.52684	31	29	9.47813	9.99274	9.49751	10.52684	10.02662	10.52684
30	9.45594	9.93072	9.47006	10.55764	10.03017	10.56862	30	30	9.47855	9.99316	9.49793	10.52726	10.02704	10.52726	30	30	9.47855	9.99316	9.49793	10.52726	10.02704	10.52726	30	30	9.47855	9.99316	9.49793	10.52726	10.02704	10.52726
31	9.45636	9.93114	9.47048	10.55806	10.03059	10.56904	29	31	9.47897	9.99358	9.49835	10.52768	10.02746	10.52768	29	31	9.47897	9.99358	9.49835	10.52768	10.02746	10.52768	29	31	9.47897	9.99358	9.49835	10.52768	10.02746	10.52768
32	9.45678	9.93156	9.47090	10.55848	10.03101																									

A Table of Artificial Sines, Tangents and Secants.

18 Degrees.

M.	Sine.	Tang.	Secant.
0	9.489982	9.978260	10.433224
1	9.490371	9.978165	10.433794
2	9.490758	9.978124	10.434365
3	9.491147	9.978083	10.434936
4	9.491534	9.978042	10.435507
5	9.491912	9.978001	10.436079
6	9.492298	9.977959	10.436651
7	9.492685	9.977918	10.437222
8	9.493071	9.977876	10.437793
9	9.493456	9.977835	10.438364
10	9.493839	9.977794	10.438935
11	9.494225	9.977752	10.439506
12	9.494610	9.977711	10.440077
13	9.494995	9.977670	10.440648
14	9.495378	9.977628	10.441219
15	9.495774	9.977587	10.441790
16	9.496154	9.977544	10.442361
17	9.496537	9.977503	10.442932
18	9.496919	9.977461	10.443503
19	9.497311	9.977419	10.444074
20	9.497682	9.977377	10.444645
21	9.498063	9.977335	10.445216
22	9.498444	9.977293	10.445787
23	9.498824	9.977251	10.446358
24	9.499205	9.977209	10.446929
25	9.499585	9.977167	10.447500
26	9.499965	9.977125	10.448071
27	9.500345	9.977083	10.448642
28	9.500721	9.977041	10.449213
29	9.501099	9.976999	10.449784
30	9.501479	9.976957	10.450355
31	9.501854	9.976914	10.450926
32	9.502231	9.976872	10.451497
33	9.502609	9.976830	10.452068
34	9.502984	9.976787	10.452639
35	9.503363	9.976745	10.453210
36	9.503739	9.976702	10.453781
37	9.504110	9.976660	10.454352
38	9.504485	9.976617	10.454923
39	9.504860	9.976574	10.455494
40	9.505234	9.976532	10.456065
41	9.505608	9.976489	10.456636
42	9.505984	9.976446	10.457207
43	9.506357	9.976404	10.457778
44	9.506727	9.976361	10.458349
45	9.507099	9.976318	10.458920
46	9.507471	9.976275	10.459491
47	9.507842	9.976232	10.460062
48	9.508214	9.976189	10.460633
49	9.508585	9.976146	10.461204
50	9.508956	9.976103	10.461775
51	9.509326	9.976060	10.462346
52	9.509695	9.976017	10.462917
53	9.510065	9.975974	10.463488
54	9.510434	9.975931	10.464059
55	9.510803	9.975888	10.464630
56	9.511172	9.975845	10.465201
57	9.511540	9.975802	10.465772
58	9.511907	9.975759	10.466343
59	9.512275	9.975716	10.466914
60	9.512642	9.975673	10.467485

Sine.

Tang.

Secant.

M.

19 Degrees.

M.	Sine.	Tang.	Secant.
0	9.512642	9.975673	10.467485
1	9.513009	9.975626	10.468056
2	9.513375	9.975583	10.468627
3	9.513741	9.975539	10.469198
4	9.514107	9.975490	10.469769
5	9.514472	9.975442	10.470340
6	9.514837	9.975398	10.470911
7	9.515201	9.975355	10.471482
8	9.515566	9.975312	10.472053
9	9.515930	9.975272	10.472624
10	9.516294	9.975233	10.473195
11	9.516657	9.975193	10.473766
12	9.517020	9.975154	10.474337
13	9.517383	9.975115	10.474908
14	9.517745	9.975075	10.475479
15	9.518107	9.975036	10.476050
16	9.518468	9.974996	10.476621
17	9.518829	9.974957	10.477192
18	9.519190	9.974917	10.477763
19	9.519551	9.974878	10.478334
20	9.519911	9.974838	10.478905
21	9.520271	9.974798	10.479476
22	9.520631	9.974758	10.480047
23	9.520990	9.974718	10.480618
24	9.521349	9.974678	10.481189
25	9.521707	9.974638	10.481760
26	9.522066	9.974598	10.482331
27	9.522424	9.974558	10.482902
28	9.522782	9.974518	10.483473
29	9.523139	9.974478	10.484044
30	9.523495	9.974437	10.484615
31	9.523852	9.974397	10.485186
32	9.524208	9.974357	10.485757
33	9.524564	9.974317	10.486328
34	9.524919	9.974277	10.486899
35	9.525275	9.974237	10.487470
36	9.525631	9.974196	10.488041
37	9.525986	9.974156	10.488612
38	9.526341	9.974116	10.489183
39	9.526696	9.974075	10.489754
40	9.527050	9.974035	10.490325
41	9.527405	9.973994	10.490896
42	9.527759	9.973954	10.491467
43	9.528113	9.973913	10.492038
44	9.528467	9.973873	10.492609
45	9.528820	9.973832	10.493180
46	9.529174	9.973792	10.493751
47	9.529527	9.973751	10.494322
48	9.529880	9.973711	10.494893
49	9.530232	9.973670	10.495464
50	9.530585	9.973630	10.496035
51	9.530937	9.973589	10.496606
52	9.531289	9.973548	10.497177
53	9.531641	9.973508	10.497748
54	9.531993	9.973467	10.498319
55	9.532345	9.973426	10.498890
56	9.532696	9.973386	10.499461
57	9.533047	9.973345	10.500032
58	9.533398	9.973305	10.500603
59	9.533749	9.973264	10.501174
60	9.534100	9.973224	10.501745

Sine.

Tang.

Secant.

M.

71 Degrees.

70 Degrees.

A Table of Artificial Sines, Tangents and Secants.

61

20 Degrees.

Min.	Sine	Tang.	Secant.
0	0.34202	0.72986	0.438035
1	0.34310	0.73040	0.43841
2	0.34418	0.73094	0.43879
3	0.34526	0.73148	0.43917
4	0.34634	0.73202	0.43955
5	0.34742	0.73256	0.43993
6	0.34850	0.73310	0.44031
7	0.34958	0.73364	0.44069
8	0.35066	0.73418	0.44107
9	0.35174	0.73472	0.44145
10	0.35282	0.73526	0.44183
11	0.35390	0.73580	0.44221
12	0.35498	0.73634	0.44259
13	0.35606	0.73688	0.44297
14	0.35714	0.73742	0.44335
15	0.35822	0.73796	0.44373
16	0.35930	0.73850	0.44411
17	0.36038	0.73904	0.44449
18	0.36146	0.73958	0.44487
19	0.36254	0.74012	0.44525
20	0.36362	0.74066	0.44563
21	0.36470	0.74120	0.44601
22	0.36578	0.74174	0.44639
23	0.36686	0.74228	0.44677
24	0.36794	0.74282	0.44715
25	0.36902	0.74336	0.44753
26	0.37010	0.74390	0.44791
27	0.37118	0.74444	0.44829
28	0.37226	0.74498	0.44867
29	0.37334	0.74552	0.44905
30	0.37442	0.74606	0.44943
31	0.37550	0.74660	0.44981
32	0.37658	0.74714	0.45019
33	0.37766	0.74768	0.45057
34	0.37874	0.74822	0.45095
35	0.37982	0.74876	0.45133
36	0.38090	0.74930	0.45171
37	0.38198	0.74984	0.45209
38	0.38306	0.75038	0.45247
39	0.38414	0.75092	0.45285
40	0.38522	0.75146	0.45323
41	0.38630	0.75200	0.45361
42	0.38738	0.75254	0.45399
43	0.38846	0.75308	0.45437
44	0.38954	0.75362	0.45475
45	0.39062	0.75416	0.45513
46	0.39170	0.75470	0.45551
47	0.39278	0.75524	0.45589
48	0.39386	0.75578	0.45627
49	0.39494	0.75632	0.45665
50	0.39602	0.75686	0.45703
51	0.39710	0.75740	0.45741
52	0.39818	0.75794	0.45779
53	0.39926	0.75848	0.45817
54	0.40034	0.75902	0.45855
55	0.40142	0.75956	0.45893
56	0.40250	0.76010	0.45931
57	0.40358	0.76064	0.45969
58	0.40466	0.76118	0.46007
59	0.40574	0.76172	0.46045
60	0.40682	0.76226	0.46083

69 Degrees.

21 Degrees.

Min.	Sine	Tang.	Secant.
0	0.354329	0.72013	0.438177
1	0.355408	0.72067	0.43855
2	0.356487	0.72121	0.43893
3	0.357566	0.72175	0.43931
4	0.358645	0.72229	0.43969
5	0.359724	0.72283	0.44007
6	0.360803	0.72337	0.44045
7	0.361882	0.72391	0.44083
8	0.362961	0.72445	0.44121
9	0.364040	0.72499	0.44159
10	0.365119	0.72553	0.44197
11	0.366198	0.72607	0.44235
12	0.367277	0.72661	0.44273
13	0.368356	0.72715	0.44311
14	0.369435	0.72769	0.44349
15	0.370514	0.72823	0.44387
16	0.371593	0.72877	0.44425
17	0.372672	0.72931	0.44463
18	0.373751	0.72985	0.44501
19	0.374830	0.73039	0.44539
20	0.375909	0.73093	0.44577
21	0.376988	0.73147	0.44615
22	0.378067	0.73201	0.44653
23	0.379146	0.73255	0.44691
24	0.380225	0.73309	0.44729
25	0.381304	0.73363	0.44767
26	0.382383	0.73417	0.44805
27	0.383462	0.73471	0.44843
28	0.384541	0.73525	0.44881
29	0.385620	0.73579	0.44919
30	0.386699	0.73633	0.44957
31	0.387778	0.73687	0.44995
32	0.388857	0.73741	0.45033
33	0.389936	0.73795	0.45071
34	0.391015	0.73849	0.45109
35	0.392094	0.73903	0.45147
36	0.393173	0.73957	0.45185
37	0.394252	0.74011	0.45223
38	0.395331	0.74065	0.45261
39	0.396410	0.74119	0.45299
40	0.397489	0.74173	0.45337
41	0.398568	0.74227	0.45375
42	0.399647	0.74281	0.45413
43	0.400726	0.74335	0.45451
44	0.401805	0.74389	0.45489
45	0.402884	0.74443	0.45527
46	0.403963	0.74497	0.45565
47	0.405042	0.74551	0.45603
48	0.406121	0.74605	0.45641
49	0.407200	0.74659	0.45679
50	0.408279	0.74713	0.45717
51	0.409358	0.74767	0.45755
52	0.410437	0.74821	0.45793
53	0.411516	0.74875	0.45831
54	0.412595	0.74929	0.45869
55	0.413674	0.74983	0.45907
56	0.414753	0.75037	0.45945
57	0.415832	0.75091	0.45983
58	0.416911	0.75145	0.46021
59	0.417990	0.75199	0.46059
60	0.419069	0.75253	0.46097

68 Degrees.

A Table of Artificial Sines, Tangents and Secants.

22 Degrees.

M.	Sine.	Tang.	Secant.
0	0.573735	0.967166	10.393590
1	0.573888	0.967115	10.393227
2	0.574000	0.967064	10.392863
3	0.574124	0.967012	10.392500
4	0.574244	0.966961	10.392137
5	0.574366	0.966910	10.391775
6	0.574477	0.966859	10.391412
7	0.574588	0.966807	10.391050
8	0.574698	0.966756	10.390688
9	0.574810	0.966704	10.390326
10	0.574921	0.966653	10.389964
11	0.575032	0.966602	10.389602
12	0.575143	0.966551	10.389241
13	0.575254	0.966500	10.388880
14	0.575365	0.966449	10.388520
15	0.575476	0.966398	10.388159
16	0.575587	0.966347	10.387799
17	0.575698	0.966296	10.387438
18	0.575809	0.966245	10.387079
19	0.575920	0.966194	10.386719
20	0.576031	0.966143	10.386359
21	0.576142	0.966092	10.386000
22	0.576253	0.966041	10.385641
23	0.576364	0.965990	10.385282
24	0.576475	0.965939	10.384923
25	0.576586	0.965888	10.384565
26	0.576697	0.965837	10.384207
27	0.576808	0.965786	10.383849
28	0.576919	0.965735	10.383491
29	0.577030	0.965684	10.383133
30	0.577141	0.965633	10.382776
31	0.577252	0.965582	10.382418
32	0.577363	0.965531	10.382061
33	0.577474	0.965480	10.381703
34	0.577585	0.965429	10.381346
35	0.577696	0.965378	10.380989
36	0.577807	0.965327	10.380632
37	0.577918	0.965276	10.380275
38	0.578029	0.965225	10.379918
39	0.578140	0.965174	10.379561
40	0.578251	0.965123	10.379204
41	0.578362	0.965072	10.378847
42	0.578473	0.965021	10.378490
43	0.578584	0.964970	10.378133
44	0.578695	0.964919	10.377776
45	0.578806	0.964868	10.377419
46	0.578917	0.964817	10.377062
47	0.579028	0.964766	10.376705
48	0.579139	0.964715	10.376348
49	0.579250	0.964664	10.375991
50	0.579361	0.964613	10.375634
51	0.579472	0.964562	10.375277
52	0.579583	0.964511	10.374920
53	0.579694	0.964460	10.374563
54	0.579805	0.964409	10.374206
55	0.579916	0.964358	10.373849
56	0.580027	0.964307	10.373492
57	0.580138	0.964256	10.373135
58	0.580249	0.964205	10.372778
59	0.580360	0.964154	10.372421
60	0.580471	0.964103	10.372064

23 Degrees.

M.	Sine.	Tang.	Secant.
0	0.591878	0.627853	10.372148
1	0.592175	0.628203	10.371797
2	0.592471	0.628554	10.371448
3	0.592769	0.628905	10.371099
4	0.593067	0.629255	10.370750
5	0.593365	0.629606	10.370401
6	0.593663	0.629956	10.370052
7	0.593961	0.630307	10.369703
8	0.594259	0.630657	10.369354
9	0.594557	0.631008	10.369005
10	0.594855	0.631358	10.368656
11	0.595153	0.631709	10.368307
12	0.595451	0.632059	10.367958
13	0.595749	0.632410	10.367609
14	0.596047	0.632760	10.367260
15	0.596345	0.633111	10.366911
16	0.596643	0.633461	10.366562
17	0.596941	0.633812	10.366213
18	0.597239	0.634162	10.365864
19	0.597537	0.634513	10.365515
20	0.597835	0.634863	10.365166
21	0.598133	0.635214	10.364817
22	0.598431	0.635564	10.364468
23	0.598729	0.635915	10.364119
24	0.599027	0.636265	10.363770
25	0.599325	0.636616	10.363421
26	0.599623	0.636966	10.363072
27	0.599921	0.637317	10.362723
28	0.600219	0.637667	10.362374
29	0.600517	0.638018	10.362025
30	0.600815	0.638368	10.361676
31	0.601113	0.638719	10.361327
32	0.601411	0.639069	10.360978
33	0.601709	0.639420	10.360629
34	0.602007	0.639770	10.360280
35	0.602305	0.640121	10.359931
36	0.602603	0.640471	10.359582
37	0.602901	0.640822	10.359233
38	0.603199	0.641172	10.358884
39	0.603497	0.641523	10.358535
40	0.603795	0.641873	10.358186
41	0.604093	0.642224	10.357837
42	0.604391	0.642574	10.357488
43	0.604689	0.642925	10.357139
44	0.604987	0.643275	10.356790
45	0.605285	0.643626	10.356441
46	0.605583	0.643976	10.356092
47	0.605881	0.644327	10.355743
48	0.606179	0.644677	10.355394
49	0.606477	0.645028	10.355045
50	0.606775	0.645378	10.354696
51	0.607073	0.645729	10.354347
52	0.607371	0.646079	10.353998
53	0.607669	0.646430	10.353649
54	0.607967	0.646780	10.353300
55	0.608265	0.647131	10.352951
56	0.608563	0.647481	10.352602
57	0.608861	0.647832	10.352253
58	0.609159	0.648182	10.351904
59	0.609457	0.648533	10.351555
60	0.609755	0.648883	10.351206

67 Degrees.

66 Degrees.

A Table of Artificial Sines, Tangents and Secants.

63

24 Degrees.

Min.	Sine	Tang.	Secant.
0	9.609313	9.60730	10.390687
1	9.609377	9.60734	10.390403
2	9.609438	9.60738	10.390120
3	9.609496	9.60742	10.389836
4	9.609553	9.60746	10.389553
5	9.609609	9.60750	10.389270
6	9.609664	9.60754	10.388987
7	9.609719	9.60758	10.388704
8	9.609773	9.60762	10.388421
9	9.609827	9.60766	10.388138
10	9.609881	9.60770	10.387855
11	9.609935	9.60774	10.387572
12	9.609988	9.60778	10.387289
13	9.610042	9.60782	10.387006
14	9.610095	9.60786	10.386723
15	9.610148	9.60790	10.386440
16	9.610201	9.60794	10.386157
17	9.610254	9.60798	10.385874
18	9.610307	9.60802	10.385591
19	9.610360	9.60806	10.385308
20	9.610413	9.60810	10.385025
21	9.610466	9.60814	10.384742
22	9.610519	9.60818	10.384459
23	9.610572	9.60822	10.384176
24	9.610625	9.60826	10.383893
25	9.610678	9.60830	10.383610
26	9.610731	9.60834	10.383327
27	9.610784	9.60838	10.383044
28	9.610837	9.60842	10.382761
29	9.610890	9.60846	10.382478
30	9.610943	9.60850	10.382195
31	9.610996	9.60854	10.381912
32	9.611049	9.60858	10.381629
33	9.611102	9.60862	10.381346
34	9.611155	9.60866	10.381063
35	9.611208	9.60870	10.380780
36	9.611261	9.60874	10.380497
37	9.611314	9.60878	10.380214
38	9.611367	9.60882	10.379931
39	9.611420	9.60886	10.379648
40	9.611473	9.60890	10.379365
41	9.611526	9.60894	10.379082
42	9.611579	9.60898	10.378799
43	9.611632	9.60902	10.378516
44	9.611685	9.60906	10.378233
45	9.611738	9.60910	10.377950
46	9.611791	9.60914	10.377667
47	9.611844	9.60918	10.377384
48	9.611897	9.60922	10.377101
49	9.611950	9.60926	10.376818
50	9.612003	9.60930	10.376535
51	9.612056	9.60934	10.376252
52	9.612109	9.60938	10.375969
53	9.612162	9.60942	10.375686
54	9.612215	9.60946	10.375403
55	9.612268	9.60950	10.375120
56	9.612321	9.60954	10.374837
57	9.612374	9.60958	10.374554
58	9.612427	9.60962	10.374271
59	9.612480	9.60966	10.373988
60	9.612533	9.60970	10.373705

24 Degrees

25 Degrees.

Min.	Sine	Tang.	Secant.
0	9.612586	9.60974	10.373422
1	9.612639	9.60978	10.373139
2	9.612692	9.60982	10.372856
3	9.612745	9.60986	10.372573
4	9.612798	9.60990	10.372290
5	9.612851	9.60994	10.372007
6	9.612904	9.60998	10.371724
7	9.612957	9.61002	10.371441
8	9.613010	9.61006	10.371158
9	9.613063	9.61010	10.370875
10	9.613116	9.61014	10.370592
11	9.613169	9.61018	10.370309
12	9.613222	9.61022	10.370026
13	9.613275	9.61026	10.369743
14	9.613328	9.61030	10.369460
15	9.613381	9.61034	10.369177
16	9.613434	9.61038	10.368894
17	9.613487	9.61042	10.368611
18	9.613540	9.61046	10.368328
19	9.613593	9.61050	10.368045
20	9.613646	9.61054	10.367762
21	9.613699	9.61058	10.367479
22	9.613752	9.61062	10.367196
23	9.613805	9.61066	10.366913
24	9.613858	9.61070	10.366630
25	9.613911	9.61074	10.366347
26	9.613964	9.61078	10.366064
27	9.614017	9.61082	10.365781
28	9.614070	9.61086	10.365498
29	9.614123	9.61090	10.365215
30	9.614176	9.61094	10.364932
31	9.614229	9.61098	10.364649
32	9.614282	9.61102	10.364366
33	9.614335	9.61106	10.364083
34	9.614388	9.61110	10.363800
35	9.614441	9.61114	10.363517
36	9.614494	9.61118	10.363234
37	9.614547	9.61122	10.362951
38	9.614600	9.61126	10.362668
39	9.614653	9.61130	10.362385
40	9.614706	9.61134	10.362102
41	9.614759	9.61138	10.361819
42	9.614812	9.61142	10.361536
43	9.614865	9.61146	10.361253
44	9.614918	9.61150	10.360970
45	9.614971	9.61154	10.360687
46	9.615024	9.61158	10.360404
47	9.615077	9.61162	10.360121
48	9.615130	9.61166	10.359838
49	9.615183	9.61170	10.359555
50	9.615236	9.61174	10.359272
51	9.615289	9.61178	10.358989
52	9.615342	9.61182	10.358706
53	9.615395	9.61186	10.358423
54	9.615448	9.61190	10.358140
55	9.615501	9.61194	10.357857
56	9.615554	9.61198	10.357574
57	9.615607	9.61202	10.357291
58	9.615660	9.61206	10.357008
59	9.615713	9.61210	10.356725
60	9.615766	9.61214	10.356442

25 Degrees

24 Degrees

25 Degrees

24 Degrees

25 Degrees

26 Degrees.

Min.	Sine.	Tang.	Secant.
0	9.641843	9.953660	9.688152
1	9.642191	9.953598	9.688302
2	9.642566	9.953537	9.688453
3	9.642961	9.953475	9.688603
4	9.643376	9.953413	9.688753
5	9.643813	9.953351	9.688903
6	9.644260	9.953288	9.689053
7	9.644728	9.953226	9.689203
8	9.645216	9.953164	9.689353
9	9.645724	9.953102	9.689503
10	9.646252	9.953040	9.689653
11	9.646800	9.952978	9.689803
12	9.647368	9.952916	9.689953
13	9.647956	9.952854	9.690103
14	9.648564	9.952792	9.690253
15	9.649192	9.952730	9.690403
16	9.649840	9.952668	9.690553
17	9.650508	9.952606	9.690703
18	9.651196	9.952544	9.690853
19	9.651904	9.952482	9.691003
20	9.652632	9.952420	9.691153
21	9.653380	9.952358	9.691303
22	9.654148	9.952296	9.691453
23	9.654936	9.952234	9.691603
24	9.655744	9.952172	9.691753
25	9.656572	9.952110	9.691903
26	9.657420	9.952048	9.692053
27	9.658288	9.951986	9.692203
28	9.659176	9.951924	9.692353
29	9.660084	9.951862	9.692503
30	9.661012	9.951800	9.692653
31	9.661960	9.951738	9.692803
32	9.662928	9.951676	9.692953
33	9.663916	9.951614	9.693103
34	9.664924	9.951552	9.693253
35	9.665952	9.951490	9.693403
36	9.667000	9.951428	9.693553
37	9.668068	9.951366	9.693703
38	9.669156	9.951304	9.693853
39	9.670264	9.951242	9.694003
40	9.671392	9.951180	9.694153
41	9.672540	9.951118	9.694303
42	9.673708	9.951056	9.694453
43	9.674896	9.950994	9.694603
44	9.676104	9.950932	9.694753
45	9.677332	9.950870	9.694903
46	9.678580	9.950808	9.695053
47	9.679848	9.950746	9.695203
48	9.681136	9.950684	9.695353
49	9.682444	9.950622	9.695503
50	9.683772	9.950560	9.695653
51	9.685120	9.950498	9.695803
52	9.686488	9.950436	9.695953
53	9.687876	9.950374	9.696103
54	9.689284	9.950312	9.696253
55	9.690712	9.950250	9.696403
56	9.692160	9.950188	9.696553
57	9.693628	9.950126	9.696703
58	9.695116	9.950064	9.696853
59	9.696624	9.950002	9.697003
60	9.698152	9.949940	9.697153

27 Degrees.

Min.	Sine.	Tang.	Secant.
0	9.657904	9.949816	9.707166
1	9.657265	9.949816	9.707478
2	9.657542	9.949754	9.707790
3	9.657790	9.949688	9.708102
4	9.658037	9.949622	9.708414
5	9.658284	9.949556	9.708726
6	9.658531	9.949490	9.709037
7	9.658778	9.949424	9.709349
8	9.659024	9.949358	9.709660
9	9.659271	9.949292	9.709971
10	9.659517	9.949226	9.710282
11	9.659763	9.949160	9.710593
12	9.660009	9.949094	9.710904
13	9.660255	9.949028	9.711215
14	9.660500	9.948962	9.711525
15	9.660746	9.948896	9.711836
16	9.660991	9.948830	9.712146
17	9.661236	9.948764	9.712456
18	9.661481	9.948698	9.712766
19	9.661726	9.948632	9.713076
20	9.661970	9.948566	9.713386
21	9.662214	9.948500	9.713696
22	9.662459	9.948434	9.714006
23	9.662703	9.948368	9.714316
24	9.662946	9.948302	9.714626
25	9.663190	9.948236	9.714936
26	9.663433	9.948170	9.715246
27	9.663676	9.948104	9.715556
28	9.663919	9.948038	9.715866
29	9.664162	9.947972	9.716176
30	9.664405	9.947906	9.716486
31	9.664648	9.947840	9.716796
32	9.664891	9.947774	9.717106
33	9.665134	9.947708	9.717416
34	9.665377	9.947642	9.717726
35	9.665619	9.947576	9.718036
36	9.665862	9.947510	9.718346
37	9.666104	9.947444	9.718656
38	9.666347	9.947378	9.718966
39	9.666589	9.947312	9.719276
40	9.666832	9.947246	9.719586
41	9.667074	9.947180	9.719896
42	9.667316	9.947114	9.720206
43	9.667559	9.947048	9.720516
44	9.667801	9.946982	9.720826
45	9.668043	9.946916	9.721136
46	9.668285	9.946850	9.721446
47	9.668527	9.946784	9.721756
48	9.668769	9.946718	9.722066
49	9.669011	9.946652	9.722376
50	9.669253	9.946586	9.722686
51	9.669495	9.946520	9.722996
52	9.669737	9.946454	9.723306
53	9.669979	9.946388	9.723616
54	9.670221	9.946322	9.723926
55	9.670463	9.946256	9.724236
56	9.670705	9.946190	9.724546
57	9.670947	9.946124	9.724856
58	9.671189	9.946058	9.725166
59	9.671431	9.945992	9.725476
60	9.671673	9.945926	9.725786

63 Degrees.

Sine. Tang. Secant. M

62 Degrees.

Sine. Tang. Secant. M

A Table of Artificial Sines, Tangents and Secants.

65

28 Degrees.

29 Degrees.

Min.	Sine.	Tang.	Secant.	Min.	Sine.	Tang.	Secant.
0	9.671609	9.943935	10.274316	0	9.685571	9.941819	10.250348
1	9.671847	9.943868	10.274979	1	9.685799	9.941747	10.250505
2	9.672084	9.943800	10.275641	2	9.686027	9.941679	10.250662
3	9.672321	9.943733	10.276303	3	9.686254	9.941609	10.250819
4	9.672558	9.943666	10.276965	4	9.686482	9.941539	10.250976
5	9.672795	9.943598	10.277627	5	9.686710	9.941469	10.251133
6	9.673032	9.943531	10.278289	6	9.686938	9.941398	10.251290
7	9.673269	9.943464	10.278951	7	9.687165	9.941328	10.251447
8	9.673506	9.943396	10.279613	8	9.687393	9.941258	10.251604
9	9.673743	9.943328	10.280275	9	9.687621	9.941187	10.251761
10	9.673980	9.943261	10.280937	10	9.687849	9.941117	10.251918
11	9.674217	9.943193	10.281600	11	9.688077	9.941047	10.252075
12	9.674454	9.943126	10.282262	12	9.688305	9.940977	10.252232
13	9.674691	9.943058	10.282924	13	9.688533	9.940907	10.252389
14	9.674928	9.942991	10.283586	14	9.688761	9.940837	10.252546
15	9.675165	9.942923	10.284248	15	9.688989	9.940767	10.252703
16	9.675402	9.942856	10.284910	16	9.689217	9.940697	10.252860
17	9.675639	9.942788	10.285572	17	9.689445	9.940627	10.253017
18	9.675876	9.942721	10.286234	18	9.689673	9.940557	10.253174
19	9.676113	9.942653	10.286896	19	9.689901	9.940487	10.253331
20	9.676350	9.942586	10.287558	20	9.690129	9.940417	10.253488
21	9.676587	9.942518	10.288220	21	9.690357	9.940347	10.253645
22	9.676824	9.942451	10.288882	22	9.690585	9.940277	10.253802
23	9.677061	9.942383	10.289544	23	9.690813	9.940207	10.253959
24	9.677298	9.942316	10.290206	24	9.691041	9.940137	10.254116
25	9.677535	9.942248	10.290868	25	9.691269	9.940067	10.254273
26	9.677772	9.942181	10.291530	26	9.691497	9.940000	10.254430
27	9.678009	9.942113	10.292192	27	9.691725	9.939930	10.254587
28	9.678246	9.942046	10.292854	28	9.691953	9.939860	10.254744
29	9.678483	9.941978	10.293516	29	9.692181	9.939790	10.254901
30	9.678720	9.941911	10.294178	30	9.692409	9.939720	10.255058
31	9.678957	9.941843	10.294840	31	9.692637	9.939650	10.255215
32	9.679194	9.941776	10.295502	32	9.692865	9.939580	10.255372
33	9.679431	9.941708	10.296164	33	9.693093	9.939510	10.255529
34	9.679668	9.941641	10.296826	34	9.693321	9.939440	10.255686
35	9.679905	9.941573	10.297488	35	9.693549	9.939370	10.255843
36	9.680142	9.941506	10.298150	36	9.693777	9.939300	10.256000
37	9.680379	9.941438	10.298812	37	9.694005	9.939230	10.256157
38	9.680616	9.941371	10.299474	38	9.694233	9.939160	10.256314
39	9.680853	9.941303	10.300136	39	9.694461	9.939090	10.256471
40	9.681090	9.941236	10.300798	40	9.694689	9.939020	10.256628
41	9.681327	9.941168	10.301460	41	9.694917	9.938950	10.256785
42	9.681564	9.941101	10.302122	42	9.695145	9.938880	10.256942
43	9.681801	9.941033	10.302784	43	9.695373	9.938810	10.257099
44	9.682038	9.940966	10.303446	44	9.695601	9.938740	10.257256
45	9.682275	9.940898	10.304108	45	9.695829	9.938670	10.257413
46	9.682512	9.940831	10.304770	46	9.696057	9.938600	10.257570
47	9.682749	9.940763	10.305432	47	9.696285	9.938530	10.257727
48	9.682986	9.940696	10.306094	48	9.696513	9.938460	10.257884
49	9.683223	9.940628	10.306756	49	9.696741	9.938390	10.258041
50	9.683460	9.940561	10.307418	50	9.696969	9.938320	10.258198
51	9.683697	9.940493	10.308080	51	9.697197	9.938250	10.258355
52	9.683934	9.940426	10.308742	52	9.697425	9.938180	10.258512
53	9.684171	9.940358	10.309404	53	9.697653	9.938110	10.258669
54	9.684408	9.940291	10.310066	54	9.697881	9.938040	10.258826
55	9.684645	9.940223	10.310728	55	9.698109	9.937970	10.258983
56	9.684882	9.940156	10.311390	56	9.698337	9.937900	10.259140
57	9.685119	9.940088	10.312052	57	9.698565	9.937830	10.259297
58	9.685356	9.940021	10.312714	58	9.698793	9.937760	10.259454
59	9.685593	9.939953	10.313376	59	9.699021	9.937690	10.259611
60	9.685830	9.939886	10.314038	60	9.699249	9.937620	10.259768

Min.	Sine.	Tang.	Secant.	Min.	Sine.	Tang.	Secant.
0	9.685571	9.941819	10.250348	0	9.685571	9.941819	10.250348
1	9.685799	9.941747	10.250505	1	9.685799	9.941747	10.250505
2	9.686027	9.941679	10.250662	2	9.686027	9.941679	10.250662
3	9.686254	9.941609	10.250819	3	9.686254	9.941609	10.250819
4	9.686482	9.941539	10.250976	4	9.686482	9.941539	10.250976
5	9.686710	9.941469	10.251133	5	9.686710	9.941469	10.251133
6	9.686938	9.941398	10.251290	6	9.686938	9.941398	10.251290
7	9.687165	9.941328	10.251447	7	9.687165	9.941328	10.251447
8	9.687393	9.941258	10.251604	8	9.687393	9.941258	10.251604
9	9.687621	9.941187	10.251761	9	9.687621	9.941187	10.251761
10	9.687849	9.941117	10.251918	10	9.687849	9.941117	10.251918
11	9.688077	9.941047	10.252075	11	9.688077	9.941047	10.252075
12	9.688305	9.940977	10.252232	12	9.688305	9.940977	10.252232
13	9.688533	9.940907	10.252389	13	9.688533	9.940907	10.252389
14	9.688761	9.940837	10.252546	14	9.688761	9.940837	10.252546
15	9.688989	9.940767	10.252703	15	9.688989	9.940767	10.252703
16	9.689217	9.940697	10.252860	16	9.689217	9.940697	10.252860
17	9.689445	9.940627	10.253017	17	9.689445	9.940627	10.253017
18	9.689673	9.940557	10.253174	18	9.689673	9.940557	10.253174
19	9.689901	9.940487	10.253331	19	9.689901	9.940487	10.253331
20	9.690129	9.940417	10.253488	20	9.690129	9.940417	10.253488
21	9.690357	9.940347	10.253645	21	9.690357	9.940347	10.253645
22	9.690585	9.940277	10.253802	22	9.690585	9.940277	10.253802
23	9.690813	9.940207	10.253959	23	9.690813	9.940207	10.253959
24	9.691041	9.940137	10.254116	24	9.691041	9.940137	10.254116
25	9.691269	9.940067	10.254273	25	9.691269	9.940067	10.254273
26	9.691497	9.940000	10.254430	26	9.691497	9.940000	10.254430
27	9.691725	9.939930	10.254587	27	9.691725	9.939930	10.254587
28	9.691953	9.939860	10.254744	28	9.691953	9.939860	10.254744
29	9.692181	9.939790	10.254901	29	9.692181	9.939790	10.254901
30	9.692409	9.939720	10.255058	30	9.692409	9.939720	10.255058
31	9.692637	9.939650	10.255215	31	9.692637	9.939650	10.255215
32	9.692865	9.939580	10.255372	32	9.692865	9.939580	10.255372
33	9.693093	9.939510	10.255529	33	9.693093	9.939510	10.255529
34	9.693321	9.939440	10.255686	34	9.693321	9.939440	10.255686
35	9.693549	9.939370	10.255843	35	9.693549	9.939370	10.255843
36	9.693777	9.939300	10.256000	36	9.693777	9.939300	10.256000
37	9.694005	9.939230	10.256157	37	9.694005	9.939230	10.256157
38	9.694233	9.939160	10.256314	38	9.694233	9.939160	10.256314
39	9.694461	9.939090	10.256471	39	9.694461	9.939090	10.256471
40	9.694689	9.939020	10.256628	40	9.694689	9.939020	10.256628
41	9.694917	9.938950	10.256785	41	9.694917	9.938950	10.256785
42	9.695145	9.938880	10.256942	42	9.695145	9.938880	10.256942
43	9.695373	9.938810	10.257099	43	9.695373	9.938810	10.257099
44	9.695601	9.938740	10.257256	44	9.695601	9.938740	10.257256
45	9.695829	9.938670	10.257413	45	9.695829	9.938670	10.257413
46	9.696057	9.938600	10.257570	46	9.696057	9.938600	10.257570
47	9.696285	9.938530	10.257727	47	9.696285	9.938530	10.257727
48	9.696513	9.938460	10.257884	48	9.696513	9.938460	10.257884
49	9.696741	9.938390	10.258041	49	9.696741	9.938390	10.258041
50	9.696969	9.938320	10.258198	50	9.696969	9.938320	10.258198
51	9.697197	9.938250	10.258355	51	9.697197	9.938250	10.258355
52	9.697425	9.938180	10.258512	52	9.697425	9.938180	10.258512
53	9.697653	9.938110	10.258669	53	9.697653	9.938110	10.258669
54	9.697881	9.938040	10.258826	54	9.697881	9.938040	10.258826
55	9.698109	9.937970	10.258983	55	9.698109	9.937970	10.258983
56	9.698337	9.937900	10.259140	56	9.698337	9.937900	10.259140
57	9.698565	9.937830	10.259297	57	9.698565	9.937830	10.259297
58	9.698793	9.937760	10.259454	58	9.698793	9.937760	10.259454
59	9.699021	9.937690	10.259611	59	9.699021	9.937690	10.259611
60	9.699249	9.937620	10.259768	60	9.699249	9.937620	10.259768

A Table of Artificial Sines, Tangents and Secants.

30 Degrees.

M.in.	Sine.	Tang.	Secant.
0	0.698970	0.937531	0.716439
1	0.699189	0.937431	0.716171
2	0.699407	0.937331	0.715903
3	0.699626	0.937231	0.715635
4	0.699844	0.937131	0.715367
5	0.700061	0.937031	0.715100
6	0.700280	0.936932	0.714832
7	0.700498	0.936832	0.714564
8	0.700716	0.936732	0.714296
9	0.700933	0.936632	0.714028
10	0.701151	0.936532	0.713760
11	0.701368	0.936432	0.713492
12	0.701585	0.936332	0.713224
13	0.701802	0.936232	0.712956
14	0.702019	0.936132	0.712688
15	0.702236	0.936032	0.712420
16	0.702452	0.935932	0.712152
17	0.702669	0.935832	0.711884
18	0.702885	0.935732	0.711616
19	0.703101	0.935632	0.711348
20	0.703317	0.935532	0.711080
21	0.703533	0.935432	0.710812
22	0.703749	0.935332	0.710544
23	0.703964	0.935232	0.710276
24	0.704179	0.935132	0.710008
25	0.704395	0.935032	0.709740
26	0.704610	0.934932	0.709472
27	0.704825	0.934832	0.709204
28	0.705040	0.934732	0.708936
29	0.705254	0.934632	0.708668
30	0.705469	0.934532	0.708400
31	0.705683	0.934432	0.708132
32	0.705897	0.934332	0.707864
33	0.706111	0.934232	0.707596
34	0.706326	0.934132	0.707328
35	0.706539	0.934032	0.707060
36	0.706753	0.933932	0.706792
37	0.706967	0.933832	0.706524
38	0.707180	0.933732	0.706256
39	0.707393	0.933632	0.705988
40	0.707606	0.933532	0.705720
41	0.707819	0.933432	0.705452
42	0.708032	0.933332	0.705184
43	0.708245	0.933232	0.704916
44	0.708457	0.933132	0.704648
45	0.708670	0.933032	0.704380
46	0.708882	0.932932	0.704112
47	0.709094	0.932832	0.703844
48	0.709306	0.932732	0.703576
49	0.709518	0.932632	0.703308
50	0.709731	0.932532	0.703040
51	0.709943	0.932432	0.702772
52	0.710155	0.932332	0.702504
53	0.710367	0.932232	0.702236
54	0.710579	0.932132	0.701968
55	0.710786	0.932032	0.701700
56	0.710997	0.931932	0.701432
57	0.711208	0.931832	0.701164
58	0.711419	0.931732	0.700896
59	0.711629	0.931632	0.700628
60	0.711839	0.931532	0.700360

59 Degree.

31 Degrees.

M.in.	Sine.	Tang.	Secant.
0	0.711839	0.931532	0.700360
1	0.712049	0.931432	0.700092
2	0.712258	0.931332	0.699824
3	0.712466	0.931232	0.699556
4	0.712675	0.931132	0.699288
5	0.712883	0.931032	0.699020
6	0.713091	0.930932	0.698752
7	0.713298	0.930832	0.698484
8	0.713505	0.930732	0.698216
9	0.713712	0.930632	0.697948
10	0.713919	0.930532	0.697680
11	0.714126	0.930432	0.697412
12	0.714332	0.930332	0.697144
13	0.714538	0.930232	0.696876
14	0.714744	0.930132	0.696608
15	0.714949	0.930032	0.696340
16	0.715154	0.929932	0.696072
17	0.715359	0.929832	0.695804
18	0.715563	0.929732	0.695536
19	0.715767	0.929632	0.695268
20	0.715971	0.929532	0.695000
21	0.716174	0.929432	0.694732
22	0.716378	0.929332	0.694464
23	0.716581	0.929232	0.694196
24	0.716784	0.929132	0.693928
25	0.716987	0.929032	0.693660
26	0.717189	0.928932	0.693392
27	0.717391	0.928832	0.693124
28	0.717593	0.928732	0.692856
29	0.717795	0.928632	0.692588
30	0.717996	0.928532	0.692320
31	0.718197	0.928432	0.692052
32	0.718398	0.928332	0.691784
33	0.718598	0.928232	0.691516
34	0.718798	0.928132	0.691248
35	0.718998	0.928032	0.690980
36	0.719197	0.927932	0.690712
37	0.719396	0.927832	0.690444
38	0.719595	0.927732	0.690176
39	0.719794	0.927632	0.689908
40	0.719992	0.927532	0.689640
41	0.720190	0.927432	0.689372
42	0.720388	0.927332	0.689104
43	0.720585	0.927232	0.688836
44	0.720782	0.927132	0.688568
45	0.720979	0.927032	0.688300
46	0.721175	0.926932	0.688032
47	0.721371	0.926832	0.687764
48	0.721567	0.926732	0.687496
49	0.721762	0.926632	0.687228
50	0.721957	0.926532	0.686960
51	0.722151	0.926432	0.686692
52	0.722345	0.926332	0.686424
53	0.722538	0.926232	0.686156
54	0.722731	0.926132	0.685888
55	0.722924	0.926032	0.685620
56	0.723116	0.925932	0.685352
57	0.723308	0.925832	0.685084
58	0.723499	0.925732	0.684816
59	0.723690	0.925632	0.684548
60	0.723880	0.925532	0.684280

58 Degree.

A Table of Artificial Sines, Tangents and Secants.

67

32 Degrees.

Min.	Sine	Tang.	Secant.
0	9.724210	9.918240	10.304111
1	9.724412	9.918414	10.303930
2	9.724614	9.918586	10.303749
3	9.724816	9.918758	10.303568
4	9.725017	9.918930	10.303387
5	9.725219	9.919102	10.303206
6	9.725420	9.919274	10.303025
7	9.725622	9.919446	10.302844
8	9.725823	9.919618	10.302663
9	9.726024	9.919790	10.302482
10	9.726225	9.919962	10.302301
11	9.726426	9.920134	10.302120
12	9.726626	9.920306	10.301939
13	9.726827	9.920478	10.301758
14	9.727027	9.920650	10.301577
15	9.727228	9.920822	10.301396
16	9.727428	9.920994	10.301215
17	9.727628	9.921166	10.301034
18	9.727828	9.921338	10.300853
19	9.728028	9.921510	10.300672
20	9.728228	9.921682	10.300491
21	9.728428	9.921854	10.300310
22	9.728628	9.922026	10.300129
23	9.728828	9.922198	10.299948
24	9.729028	9.922370	10.299767
25	9.729228	9.922542	10.299586
26	9.729428	9.922714	10.299405
27	9.729628	9.922886	10.299224
28	9.729828	9.923058	10.299043
29	9.730028	9.923230	10.298862
30	9.730228	9.923402	10.298681
31	9.730428	9.923574	10.298500
32	9.730628	9.923746	10.298319
33	9.730828	9.923918	10.298138
34	9.731028	9.924090	10.297957
35	9.731228	9.924262	10.297776
36	9.731428	9.924434	10.297595
37	9.731628	9.924606	10.297414
38	9.731828	9.924778	10.297233
39	9.732028	9.924950	10.297052
40	9.732228	9.925122	10.296871
41	9.732428	9.925294	10.296690
42	9.732628	9.925466	10.296509
43	9.732828	9.925638	10.296328
44	9.733028	9.925810	10.296147
45	9.733228	9.925982	10.295966
46	9.733428	9.926154	10.295785
47	9.733628	9.926326	10.295604
48	9.733828	9.926498	10.295423
49	9.734028	9.926670	10.295242
50	9.734228	9.926842	10.295061
51	9.734428	9.927014	10.294880
52	9.734628	9.927186	10.294699
53	9.734828	9.927358	10.294518
54	9.735028	9.927530	10.294337
55	9.735228	9.927702	10.294156
56	9.735428	9.927874	10.293975
57	9.735628	9.928046	10.293794
58	9.735828	9.928218	10.293613
59	9.736028	9.928390	10.293432
60	9.736228	9.928562	10.293251

57 Degrees.

33 Degrees.

Min.	Sine	Tang.	Secant.
0	9.716103	9.912517	10.174781
1	9.716303	9.912690	10.174599
2	9.716503	9.912862	10.174418
3	9.716703	9.913034	10.174237
4	9.716903	9.913206	10.174056
5	9.717103	9.913378	10.173875
6	9.717303	9.913550	10.173694
7	9.717503	9.913722	10.173513
8	9.717703	9.913894	10.173332
9	9.717903	9.914066	10.173151
10	9.718103	9.914238	10.172970
11	9.718303	9.914410	10.172789
12	9.718503	9.914582	10.172608
13	9.718703	9.914754	10.172427
14	9.718903	9.914926	10.172246
15	9.719103	9.915098	10.172065
16	9.719303	9.915270	10.171884
17	9.719503	9.915442	10.171703
18	9.719703	9.915614	10.171522
19	9.719903	9.915786	10.171341
20	9.720103	9.915958	10.171160
21	9.720303	9.916130	10.170979
22	9.720503	9.916302	10.170798
23	9.720703	9.916474	10.170617
24	9.720903	9.916646	10.170436
25	9.721103	9.916818	10.170255
26	9.721303	9.916990	10.170074
27	9.721503	9.917162	10.169893
28	9.721703	9.917334	10.169712
29	9.721903	9.917506	10.169531
30	9.722103	9.917678	10.169350
31	9.722303	9.917850	10.169169
32	9.722503	9.918022	10.168988
33	9.722703	9.918194	10.168807
34	9.722903	9.918366	10.168626
35	9.723103	9.918538	10.168445
36	9.723303	9.918710	10.168264
37	9.723503	9.918882	10.168083
38	9.723703	9.919054	10.167902
39	9.723903	9.919226	10.167721
40	9.724103	9.919398	10.167540
41	9.724303	9.919570	10.167359
42	9.724503	9.919742	10.167178
43	9.724703	9.919914	10.166997
44	9.724903	9.920086	10.166816
45	9.725103	9.920258	10.166635
46	9.725303	9.920430	10.166454
47	9.725503	9.920602	10.166273
48	9.725703	9.920774	10.166092
49	9.725903	9.920946	10.165911
50	9.726103	9.921118	10.165730
51	9.726303	9.921290	10.165549
52	9.726503	9.921462	10.165368
53	9.726703	9.921634	10.165187
54	9.726903	9.921806	10.165006
55	9.727103	9.921978	10.164825
56	9.727303	9.922150	10.164644
57	9.727503	9.922322	10.164463
58	9.727703	9.922494	10.164282
59	9.727903	9.922666	10.164101
60	9.728103	9.922838	10.163920

56 Degrees.

A Table of Artificial Sines, Tangents and Secants.

34 Degrees.

M.	Sine	Tang.	Secant.
0	9.747363	9.918174	9.828987
1	9.747749	9.918486	9.829260
2	9.748136	9.918800	9.829534
3	9.748523	9.919115	9.829808
4	9.748910	9.919431	9.830082
5	9.749297	9.919747	9.830356
6	9.749684	9.920063	9.830630
7	9.750071	9.920379	9.830904
8	9.750458	9.920695	9.831178
9	9.750845	9.921011	9.831452
10	9.751232	9.921327	9.831726
11	9.751619	9.921643	9.832000
12	9.752006	9.921959	9.832274
13	9.752393	9.922275	9.832548
14	9.752780	9.922591	9.832822
15	9.753167	9.922907	9.833096
16	9.753554	9.923223	9.833370
17	9.753941	9.923539	9.833644
18	9.754328	9.923855	9.833918
19	9.754715	9.924171	9.834192
20	9.755102	9.924487	9.834466
21	9.755489	9.924803	9.834740
22	9.755876	9.925119	9.835014
23	9.756263	9.925435	9.835288
24	9.756650	9.925751	9.835562
25	9.757037	9.926067	9.835836
26	9.757424	9.926383	9.836110
27	9.757811	9.926699	9.836384
28	9.758198	9.927015	9.836658
29	9.758585	9.927331	9.836932
30	9.758972	9.927647	9.837206
31	9.759359	9.927963	9.837480
32	9.759746	9.928279	9.837754
33	9.760133	9.928595	9.838028
34	9.760520	9.928911	9.838302
35	9.760907	9.929227	9.838576
36	9.761294	9.929543	9.838850
37	9.761681	9.929859	9.839124
38	9.762068	9.930175	9.839398
39	9.762455	9.930491	9.839672
40	9.762842	9.930807	9.839946
41	9.763229	9.931123	9.840220
42	9.763616	9.931439	9.840494
43	9.764003	9.931755	9.840768
44	9.764390	9.932071	9.841042
45	9.764777	9.932387	9.841316
46	9.765164	9.932703	9.841590
47	9.765551	9.933019	9.841864
48	9.765938	9.933335	9.842138
49	9.766325	9.933651	9.842412
50	9.766712	9.933967	9.842686
51	9.767099	9.934283	9.842960
52	9.767486	9.934599	9.843234
53	9.767873	9.934915	9.843508
54	9.768260	9.935231	9.843782
55	9.768647	9.935547	9.844056
56	9.769034	9.935863	9.844330
57	9.769421	9.936179	9.844604
58	9.769808	9.936495	9.844878
59	9.770195	9.936811	9.845152
60	9.770582	9.937127	9.845426

55 Degrees.

35 Degrees.

M.	Sine	Tang.	Secant.
0	9.758591	9.913364	9.845227
1	9.758977	9.913676	9.845501
2	9.759364	9.913988	9.845775
3	9.759751	9.914300	9.846049
4	9.760138	9.914612	9.846323
5	9.760525	9.914924	9.846597
6	9.760912	9.915236	9.846871
7	9.761299	9.915548	9.847145
8	9.761686	9.915860	9.847419
9	9.762073	9.916172	9.847693
10	9.762460	9.916484	9.847967
11	9.762847	9.916796	9.848241
12	9.763234	9.917108	9.848515
13	9.763621	9.917420	9.848789
14	9.764008	9.917732	9.849063
15	9.764395	9.918044	9.849337
16	9.764782	9.918356	9.849611
17	9.765169	9.918668	9.849885
18	9.765556	9.918980	9.850159
19	9.765943	9.919292	9.850433
20	9.766330	9.919604	9.850707
21	9.766717	9.919916	9.850981
22	9.767104	9.920228	9.851255
23	9.767491	9.920540	9.851529
24	9.767878	9.920852	9.851803
25	9.768265	9.921164	9.852077
26	9.768652	9.921476	9.852351
27	9.769039	9.921788	9.852625
28	9.769426	9.922100	9.852899
29	9.769813	9.922412	9.853173
30	9.770200	9.922724	9.853447
31	9.770587	9.923036	9.853721
32	9.770974	9.923348	9.853995
33	9.771361	9.923660	9.854269
34	9.771748	9.923972	9.854543
35	9.772135	9.924284	9.854817
36	9.772522	9.924596	9.855091
37	9.772909	9.924908	9.855365
38	9.773296	9.925220	9.855639
39	9.773683	9.925532	9.855913
40	9.774070	9.925844	9.856187
41	9.774457	9.926156	9.856461
42	9.774844	9.926468	9.856735
43	9.775231	9.926780	9.857009
44	9.775618	9.927092	9.857283
45	9.776005	9.927404	9.857557
46	9.776392	9.927716	9.857831
47	9.776779	9.928028	9.858105
48	9.777166	9.928340	9.858379
49	9.777553	9.928652	9.858653
50	9.777940	9.928964	9.858927
51	9.778327	9.929276	9.859201
52	9.778714	9.929588	9.859475
53	9.779101	9.929900	9.859749
54	9.779488	9.930212	9.860023
55	9.779875	9.930524	9.860297
56	9.780262	9.930836	9.860571
57	9.780649	9.931148	9.860845
58	9.781036	9.931460	9.861119
59	9.781423	9.931772	9.861393
60	9.781810	9.932084	9.861667

54 Degrees.

A Table of Artificial Sines, Tangents and Secants.

69

36 Degrees.

N	Sine	Tang.	Secant.
1	9.760219	9.907958	9.861261
2	9.760312	9.907866	9.861527
3	9.760416	9.907774	9.861792
4	9.760520	9.907682	9.862058
5	9.760623	9.907590	9.862323
6	9.760727	9.907498	9.862589
7	9.760830	9.907406	9.862854
8	9.760933	9.907314	9.863119
9	9.761036	9.907221	9.863385
10	9.761139	9.907129	9.863650
11	9.761242	9.907037	9.863915
12	9.761345	9.906945	9.864180
13	9.761448	9.906853	9.864445
14	9.761551	9.906760	9.864710
15	9.761654	9.906667	9.864975
16	9.761757	9.906575	9.865240
17	9.761860	9.906483	9.865505
18	9.761963	9.906390	9.865770
19	9.762066	9.906298	9.866035
20	9.762169	9.906206	9.866300
21	9.762272	9.906114	9.866565
22	9.762375	9.906022	9.866830
23	9.762478	9.905930	9.867095
24	9.762581	9.905838	9.867360
25	9.762684	9.905746	9.867625
26	9.762787	9.905654	9.867890
27	9.762890	9.905562	9.868155
28	9.762993	9.905470	9.868420
29	9.763096	9.905378	9.868685
30	9.763199	9.905286	9.868950
31	9.763302	9.905194	9.869215
32	9.763405	9.905102	9.869480
33	9.763508	9.905010	9.869745
34	9.763611	9.904918	9.870010
35	9.763714	9.904826	9.870275
36	9.763817	9.904734	9.870540
37	9.763920	9.904642	9.870805
38	9.764023	9.904550	9.871070
39	9.764126	9.904458	9.871335
40	9.764229	9.904366	9.871600
41	9.764332	9.904274	9.871865
42	9.764435	9.904182	9.872130
43	9.764538	9.904090	9.872395
44	9.764641	9.904000	9.872660
45	9.764744	9.903908	9.872925
46	9.764847	9.903816	9.873190
47	9.764950	9.903724	9.873455
48	9.765053	9.903632	9.873720
49	9.765156	9.903540	9.873985
50	9.765259	9.903448	9.874250
51	9.765362	9.903356	9.874515
52	9.765465	9.903264	9.874780
53	9.765568	9.903172	9.875045
54	9.765671	9.903080	9.875310
55	9.765774	9.902988	9.875575
56	9.765877	9.902896	9.875840
57	9.765980	9.902804	9.876105
58	9.766083	9.902712	9.876370
59	9.766186	9.902620	9.876635
60	9.766289	9.902528	9.876900
Sine.		Tang.	Secant.

53 Degrees

37 Degrees.

N	Sine	Tang.	Secant.
1	9.779013	9.902349	9.877114
2	9.779013	9.902349	9.877114
3	9.779013	9.902349	9.877114
4	9.779013	9.902349	9.877114
5	9.779013	9.902349	9.877114
6	9.779013	9.902349	9.877114
7	9.779013	9.902349	9.877114
8	9.779013	9.902349	9.877114
9	9.779013	9.902349	9.877114
10	9.779013	9.902349	9.877114
11	9.779013	9.902349	9.877114
12	9.779013	9.902349	9.877114
13	9.779013	9.902349	9.877114
14	9.779013	9.902349	9.877114
15	9.779013	9.902349	9.877114
16	9.779013	9.902349	9.877114
17	9.779013	9.902349	9.877114
18	9.779013	9.902349	9.877114
19	9.779013	9.902349	9.877114
20	9.779013	9.902349	9.877114
21	9.779013	9.902349	9.877114
22	9.779013	9.902349	9.877114
23	9.779013	9.902349	9.877114
24	9.779013	9.902349	9.877114
25	9.779013	9.902349	9.877114
26	9.779013	9.902349	9.877114
27	9.779013	9.902349	9.877114
28	9.779013	9.902349	9.877114
29	9.779013	9.902349	9.877114
30	9.779013	9.902349	9.877114
31	9.779013	9.902349	9.877114
32	9.779013	9.902349	9.877114
33	9.779013	9.902349	9.877114
34	9.779013	9.902349	9.877114
35	9.779013	9.902349	9.877114
36	9.779013	9.902349	9.877114
37	9.779013	9.902349	9.877114
38	9.779013	9.902349	9.877114
39	9.779013	9.902349	9.877114
40	9.779013	9.902349	9.877114
41	9.779013	9.902349	9.877114
42	9.779013	9.902349	9.877114
43	9.779013	9.902349	9.877114
44	9.779013	9.902349	9.877114
45	9.779013	9.902349	9.877114
46	9.779013	9.902349	9.877114
47	9.779013	9.902349	9.877114
48	9.779013	9.902349	9.877114
49	9.779013	9.902349	9.877114
50	9.779013	9.902349	9.877114
51	9.779013	9.902349	9.877114
52	9.779013	9.902349	9.877114
53	9.779013	9.902349	9.877114
54	9.779013	9.902349	9.877114
55	9.779013	9.902349	9.877114
56	9.779013	9.902349	9.877114
57	9.779013	9.902349	9.877114
58	9.779013	9.902349	9.877114
59	9.779013	9.902349	9.877114
60	9.779013	9.902349	9.877114
Sine		Tang.	Secant.

52 Degrees.

A Table of Artificial Sines, Tangents and Secants.

38 Degrees.

Min.	Sine	Tang.	Secant.	Min.
0	9.789342	9.896532	9.892810	10.107190
1	9.789504	9.896433	9.893070	10.106930
2	9.789665	9.896333	9.893331	10.106669
3	9.789827	9.896236	9.893591	10.106409
4	9.789988	9.896137	9.893851	10.106149
5	9.790149	9.896038	9.894111	10.105889
6	9.790310	9.895939	9.894371	10.105628
7	9.790471	9.895840	9.894632	10.105368
8	9.790632	9.895741	9.894892	10.105108
9	9.790793	9.895641	9.895152	10.104848
10	9.790954	9.895542	9.895412	10.104588
11	9.791115	9.895443	9.895672	10.104328
12	9.791275	9.895343	9.895932	10.104068
13	9.791436	9.895244	9.896192	10.103808
14	9.791596	9.895144	9.896452	10.103548
15	9.791757	9.895045	9.896712	10.103288
16	9.791917	9.894945	9.896971	10.103028
17	9.792077	9.894846	9.897231	10.102768
18	9.792237	9.894746	9.897491	10.102508
19	9.792397	9.894646	9.897751	10.102248
20	9.792557	9.894546	9.898010	10.101988
21	9.792716	9.894446	9.898270	10.101728
22	9.792876	9.894346	9.898530	10.101468
23	9.793035	9.894246	9.898789	10.101208
24	9.793195	9.894146	9.899049	10.100948
25	9.793354	9.894046	9.899308	10.100688
26	9.793513	9.893946	9.899568	10.100428
27	9.793673	9.893846	9.899827	10.100168
28	9.793832	9.893745	9.900086	10.099908
29	9.793991	9.893645	9.900345	10.099648
30	9.794150	9.893544	9.900605	10.099388
31	9.794308	9.893444	9.900864	10.099128
32	9.794467	9.893343	9.901124	10.098868
33	9.794626	9.893243	9.901383	10.098608
34	9.794784	9.893142	9.901642	10.098348
35	9.794942	9.893041	9.901901	10.098088
36	9.795101	9.892940	9.902160	10.097828
37	9.795259	9.892839	9.902419	10.097568
38	9.795417	9.892738	9.902678	10.097308
39	9.795575	9.892637	9.902937	10.097048
40	9.795733	9.892536	9.903195	10.096788
41	9.795891	9.892435	9.903455	10.096528
42	9.796049	9.892335	9.903714	10.096268
43	9.796206	9.892233	9.903973	10.096008
44	9.796364	9.892132	9.904232	10.095748
45	9.796521	9.892030	9.904491	10.095488
46	9.796679	9.891929	9.904750	10.095228
47	9.796836	9.891827	9.905009	10.094968
48	9.796993	9.891726	9.905267	10.094708
49	9.797150	9.891624	9.905526	10.094448
50	9.797307	9.891523	9.905784	10.094188
51	9.797464	9.891421	9.906043	10.093928
52	9.797621	9.891319	9.906302	10.093668
53	9.797777	9.891217	9.906560	10.093408
54	9.797934	9.891115	9.906819	10.093148
55	9.798091	9.891013	9.907077	10.092888
56	9.798247	9.890911	9.907336	10.092628
57	9.798403	9.890809	9.907594	10.092368
58	9.798560	9.890707	9.907852	10.092108
59	9.798717	9.890605	9.908111	10.091848
60	9.798872	9.890503	9.908369	10.091588
	Sine	Tang.	Secant.	

39 Degrees.

Min.	Sine	Tang.	Secant.	Min.
0	9.798872	9.890503	9.908369	10.091611
1	9.799028	9.890400	9.908627	10.091328
2	9.799184	9.890298	9.908886	10.091068
3	9.799339	9.890195	9.909144	10.090808
4	9.799495	9.890093	9.909402	10.090548
5	9.799651	9.889990	9.909660	10.090288
6	9.799806	9.889888	9.909918	10.090028
7	9.799962	9.889785	9.910177	10.089768
8	9.800117	9.889682	9.910435	10.089508
9	9.800272	9.889579	9.910693	10.089248
10	9.800427	9.889476	9.910951	10.088988
11	9.800582	9.889374	9.911209	10.088728
12	9.800737	9.889271	9.911467	10.088468
13	9.800892	9.889168	9.911724	10.088208
14	9.801047	9.889064	9.911982	10.087948
15	9.801201	9.888961	9.912240	10.087688
16	9.801356	9.888858	9.912498	10.087428
17	9.801511	9.888755	9.912756	10.087168
18	9.801666	9.888652	9.913014	10.086908
19	9.801819	9.888548	9.913271	10.086648
20	9.801973	9.888444	9.913529	10.086388
21	9.802128	9.888341	9.913787	10.086128
22	9.802282	9.888237	9.914044	10.085868
23	9.802435	9.888133	9.914302	10.085608
24	9.802589	9.888029	9.914560	10.085348
25	9.802742	9.887926	9.914817	10.085088
26	9.802897	9.887822	9.915075	10.084828
27	9.803050	9.887718	9.915332	10.084568
28	9.803204	9.887614	9.915590	10.084308
29	9.803357	9.887510	9.915847	10.084048
30	9.803510	9.887406	9.916104	10.083788
31	9.803664	9.887302	9.916361	10.083528
32	9.803817	9.887198	9.916618	10.083268
33	9.803970	9.887093	9.916875	10.083008
34	9.804123	9.886989	9.917132	10.082748
35	9.804276	9.886885	9.917389	10.082488
36	9.804428	9.886780	9.917646	10.082228
37	9.804581	9.886676	9.917903	10.081968
38	9.804734	9.886571	9.918160	10.081708
39	9.804886	9.886466	9.918417	10.081448
40	9.805038	9.886362	9.918674	10.081188
41	9.805191	9.886257	9.918931	10.080928
42	9.805343	9.886152	9.919188	10.080668
43	9.805495	9.886047	9.919445	10.080408
44	9.805647	9.885942	9.919702	10.080148
45	9.805799	9.885837	9.920216	10.079888
46	9.805951	9.885732	9.920473	10.079628
47	9.806103	9.885627	9.920730	10.079368
48	9.806255	9.885521	9.920987	10.079108
49	9.806406	9.885416	9.921244	10.078848
50	9.806558	9.885311	9.921501	10.078588
51	9.806709	9.885206	9.921758	10.078328
52	9.806860	9.885100	9.922015	10.078068
53	9.807011	9.884995	9.922272	10.077808
54	9.807163	9.884889	9.922529	10.077548
55	9.807314	9.884784	9.922786	10.077288
56	9.807465	9.884678	9.923043	10.077028
57	9.807616	9.884572	9.923300	10.076768
58	9.807767	9.884466	9.923557	10.076508
59	9.807917	9.884359	9.923814	10.076248
60	9.808068	9.884254	9.924071	10.075988
	Sine	Tang.	Secant.	M

51 Degrees.

50 Degrees.

A Table of Artificial Sines, Tangents and Secants.

71

40 Degrees.

41 Degrees.

Min.	Sine.	Tang.	Secant.
0	9.80866	9.884754	10.11316
1	9.80881	9.88484	10.11316
2	9.80896	9.88493	10.11316
3	9.80911	9.88502	10.11316
4	9.80926	9.88511	10.11316
5	9.80941	9.88520	10.11316
6	9.80956	9.88529	10.11316
7	9.80971	9.88538	10.11316
8	9.80986	9.88547	10.11316
9	9.80999	9.88556	10.11316
10	9.81014	9.88565	10.11316
11	9.81029	9.88574	10.11316
12	9.81044	9.88583	10.11316
13	9.81059	9.88592	10.11316
14	9.81074	9.88601	10.11316
15	9.81089	9.88610	10.11316
16	9.81104	9.88619	10.11316
17	9.81119	9.88628	10.11316
18	9.81134	9.88637	10.11316
19	9.81149	9.88646	10.11316
20	9.81164	9.88655	10.11316
21	9.81179	9.88664	10.11316
22	9.81194	9.88673	10.11316
23	9.81209	9.88682	10.11316
24	9.81224	9.88691	10.11316
25	9.81239	9.88700	10.11316
26	9.81254	9.88709	10.11316
27	9.81269	9.88718	10.11316
28	9.81284	9.88727	10.11316
29	9.81299	9.88736	10.11316
30	9.81314	9.88745	10.11316
31	9.81329	9.88754	10.11316
32	9.81344	9.88763	10.11316
33	9.81359	9.88772	10.11316
34	9.81374	9.88781	10.11316
35	9.81389	9.88790	10.11316
36	9.81404	9.88799	10.11316
37	9.81419	9.88808	10.11316
38	9.81434	9.88817	10.11316
39	9.81449	9.88826	10.11316
40	9.81464	9.88835	10.11316
41	9.81479	9.88844	10.11316
42	9.81494	9.88853	10.11316
43	9.81509	9.88862	10.11316
44	9.81524	9.88871	10.11316
45	9.81539	9.88880	10.11316
46	9.81554	9.88889	10.11316
47	9.81569	9.88898	10.11316
48	9.81584	9.88907	10.11316
49	9.81599	9.88916	10.11316
50	9.81614	9.88925	10.11316
51	9.81629	9.88934	10.11316
52	9.81644	9.88943	10.11316
53	9.81659	9.88952	10.11316
54	9.81674	9.88961	10.11316
55	9.81689	9.88970	10.11316
56	9.81704	9.88979	10.11316
57	9.81719	9.88988	10.11316
58	9.81734	9.88997	10.11316
59	9.81749	9.89006	10.11316
60	9.81764	9.89015	10.11316

Min.	Sine.	Tang.	Secant.
0	9.81779	9.89024	10.11316
1	9.81794	9.89033	10.11316
2	9.81809	9.89042	10.11316
3	9.81824	9.89051	10.11316
4	9.81839	9.89060	10.11316
5	9.81854	9.89069	10.11316
6	9.81869	9.89078	10.11316
7	9.81884	9.89087	10.11316
8	9.81899	9.89096	10.11316
9	9.81914	9.89105	10.11316
10	9.81929	9.89114	10.11316
11	9.81944	9.89123	10.11316
12	9.81959	9.89132	10.11316
13	9.81974	9.89141	10.11316
14	9.81989	9.89150	10.11316
15	9.82004	9.89159	10.11316
16	9.82019	9.89168	10.11316
17	9.82034	9.89177	10.11316
18	9.82049	9.89186	10.11316
19	9.82064	9.89195	10.11316
20	9.82079	9.89204	10.11316
21	9.82094	9.89213	10.11316
22	9.82109	9.89222	10.11316
23	9.82124	9.89231	10.11316
24	9.82139	9.89240	10.11316
25	9.82154	9.89249	10.11316
26	9.82169	9.89258	10.11316
27	9.82184	9.89267	10.11316
28	9.82199	9.89276	10.11316
29	9.82214	9.89285	10.11316
30	9.82229	9.89294	10.11316
31	9.82244	9.89303	10.11316
32	9.82259	9.89312	10.11316
33	9.82274	9.89321	10.11316
34	9.82289	9.89330	10.11316
35	9.82304	9.89339	10.11316
36	9.82319	9.89348	10.11316
37	9.82334	9.89357	10.11316
38	9.82349	9.89366	10.11316
39	9.82364	9.89375	10.11316
40	9.82379	9.89384	10.11316
41	9.82394	9.89393	10.11316
42	9.82409	9.89402	10.11316
43	9.82424	9.89411	10.11316
44	9.82439	9.89420	10.11316
45	9.82454	9.89429	10.11316
46	9.82469	9.89438	10.11316
47	9.82484	9.89447	10.11316
48	9.82499	9.89456	10.11316
49	9.82514	9.89465	10.11316
50	9.82529	9.89474	10.11316
51	9.82544	9.89483	10.11316
52	9.82559	9.89492	10.11316
53	9.82574	9.89501	10.11316
54	9.82589	9.89510	10.11316
55	9.82604	9.89519	10.11316
56	9.82619	9.89528	10.11316
57	9.82634	9.89537	10.11316
58	9.82649	9.89546	10.11316
59	9.82664	9.89555	10.11316
60	9.82679	9.89564	10.11316

40 Degrees.

41 Degrees.

A Table of Artificial Sines, Tangents and Secants.

42 Degrees.

Min.	Sine.	Tang.	Secant.
0	9.825511	9.871073	10.045651
1	9.825611	9.871060	10.045693
2	9.825711	9.871046	10.045735
3	9.825811	9.871031	10.045777
4	9.825911	9.871016	10.045819
5	9.826011	9.871001	10.045861
6	9.826111	9.870986	10.045903
7	9.826211	9.870971	10.045945
8	9.826311	9.870956	10.045987
9	9.826411	9.870941	10.046029
10	9.826511	9.870926	10.046071
11	9.826611	9.870911	10.046113
12	9.826711	9.870896	10.046155
13	9.826811	9.870881	10.046197
14	9.826911	9.870866	10.046239
15	9.827011	9.870851	10.046281
16	9.827111	9.870836	10.046323
17	9.827211	9.870821	10.046365
18	9.827311	9.870806	10.046407
19	9.827411	9.870791	10.046449
20	9.827511	9.870776	10.046491
21	9.827611	9.870761	10.046533
22	9.827711	9.870746	10.046575
23	9.827811	9.870731	10.046617
24	9.827911	9.870716	10.046659
25	9.828011	9.870701	10.046701
26	9.828111	9.870686	10.046743
27	9.828211	9.870671	10.046785
28	9.828311	9.870656	10.046827
29	9.828411	9.870641	10.046869
30	9.828511	9.870626	10.046911
31	9.828611	9.870611	10.046953
32	9.828711	9.870596	10.046995
33	9.828811	9.870581	10.047037
34	9.828911	9.870566	10.047079
35	9.829011	9.870551	10.047121
36	9.829111	9.870536	10.047163
37	9.829211	9.870521	10.047205
38	9.829311	9.870506	10.047247
39	9.829411	9.870491	10.047289
40	9.829511	9.870476	10.047331
41	9.829611	9.870461	10.047373
42	9.829711	9.870446	10.047415
43	9.829811	9.870431	10.047457
44	9.829911	9.870416	10.047499
45	9.830011	9.870401	10.047541
46	9.830111	9.870386	10.047583
47	9.830211	9.870371	10.047625
48	9.830311	9.870356	10.047667
49	9.830411	9.870341	10.047709
50	9.830511	9.870326	10.047751
51	9.830611	9.870311	10.047793
52	9.830711	9.870296	10.047835
53	9.830811	9.870281	10.047877
54	9.830911	9.870266	10.047919
55	9.831011	9.870251	10.047961
56	9.831111	9.870236	10.048003
57	9.831211	9.870221	10.048045
58	9.831311	9.870206	10.048087
59	9.831411	9.870191	10.048129
60	9.831511	9.870176	10.048171

47 Degrees.

43 Degrees.

Min.	Sine.	Tang.	Secant.
0	9.831711	9.870161	10.048213
1	9.831811	9.870146	10.048255
2	9.831911	9.870131	10.048297
3	9.832011	9.870116	10.048339
4	9.832111	9.870101	10.048381
5	9.832211	9.870086	10.048423
6	9.832311	9.870071	10.048465
7	9.832411	9.870056	10.048507
8	9.832511	9.870041	10.048549
9	9.832611	9.870026	10.048591
10	9.832711	9.870011	10.048633
11	9.832811	9.869996	10.048675
12	9.832911	9.869981	10.048717
13	9.833011	9.869966	10.048759
14	9.833111	9.869951	10.048801
15	9.833211	9.869936	10.048843
16	9.833311	9.869921	10.048885
17	9.833411	9.869906	10.048927
18	9.833511	9.869891	10.048969
19	9.833611	9.869876	10.049011
20	9.833711	9.869861	10.049053
21	9.833811	9.869846	10.049095
22	9.833911	9.869831	10.049137
23	9.834011	9.869816	10.049179
24	9.834111	9.869801	10.049221
25	9.834211	9.869786	10.049263
26	9.834311	9.869771	10.049305
27	9.834411	9.869756	10.049347
28	9.834511	9.869741	10.049389
29	9.834611	9.869726	10.049431
30	9.834711	9.869711	10.049473
31	9.834811	9.869696	10.049515
32	9.834911	9.869681	10.049557
33	9.835011	9.869666	10.049599
34	9.835111	9.869651	10.049641
35	9.835211	9.869636	10.049683
36	9.835311	9.869621	10.049725
37	9.835411	9.869606	10.049767
38	9.835511	9.869591	10.049809
39	9.835611	9.869576	10.049851
40	9.835711	9.869561	10.049893
41	9.835811	9.869546	10.049935
42	9.835911	9.869531	10.049977
43	9.836011	9.869516	10.050019
44	9.836111	9.869501	10.050061
45	9.836211	9.869486	10.050103
46	9.836311	9.869471	10.050145
47	9.836411	9.869456	10.050187
48	9.836511	9.869441	10.050229
49	9.836611	9.869426	10.050271
50	9.836711	9.869411	10.050313
51	9.836811	9.869396	10.050355
52	9.836911	9.869381	10.050397
53	9.837011	9.869366	10.050439
54	9.837111	9.869351	10.050481
55	9.837211	9.869336	10.050523
56	9.837311	9.869321	10.050565
57	9.837411	9.869306	10.050607
58	9.837511	9.869291	10.050649
59	9.837611	9.869276	10.050691
60	9.837711	9.869261	10.050733

46 Degrees.

44 Degrees.

Angle	Sine	Tang.	Secant.
0	0.00000	0.00000	1.00000
1	0.01745	0.03492	1.00309
2	0.03490	0.06976	1.00618
3	0.05234	0.10460	1.00927
4	0.06976	0.13944	1.01236
5	0.08718	0.17428	1.01545
6	0.10460	0.20912	1.01854
7	0.12202	0.24396	1.02163
8	0.13944	0.27880	1.02472
9	0.15686	0.31364	1.02781
10	0.17428	0.34848	1.03090
11	0.19170	0.38332	1.03399
12	0.20912	0.41816	1.03708
13	0.22654	0.45300	1.04017
14	0.24396	0.48784	1.04326
15	0.26138	0.52268	1.04635
16	0.27880	0.55752	1.04944
17	0.29622	0.59236	1.05253
18	0.31364	0.62720	1.05562
19	0.33106	0.66204	1.05871
20	0.34848	0.69688	1.06180
21	0.36590	0.73172	1.06489
22	0.38332	0.76656	1.06798
23	0.40074	0.80140	1.07107
24	0.41816	0.83624	1.07416
25	0.43558	0.87108	1.07725
26	0.45300	0.90592	1.08034
27	0.47042	0.94076	1.08343
28	0.48784	0.97560	1.08652
29	0.50526	1.01044	1.08961
30	0.52268	1.04528	1.09270
31	0.54010	1.08012	1.09579
32	0.55752	1.11496	1.09888
33	0.57494	1.14980	1.10197
34	0.59236	1.18464	1.10506
35	0.60978	1.21948	1.10815
36	0.62720	1.25432	1.11124
37	0.64462	1.28916	1.11433
38	0.66204	1.32400	1.11742
39	0.67946	1.35884	1.12051
40	0.69688	1.39368	1.12360
41	0.71430	1.42852	1.12669
42	0.73172	1.46336	1.12978
43	0.74914	1.49820	1.13287
44	0.76656	1.53304	1.13596
45	0.78398	1.56788	1.13905
46	0.80140	1.60272	1.14214
47	0.81882	1.63756	1.14523
48	0.83624	1.67240	1.14832
49	0.85366	1.70724	1.15141
50	0.87108	1.74208	1.15450
51	0.88850	1.77692	1.15759
52	0.90592	1.81176	1.16068
53	0.92334	1.84660	1.16377
54	0.94076	1.88144	1.16686
55	0.95818	1.91628	1.16995
56	0.97560	1.95112	1.17304
57	0.99302	1.98596	1.17613
58	1.01044	2.02080	1.17922
59	1.02786	2.05564	1.18231
60	1.04528	2.09048	1.18540
Sine.	Tang.	Secant.	M

A Table of Angles, which every Rhomb (or point of the Compass) maketh with the Meridian.

<i>North.</i>	<i>South.</i>	<i>Point.</i>	<i>D. M.</i>	<i>North.</i>	<i>South.</i>
		$\frac{1}{4}$	02 49		
		$\frac{1}{2}$	05 37		
		$\frac{3}{4}$	08 26		
N. by E.	S. by East.	1	11 15	N. by W.	S. by W.
		$\frac{1}{2}$	14 04		
		$\frac{1}{2}$	16 52		
		$\frac{1}{2}$	19 41		
N. N. E.	S. S. E.	2	22 30	N. N. W.	S. S. W.
		$\frac{1}{2}$	25 19		
		$\frac{1}{2}$	28 07		
		$\frac{1}{2}$	30 56		
N. E. by N.	S. E. by S.	3	33 45	N. W. by N.	S. W. by S.
		$\frac{1}{2}$	36 34		
		$\frac{1}{2}$	39 22		
		$\frac{1}{2}$	42 11		
No. East.	So. East.	4	45 00	N. West.	So. West.
		$\frac{1}{2}$	47 49		
		$\frac{1}{2}$	50 38		
		$\frac{1}{2}$	53 26		
N. E. by E.	S. E. by E.	5	56 15	N. W. by W.	S. W. by W.
		$\frac{1}{2}$	59 04		
		$\frac{1}{2}$	61 53		
		$\frac{1}{2}$	64 41		
E. N. E.	E. S. E.	6	67 30	W. N. W.	W. S. W.
		$\frac{1}{2}$	70 19		
		$\frac{1}{2}$	73 08		
		$\frac{1}{2}$	75 56		
E. by N.	E. by S.	7	78 45	W. by N.	W. by S.
		$\frac{1}{2}$	81 34		
		$\frac{1}{2}$	84 23		
		$\frac{1}{2}$	87 11		
East.	East.	8	90 00	West.	West.

F I N I S:







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